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ORIGINAL ARTICLE

Evaluating the vertical vibration response of footbridges using a response spectrum approach

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Abstract In this paper, the vertical vibration response of footbridges subjected to dynamic loads induced by walking humans is assessed via a response spectrum approach. The dynamic walking load in the vertical direction is applied on the bridges using two different loading schemes: (1) a stationary load at the mid-span and (2) a moving load across the bridge. The response spectrum analysis is carried out using a Generalized Single Degree of Freedom procedure which has been verified by comparing its predictions with the results of a Multi-Degrees of Freedom modeling.

The results obtained indicated that the response spectrum approach is capable of accurately predicting the footbridge vibration response. The results obtained also indicated that the main parameters that affect the induced accelerations in footbridges due to the human walking loads in the vertical direction are the footbridge mass and damping ratio.

1. Introduction

The construction of increasingly slender footbridges recently has led to their vulnerability to dynamic loads with low magnitude such as the pedestrian walking loads. This phenomenon occurs due to the resonant vibration effects in footbridges with fundamental frequencies close to the frequency of the pedestrian walking load. In this case, the dynamic load will have a great effect on the functionality of the bridge as the human beings are very sensitive to vibration levels as low as 0.001 mm [1]. This matter is mainly a serviceability problem rather than a strength problem as the vibration induced forces are far below the forces required to affect the structure safety.

The dynamic force produced by humans consists of three components; vertical, lateral and longitudinal. These forces are produced due to accelerating and decelerating of the mass of their bodies. The vertical component was the most investigated, as it has the highest magnitude.

Tests were conducted to describe the human-induced dynamic load in the vertical direction due to continuous steps by Blanchard et al. [2], Rainer et al. [3], Ebrahimpour et al. [4] and Ebrahimpour et al. [5]. The time history of the force was found to be nearly periodic. Deterministic force model of the pedestrian load in the form of Fourier series is considered by Blanchard et al. [2], Bachmann and Ammann [6], Rainer et al. [3], Kerr [7] and Li et al. [8].

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Dynamic analysis of footbridges under pedestrian walking loads is conducted using a finite element approach [9,10]. Hauksson has used the ABAQUS finite element package, while Figueiredo et al. have used the ANSYS program. The finite element analysis of footbridges required representing the bridge slab by shell elements and the girders by three-dimensional beam elements. This approach is time consuming due to the large number of degrees of freedom required to perform the analysis.

Hauksson [9] has used a Generalized Single Degree of Freedom (GSDF) model to calculate the dynamic response of the London Millennium Bridge when subjected to dynamic loading. He stated that the GSDF modeling of the London Millennium Bridge showed similar results as those obtained by the finite element modeling. He concluded that a simple Single Degree of Freedom (SDF) model can be adequate to perform a dynamic analysis of a footbridge subjected to pedestrian walking loads.

The response spectrum is a useful tool for predicting the structural response to dynamic load as long as the structure can behave principally as a SDF system. It is used extensively in seismic analysis and design of structures to present the peak structural response due to earthquake loading.

The response spectrum approach provides the peak vibration response for a whole spectrum of structures having a specific fundamental frequency range. The peak response of the structure can be estimated by reading the value from the response spectrum for the appropriate frequency. In most building codes in seismic regions, this value forms the basis for calculating the forces that a structure must be designed to resist.

The objective of this study is to use the response spectrum approach to predict the vertical acceleration response of footbridges subjected different types of walking load models and to determine the main parameters that affect the induced accelerations in footbridges due to the human walking loads in the vertical direction.

2. Selected footbridges

Five footbridges were selected having spans that range from 15 m to 35 m. The bridge dimensions are selected identical to those presented by Figueiredo et al. [10]. The structural system of the bridges consists of two simply supported steel main girders with a 10 cm concrete slab. The bridge width is 250 cm as shown in Fig. 1.

