New 1-D Method for the Prediction of Axial-Flow Compressors Off-Design Performance

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Abstract

The development of advanced turbomachinery relies heavily on the use of 3-D CFD methods. Fast and efficient 1-D and 2-D methods are used at early design stages in order to explore the design space of a new configuration. A new method has been developed to predict performance characteristics of axial-flow compressors in the following conditions: stable off-design; part-span and full-span rotating stall; reverse flow. The new method is based on the classical empirical 1-D method proposed by L.E. Olshteyn. An empirical model for a stall/unstall hysteresis loop has been developed. The developed method has been implemented in COMPRESSOR\_S simulation software. The results for three stage axial compressor have been produced and compared against the experimental data published by Gamache.

Keywords: axial compressor; off-design; stall; reverse flow

1. Introduction

Gas turbine engines that power both civil and military aircraft are common machines in which axial-flow compressors are found. One of the ongoing trends in aircraft engine design is the substantial reduction of production time and costs. The design process of a modern axial compressor is very complex process, which is divided into many stages and to a large extent is performed using computer simulations. It starts with fairly primitive 1-D and 2-D methods, and ends with sophisticated 3-D simulations with use Computational Fluid Dynamics (CFD). Therefore, design process is heavily dependent on fast and accurate computer programs that can exploit a large design space in
the shortest time possible. Although, 3-D CFD methods are very efficient, they can only be used on final stages of the design process as it takes a plenty of a computational time for a solution to converge and there is the strong need for a large amount of initial data. Fast and efficient 1-D and 2-D methods at early design stages are used in order to explore the design space of a new configuration. The purpose of the approach being used for performance analysis is modeling of axial-flow compressor based on empirical correlations and classical thermodynamic and aerodynamic laws and the minimization of compressor modelling in CFD software. Finding better correlations and methods on how one can model a compressor will result in less time for fine tuning in advanced CFD programs and a hence time and cost saving.

A performance prediction of axial compressors can be done by a variety of analytical and empirical methods. One of the most popular is the stage-stacking method introduced by A. R. Howell et al. in 1978 [1] and revealed by Jack et al. [2]. Another wide-spread method is a streamline curvature (SLC) throughflow method. It is mainly used at the preliminary design stage for specifying the target aerodynamic performances to be achieved by the blading and gives a first insight of the global component functioning [3]. For over 30 years SLC was a dominant numerical approach and the most important tool for the axial compressor design [4]. Both methods are rather effective, but their effectiveness is limited by empirical models being used in calculation procedures (reference and off-design flow angles, loss models). The development of accurate off-design empirical models is a big problem and new models are being developed to date.

The aim of this work is to develop an integrated axial compressor one-dimensional model being capable of: off-design performance prediction in stable range of operation; stall performance prediction; reverse-flow performance prediction. The will be useful for aero engines stall and surge analysis.

2. Compressor-off design prediction method

An original method for axial flow compressor off-design performance prediction based on compressor stage generalized functions was developed in the P.I. Baranov Central Institute for Aviation Motors (Russia) under scientific supervision of L. E. Ol’shtein. This is a simple meanline method, there is no need for 2-D analysis and spanwise integration of airflow parameters [5-7]. Generalized functions are obtained from a statistical analysis of significant amount of individual compressor stage performance maps.

The method requires a nominal (reference) on a speed line being analyzed. The nominal point for the preferred speed line in Ol’shtein’s method is considered as maximal isentropic efficiency point and is determined by an empirical model. Assume that the angles $\alpha_1 = \alpha_{10}$ (rotor row inlet absolute flow angle) and $\beta_2 = \beta_{20}$ (rotor row outlet relative flow angle) are constant both for the nominal design point and the off-design points (in a wide range of incidence angles up to surge line) [5, 6]. In accordance with the elementary stage velocity diagram the stage loading coefficient for the nominal point and for off-design conditions can be written as

$$\bar{H}_{d0} = 1 - \bar{c}_a (\operatorname{ctg} \alpha_1 + \operatorname{ctg} \beta_2), \quad \bar{H}_{d} = 1 - \bar{c}_a (\operatorname{ctg} \alpha_1 + \operatorname{ctg} \beta_2).$$

(1)

where $\bar{c}_a$ - is a flow coefficient. The nominal point is indexed 0, off-design point is without an index. The relationship between stage loading, isentropic stage loading, and isentropic efficiency for the design and off-design points can be described by the following equation:

$$\frac{\bar{H}_{d0}}{\bar{H}_{d} \eta_0} \cong \frac{\bar{H}_{d0} \eta}{\bar{H}_{d} \eta_0}. \quad (2)$$