The steel used in the bridges has a 2.05×10^5 MPa modulus of elasticity, a 300 MPa yield strength, and a Poison ratio of 0.3. For the concrete slab, a 3.84×10^4 MPa modulus of elasticity is used, maximum compressive strength of 30 MPa and a 0.2 Poison ratio. Table 1 shows the properties of the main girder of each bridge used in the analysis. It is also worth mentioning that these sections meet the stress and deflection requirements of the Egyptian code.

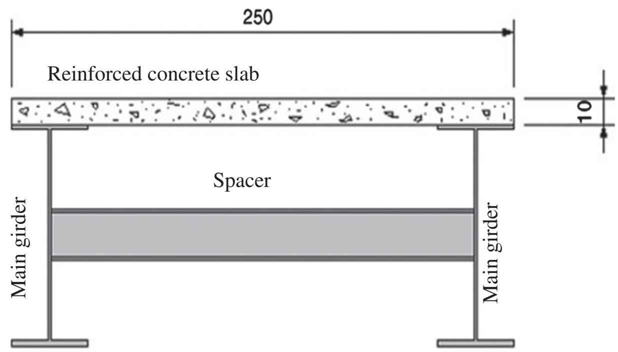


Figure 1 Typical bridge cross-section (Figueiredo et al. [10]).

Table 1 The main girder dimensions of the selected bridges (Figueiredo et al. [10]).

Span (m)	15	20	25	30	35
Height (mm)	550	700	900	1100	1200
Flange width (mm)	250	320	350	400	450
Flange thickness (mm)	19	22.4	25	25	31.5
Web thickness (mm)	6.3	8	8	9.5	9.5

2.1. Modeling of human walking loads in the vertical direction

The human walking forces of one person are considered periodic and are represented by the following Fourier series:

$$F(t) = P\left[\sum_{i}^{n}\alpha_i \cos(2\pi i f_s t + \varphi)\right] \quad (1)$$

where $F(t)$ is the human-induced dynamic load, P is the pedestrian weight (is assumed equal to 0.70 kN in this study), α_i is the dynamic load factor (DLF) of the ith harmonic, f_s is the pedestrian step-frequency (Hz), and φ is the phase shift of the ith harmonic. The frequencies of the load harmonics are the multiples of the pedestrian step-frequency. The pedestrian load parameters of Eq. (1) proposed by Murray et al. [11] and summarized in Table 2 are adopted in the current study. These parameters are based on using only the first four harmonics of Eq. (1).

The human walking forces of one person in the vertical direction are assumed to be applied on the footbridge in two different loading cases; V1 and V2.

In V1, the human-induced dynamic load in the vertical direction is considered stationary at the mid-span of the bridge. A walking frequency is chosen for each load harmonic from the walking frequency ranges shown in Table 2, such that it or its multiples in the following harmonics coincides with the bridge fundamental frequency in the vertical direction. Consequently, the bridge would be in the state of resonance with one of the four load harmonics. For the 30 m span bridge which has a 3.76 Hz fundamental frequency, the load frequencies of the four load harmonics would be 1.88, 3.76, 5.64, and 7.52 Hz and resonance occurs with the second load harmonic.

In load case V2, the walking load is considered moving across the bridge with a certain velocity. Table 3 shows the walking velocities and step sizes corresponding to certain

Table 2 The pedestrian load parameters as given by Murray et al. [11].

Harmonic i	Frequency range (Hz)	α_i	ϕ
1	$f_s = 1.6–2.2$	0.5	0
2	$2f_s = 3.2–4.4$	0.2	$\pi/2$
3	$3f_s = 4.8–6.6$	0.1	$\pi/2$
4	$4f_s = 6.4–8.8$	0.05	$\pi/2$

Table 3 Characteristics of the walking pedestrian (Murray et al. [11]).

Activity	Velocity (m/s)	Step size (m)	Step-frequency (Hz)
Slow walking	1.1	0.6	1.7
Normal walking	1.5	0.75	2.0
Fast walking	2.2	1.0	2.3

Table 4 Mass and inertia properties of the bridge modeles.