Using equations (1) – (2) two dimensionless groups are obtained:

$$K_1 = \bar{H} \eta - \bar{c}_a \bar{H}_0 \eta_0, \quad K_2 = \bar{H} - \bar{H}_0 \frac{\bar{c}_a}{\bar{c}_a_0}. \quad (3)$$
The complex relationships \( K_1 = f\left( \frac{\bar{C}_a}{C_{a0}}, M_u \right) \) and \( K_2 = f\left( \frac{\bar{C}_a}{C_{a0}}, M_u \right) \) are generalized performance characteristics of an individual compressor stage. Variation of dimensionless groups \( K_1 \) and \( K_2 \) with \( \alpha \) criterions (which determine compressor stage operating conditions), are shown on figure 1 [8]. Stage pressure ratio and isentropic efficiency for off-design condition are determined as the follows:

\[
\pi = \left( 1 + \frac{K_2 + \bar{H}_{0} \frac{\bar{C}_a}{C_{a0}}}{\frac{k k - 1 R \cdot T^*}{C_{a0}}} \right)^{\frac{k}{k - 1}} \quad \eta = \frac{K_2 + \bar{H}_{0} \frac{\bar{C}_a}{C_{a0}}}{K_1 + \frac{\bar{H}_{0} \frac{\bar{C}_a}{C_{a0}}}{\eta_{0} C_{a0}}}
\]

(4)

![Fig. 1. Compressor individual stage generalized functions](image)

The next problem in the prediction of the compressor off-design performance is the determination of a surge line. Compressor off-design performance prediction is based on a simplification that the flow outlet angles do not change appreciably in a wide range of operating conditions characterized by incidence angles. This simplification suggests the critical incidence angle as a criterion for a compressor surge line prediction. The critical incidence angle for each blade row is being calculated with use Howell’s method [9]. The nominal deflection in the compressor cascade must be set in predicting a compressor surge line. The nominal deflection is related to maximal deflection in the compressor cascade as the follows \( \Delta \beta_0 = 0.8 \cdot \Delta \beta_{\text{max}} \). This nominal deflection has a corresponding nominal incidence angle \( i_{\text{nom}} \).

Compressor cascade stall limit is determined by a maximum point on the performance curve \( \frac{\Delta \beta}{\Delta \beta_{0}} = f\left( \frac{\left( i - i_{0} \right)}{\Delta \beta_{0}} \right) \), that is related to flow the separation on the suction surface. Flow separation causes the significant decrease in the cascade deflection and increases in pressure losses. Thus, analysis of Howell’s compressor cascade performance curves gives the justifiable criterion for the meanline compressor surge line prediction. Two empirical models have been used – the critical incidence angle (for the surge line prediction) and the stall onset incidence angle:

\[
\frac{\left( i_{\text{crit}} - i_{0} \right)}{\Delta \beta_{0}} \approx 0.4, \quad \frac{\left( i_{\text{stall onset}} - i_{0} \right)}{\Delta \beta_{0}} \approx 0.2.
\]

(5)

3. Compressor stalled and reverse flow performance prediction

The developed method for the stalled compressor performance prediction is based on the statistical analysis of a large set of experimental compressor stage individual maps. Axial compressor part-span stall performance
experimental data was obtained from NASA, General Electric, Pratt&Whitney, University of Cambridge, Massachusetts Institute of Technology, Cornell University [10-15]. Statistical analysis is used for the determination of correction relationships for a calculation of the load coefficient and isentropic efficiency, fig. 2. The model for part-span stall compressor performance prediction:

1. Firstly parameters on the surge line for preferred rotational speed are determined: the flow coefficient \( \bar{\nu} = \frac{c_u}{c_{stall}} \), isentropic efficiency \( \eta_{stall} \) and load coefficient \( \overline{H}_{stall} \).

2. Compressor parameters \( \overline{H}_{stable}, \overline{H}_{stall} \) and \( \eta_{stable} \) for an off-design condition are calculated with use of equations (1-4) for the preferred flow coefficient.

3. Correction relationships are used for the load coefficient (fig. 2) and isentropic efficiency considering part-span stall in the compressor (determined by means of statistical analysis):

   \[
   \eta = -0.2482 \cdot (\bar{\nu})^2 - 0.3362 \cdot (\bar{\nu}) + 0.5847. \tag{6}
   \]

4. Real performance data is determined with use of correction relationships:

   \[
   \overline{H} = \overline{H}_{stable} - \delta \overline{H} \cdot \overline{H}_{stall} \cdot \eta_{stall}, \quad \eta = (1 - \delta \eta) \cdot \eta_{stall}. \tag{7}
   \]

5. Stage pressure ratio and isentropic efficiency are determined using equation (4).

Results of the compressor full span stall experimental research conducted in NASA, General Electric, Pratt&Whitney, Snecma [16-18] are used for the development of compressor off-design performance prediction method. The classical criterion is used for the transition from part-span to full-span stall. Rotating stall with blockage less than 30 percent will be assumed to be part-span and stall having blockages greater to be full-span. Another criterion for the transition from one model to another one is hub-to-tip ratio which greater than 0.65.