Span (m)	Mass (ton/m)	I (m^4)
15	0.829	0.003734776
20	0.938	0.008064
25	1.013	0.0152703
30	1.103	0.02524986
35	1.228	0.038357

walking frequencies proposed by Murray et al. [11]. Modeling of the load case V2 is carried out by calculating the walking frequency based on the bridge fundamental frequency and then calculating the step size and the walking velocity by linear interpolation of the data presented in Table 3.

2.2. Structural models of footbridges

The responses of the footbridges to dynamic loads induced by walking humans are evaluated in this study using two types of structural models: (1) a Multi-Degrees of Freedom (MDF) modeling and (2) a simplified Generalized Single Degree of Freedom (GSDF) approach.

The MDF modeling of the footbridges is carried out using a two-dimensional (2-D) beam element that has four degrees of freedom (lateral transition and rotation at each node). The bridge is modeled as a number of equal-length beam elements, and the dynamic load is assumed acting on the nodes connecting the beam elements to simulate the case in which the dynamic load is moving across the footbridge. To determine the minimum number of beam elements that can be used in the MDF model, while maintaining the model accuracy, the solution is repeated with larger number of elements until the results of two successive solutions are close enough.

The moment of inertia of each frame element is considered equal to that of the actual composite section of the bridge. The mass and inertia properties of the five footbridge models considered in this study are summarized in Table 4.

In the GSDF model, the footbridge is assumed to be excited by the shape function ($\psi(x) = Sin(\pi x/l)$). The GSDOF mass ($\bar{m}$), stiffness ($\overline{K}$), frequency (ω_1), damping coefficient ($\overline{C}$), and load function $\overline{F}(t)$ are calculated as presented in Table 5.

Table 6 Fundamental frequencies obtained using the MDF and the GSDF approaches.

Frequency (Hz)		MDF	GSDF
Span (m)	15	6.71	6.71
	20	5.21	5.21
	25	4.42	4.42
	30	3.79	3.78
	35	3.24	3.24

where l is the bridge span, m is the bridge mass per unit length, E is the modulus of elasticity, I is the cross-section moment of inertia, ε is the damping ratio, and t_d is the time taken to cross the bridge. The equation of motion of the GSDF is solved numerically for the bridges under study using Newark β method.

2.3. The fundamental frequency of the footbridges

The frequency equation of the GSDF model presented in Table 5 is exactly the same as the equation provided by BD 37/01 which can be written as:

$$f_s = \frac{C^2}{2\pi l^2}\sqrt{\frac{EIg}{W}} \tag{2}$$

where g is the gravitational acceleration (m/s^2), l is the bridge span (m), C is a configuration factor ($C = \pi$, for single span bridges), E is the modulus of elasticity (kN/m^2), I is the moment of inertia of the cross-section at mid-span (m^4) and W is the weight per unit length of the full cross-section at mid-span (kN/m).

The fundamental frequencies of the five footbridges with spans 15, 20, 25, 30 and 35 m. are calculated using the MDF and the GSDF models and are presented in Table 6. The results presented indicate that the frequency predictions of the MDF and the GSDF are almost identical.

2.4. Acceleration response of footbridges

The acceleration is used as a measure of the footbridge vibration. The maximum acceleration at mid-span for each model is obtained as shown in Table 7. Also, the maximum acceleration for each bridge is calculated using a method developed by Allen and Murray [12] and used by the AISC standards which is represented by the following equation:

$$a = \frac{P}{M\varepsilon} \tag{3}$$

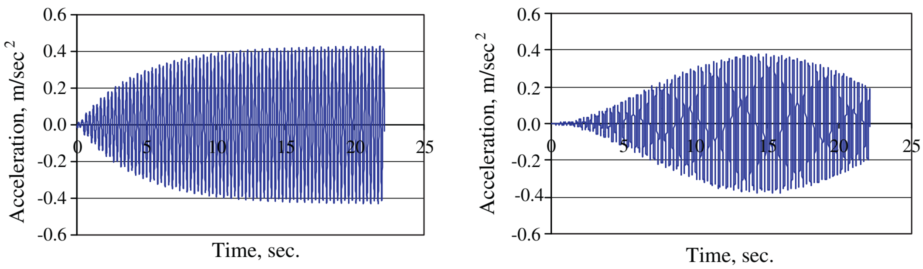
where P is the force magnitude depending on the DLF, M is the total mass of the bridge, and ε is the damping ratio of the bridge.

Table 5 Parameters of the GSDF.

Mass ($\bar{m}$)	Stiffness ($\overline{K}$)	Frequency (ω_1)	Damping Coefficient ($\overline{C}$)	GSDF load function ($\overline{F}(t)$)	
				Stationary	Moving
$\bar{m} = \frac{ml}{2}$	$\overline{K} = \frac{\pi^4 EI}{2l^3}$	$\omega_1 = \sqrt{\frac{\pi^4 EI}{l^4 m}}$	$\overline{C} = 2\bar{m}\omega_n\varepsilon$	$\overline{F}(t) = F(t)$	$\overline{F}(t) = F(t)\sin\left(\frac{\pi t}{t_d}\right)$

Table 7 Maximum accelerations at mid-span calculated using different approaches.

Span (m)		15	20	25	30	35
Acceleration (m/s^2)	MDF, V1	0.362	0.387	0.517	0.429	0.332
	GSDF, V1	0.361	0.387	0.517	0.428	0.332
	MDF, V2	0.335	0.345	0.432	0.375	0.305
	GSDF, V2	0.335	0.344	0.431	0.375	0.305
	Allen and Murray equation	0.281	0.373	0.553	0.423	0.326



(a) Load case V1.

(b) Load case V2.

Figure 2 The 30 m span footbridge acceleration response at resonance.

The acceleration responses of the MDF bridge models considered in this study are determined using modal dynamic analyses with modal damping ratios of 1.0% for all modes. The number of beam elements of the MDF bridge models is selected by repeating the analysis with larger number of elements until the results of two successive solutions are close enough. The moving dynamic loads on the MDF bridge models are represented using the data shown in Table 3.

The results obtained by the MDF and the GSDF models along with the equation of Allen and Murray are presented in Table 7 for the load cases V1 and V2. The results of the MDF models are considered the references for evaluating the accuracy of the GSDF models.

The data presented in Table 7 indicate that the accelerations obtained by the GSDF models for load cases V1 and V2 are almost identical to the corresponding accelerations obtained using the MDF models. These results denote that the GSDF models of footbridges are capable of exactly predicting the footbridge maximum acceleration responses. This trend is attributed to the fact that the bridge dynamic response during the resonance behavior is totally governed by the response of the fundamental mode of vibration and that the response contributions of other modes to the bridge dynamic response are almost null.

The accelerations obtained for load case V2 are always lower than those of load case V1. The load case V2 simulates the real case when the pedestrian cross the footbridge with a certain velocity, and as such, the effect of the human-induced dynamic load is at its peak only at the moment when the pedestrian crosses through the mid-span of the footbridge.

The acceleration results calculated according to the Allen and Murray equation tend to be higher than the acceleration results of load case V2. They also tend to be more close to the acceleration predictions of the loading case V1. This can be attributed to the approximate nature of the empirical equation of the Allen and Murray which was developed considering the load stationary at the mid-span of the footbridge. The only exception to this trend is in the case of the 15 m span footbridge, where the maximum acceleration calculated according to the Allen and Murray equation significantly underestimated the acceleration predictions of the load cases V1 and V2.

Fig. 2 represents the relations between the acceleration at mid-span and time for the 30 m span footbridge, for the two load cases V1 and V2. In Fig. 2a, the maximum acceleration appears to be constant as time increases; because the pedestrian is considered at the mid-span of the bridge. In Fig. 2b, the pedestrian is considered moving across the footbridge, the acceleration response increases as the pedestrian walks closer to the mid-span and then decreases as he walks away from the mid-span of the footbridge.