![Fig. 2. Correction relationships for calculation of compressor stage load coefficient and isentropic efficiency in part-span stall](image)

One of the features of full-span stall in a compressor is that the performance curve can exhibit a large discontinuity where the pressure rise and mass flow jump to significantly reduced values. The abrupt performance characteristics are modelled by a set of correction relationships identified via analysis of experimental data.

\[
\overline{H} = \overline{H}_0 \left( \frac{c_u}{c_{u0}} \right) + K_2 = \overline{H}_0 \left( \frac{c_u}{c_{u0}} \right) - 0.2367 \cdot \left( \frac{c_u}{c_{u0}} \right)^3 - 0.0545 \cdot \left( \frac{c_u}{c_{u0}} \right) + 0.2908. \tag{8}
\]

\[
(i < i_{crit}): \overline{H} = \overline{H}_0 \left( \frac{c_u}{c_{u0}} \right) + K_2 - K_3; K_3 = \overline{H}_{stallset} \left( \frac{c_u}{c_{u0}} \right) - 0.391 \cdot \left( \frac{c_u}{c_{u0}} \right) + 0.394. \tag{9}
\]
\[ H = \frac{c_\alpha}{c_{\alpha 0}} + K_2 - K_3 \cdot K_4 = H_{\text{stall}} \cdot (1.975 \cdot M_u^3 - 2.704 \cdot M_u^2 + 0.329 \cdot M_u + 0.5). \] (10)

The developed method for the reverse-flow performance prediction is based on data obtained in NASA, Massachusetts Institute of Technology, University of Cambridge, Cranfield University [19-21]. The reference point for the reverse-flow performance prediction is located on a surge line for the preferred rotational speed. A virtual value of load coefficient for a zero flow coefficient is also required. This virtual point is being determined during the stalled performance prediction. Reverse-flow performance prediction is based on the following empirical model:

\[ H = H_{G-O} \cdot \left( 12.6110 \cdot (\nu) - 8.1577 \cdot (\nu) + 1.1592 \right), \quad \eta = \eta_{\text{stall}} \cdot \left( -0.2482 \cdot (\nu) - 0.3362 \cdot (\nu) + 0.5847 \right). \] (11)

An empirical model is presented for a stall/unstall hysteresis loop determination:

\[ \frac{H_{\text{crit}} - H_{\text{unstall}}}{H_{\text{crit}}} = 0.623 - 0.42 \cdot 10^{-4} \cdot \left( \frac{b}{t} \right) - 4.3 \cdot 10^{-7} \cdot \text{Re} + 0.12 \cdot 10^{-5} \cdot \text{Re} \left( \frac{b}{t} \right). \] (12)

where \( b/t \) is a cascade solidity, Re is a blade chord Reynolds number.

Each empirical model from the developed set is verified via a comparison against individual compressor stages performance experimental data. Developed empirical models are implemented in the COMPRESSOR_S software tool for the compressor preliminary design.

### 4. Validation of software tool

Robert Gamache’s PhD thesis in 1985 [22] is still the only detailed investigation of the compressor reverse flow operation, in which a complete set of overall performance characteristics has been presented. In his work, experiments were carried out on two builds of three-stage, constant annulus compressors preceded by an IGV, but each compressors with different blade reaction. The compressor was operated at the stable off-design modes, stall and steady reverse flow with use an auxiliary fan to draw the air backwards while a conical nozzle was each time adjusted to give the desirable mass flow rate. Abovementioned work is a unique data set for the validation of the developed method. The comparison of experimental and numerical results for 3-stage compressor and one of individual stages is shown on fig. 3.

From fig. 3 it can be found that the developed set of models is characterized by high accuracy in a wide range of operation conditions. Numerical results also show a stage-by-stage unstall of 3-stage axial compressor.

One of the features of the developed method is the integrated approach – the method is capable of the off-design performance prediction in a wide range of operating conditions (stable off-design, part-span and full-span stall, reverse flow) based on a limited data set (meanline design parameters). For the stable operational conditions the off design performance and the surge line prediction calculation error of model is less than 5%. Maximal error for stalled and reverse-flow conditions performance prediction is about 7%. Obtained results show high accuracy level of designed software tool for the compressor preliminary design in despite of the simplicity of empirical models is being used. COMPRESSOR_S software tool is useful for the generation of compressor static performance characteristics for dynamic analysis of compression systems stall and surge. Generated compressor static performance data can also be used for gas turbine engines stall and surge analysis.
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References