2.5. The response spectrum of footbridges subjected to the walking loads

The effect of the bridge dynamic properties (e.g., the natural frequency, the mass, and the damping ratio) on the maximum acceleration of the bridge is investigated in this study via a response spectrum approach. The response spectrum is a plot of the peak value of a response quantity as a function of the natural frequency of the structure. In this study, the response quantity considered is a pseudo force (F), where:

$$F = \text{GSDF maximum acceleration} \times \text{GSDF mass} \tag{4}$$

The quantity F has the units of force. For GSDF systems with a damping ratio ζ and a natural frequency ω, the pseudo force response is independent on the GSDF mass and therefore only one F–ω response curve is present for all GSDF system with a specified damping ratio ζ. In other words, all GSDF systems that have different masses but constant frequency and damping ratio will have the same pseudo force. The response spectra are developed for different damping ratios as shown in Fig. 3. The pedestrian dynamic load is considered stationary at the mid-span of the footbridge and the pedestrian weight is assumed equal to 0.70 kN.

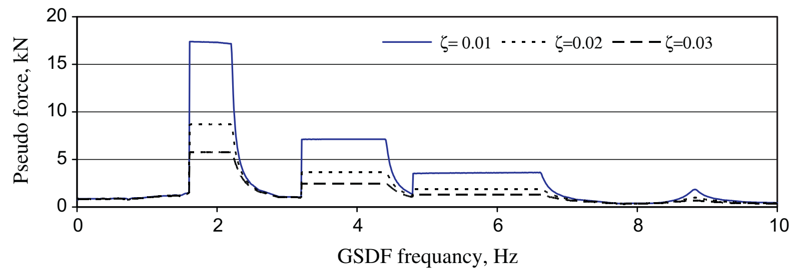


Figure 3 The pseudo force response spectra for the human walking load.

Table 8 Maximum accelerations at mid-span of the footbridge models calculated using different approaches.

Span (m)		15	20	25	30	35
Acceleration (m/s^2)	MDF Results	0.362	0.387	0.517	0.429	0.332
	Response Spectrum Predictions	0.374	0.379	0.527	0.431	0.332
	Allen and Murray equation	0.281	0.373	0.553	0.423	0.326

The GSDF maximum acceleration is calculated from the response spectrum as:

$$\text{GSDF maximum acceleration} = \frac{F}{(\text{mL}/2)} \tag{5}$$

Table 8 summarized the acceleration responses of the footbridge models considered in this study when subjected to a stationary load at the mid-span of the bridge and with considering modal damping ratios of 1.0% for all modes. The results presented in the table are obtained using the MDF approach, the response spectrum procedure (Fig. 3) and the equation of Allen and Murray.

The results presented in Table 8 shows that the response spectrum predictions are very close to those of the MDF approach. The results of the MDF models are considered the references for evaluating the accuracy of the results of both the response spectrum procedure and the equation of Allen and Murray. The data presented in table indicates that the response spectrum predictions are more accurate than the results obtained using the equation of Allen and Murray.

The fundamental frequency of the bridge has a great effect on the induced maximum acceleration as shown in Fig. 3. The applied dynamic load consists of four harmonics that have the frequency ranges presented in Table 2. The bridge response decreases when the bridge frequency lies out of the loading frequency ranges. The bridge response has its peak value when the bridge frequency lies in the frequency range of the first harmonic (1.6–2.2 Hz). This is attributed to the fact that the first harmonic has the largest contribution to the applied dynamic load. Fig. 3 also shows that the maximum acceleration decreases as the bridge damping ratio increases. Eq. (5) indicates that mass of the footbridge clearly affects the induced maximum acceleration, as the bridge mass increases the maximum acceleration decreases.

3. Conclusions

- The results presented in this study indicated that the GSDF models of footbridges are capable of exactly predicting the footbridge maximum acceleration responses. This trend is attributed to the fact that the bridge dynamic response during the resonance behavior is totally governed by the response of the fundamental mode of vibration and that the response contributions of other modes to the bridge dynamic response are almost null.
- The response spectrum approach is capable of accurately presenting the vibration responses of footbridges that have a specified range of fundamental frequencies. The analysis conducted in this study indicated that the response spectrum predictions are more accurate than the results obtained using the equation of Allen and Murray.

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