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The small world phenomenon and assortative mixing in Polish corporate board and director networks

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HIGHLIGHTS

- Both networks are distant to random displaying the small-world properties.
- Both networks are more compact than the classical random graph.
- Both networks display assortative mixing.

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ABSTRACT

This paper investigates the corporate board and director networks in the Polish capital market in 2014. We examined real board and director networks in comparison with networks that were randomly constructed. Through empirical analyses, we demonstrated that the real networks have the characteristics of small-world networks. In addition, the networks are assortative and highly clustered, which imposes certain behaviors on them.

1. Introduction

Considerable attention has been paid to real world networks, such as corporate board and director networks [1–5]. This assertion also pertains to econophysics [4,6,7]. The analyses in this vein inform us about the organization of business networks, including how information is disseminated and how economic power is concentrated [8–13]. Therefore, in this paper, we will uncover the properties of board and director networks, focusing on investigating the small-world properties [14–16] and assortativity mixing [17] that might determine their behaviors. At the same time, we will try to shed light on the constraints on the interpretation of specific quantities that are inherited from the construction of specific metrics relating to features of the board and director networks.

Analyses of small-world effect in corporate and director networks have been conducted for the US [8,11,12,16,18–22]; UK [19,22]; Australia [21]; Canada [22]; Italy [8,11,20,22]; Germany [19,22]; the Netherlands and Switzerland [22,23]; Denmark, Norway, Sweden [22,24]; South Africa [25]; and Spain, France, Brazil, Chile, Israel, Korea, Mexico, Taiwan [22]. Analyses of assortativity in corporate and director networks have been conducted for the US [11,12,19,20]; Italy [11,20]; UK and Germany [19]; South Africa [25].

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Table 1
Average board size and average number of directorship held in Poland and other countries.

Country (year)	Average board size	Average number of directorship held
Poland (2014)	7.8	1.2
UK (2002) [19]	6.51	1.84
Germany (2008) [26]	13.3	1.12
Germany (2002) [19]	6.33	1.45
Italy (1998–2011) [27]	⟨9.57–10.41⟩	⟨1.20–1.27⟩
Italy (2008) [28]	10.16	1.54
Switzerland (2000) [23]	9.5	–
Netherlands (2001) [23]	8.2	–
US (2003) [19]	9.97	1.63
US (1995) [29]	–	1.6
New Zealand (1993) [30]	6.14	1.22
Australia (1991) [30]	8.37	1.19
South Africa (2008) [25]	8.56	1.28
France (1999) [31]	9.48	–

Entries '–' indicate missing data.

Table 2
Global indicators for the board and director networks.

Variable	Board		Director	
	Entire network	Largest component	Entire network	Largest component
N	903	518	5943	3282
E	1279	1185	26 121	16 336
k	2558	2370	52 242	32 672
$\langle k \rangle$	2.83	4.58	8.79	9.94
Max k	19	19	76	76
Number of pendant nodes	159	88	14	1
N_{LC}/N	57.36%		55.22%	
Number of isolated nodes	265		4	
Number of connected nodes	638		5939	
Inclusiveness	70.65%		99.93%	
c	310		310	
$(c - 1/N - 1)$	0.34		0.05	

N = Number of nodes, E = Number of edges, k = Degree, $\langle k \rangle$ = Mean degree, Max k = Maximum degree, N_{LC}/N = Fraction of vertices in the largest component, c = Number of components, $(c - 1/N - 1)$ = Component ratio.

The paper is organized as follows: Section 2 introduces the characteristics of the data set used in this paper. In Section 3, we inspect the board and director networks from a global perspective. In Section 4, the small-world properties of both networks are investigated. Finally, in Section 5, assortativity is explored. The paper ends with a summary of its conclusions.

2. The data set

We analyzed the composition of corporate board and director networks in Poland in 2014. We obtained corporate board information on the 903 companies listed on the main market at the Warsaw Stock Exchange (461 companies) and on the NewConnect market (442 firms) in October 2014. This data was obtained from Notoria database and checked for consistency. Using this data, we created a corporate board network with 903 vertices/companies and a director network with 5943 vertices. In the *board network*, two vertices (representing boards or companies) are connected by an edge if they have at least one director in common. In the *director network*, when two vertices represent directors who are both members of a particular board, an edge is established between them. When two firms have at least one director in common, the directorates are called interlocking [1]. The two aforementioned networks are undirected and unweighted. The average board of directors in our data set is composed of 7.8 directors and the average director holds 1.2 directorships, which implies that there are interlocking directorates, and it can be compared with other countries (Table 1). Most directors sit on only one board. Out of the 5943 directors, 5224 (approximately 88%) hold only one board position.

3. The global view of the corporate board and director networks: statistical properties

Many indicators have been proposed for characterizing the networks of boards and directors. In an initial inspection, we consider the basic indicators needed to understand the two networks. We report them in Table 2. From a global point of view (Table 2) we first observe that the fraction of the vertices in the largest component, N_{LC}/N, of each network is larger than 0.55. The presence of the largest component prevails in real-world data networks [4,8]. In the case of the corporate board network, the vertices that are not part of the largest component belong to 309 small components, 265 of which consist of one vertex (an isolated vertex). Interestingly, in the director network, the vertices that are not part of the largest component

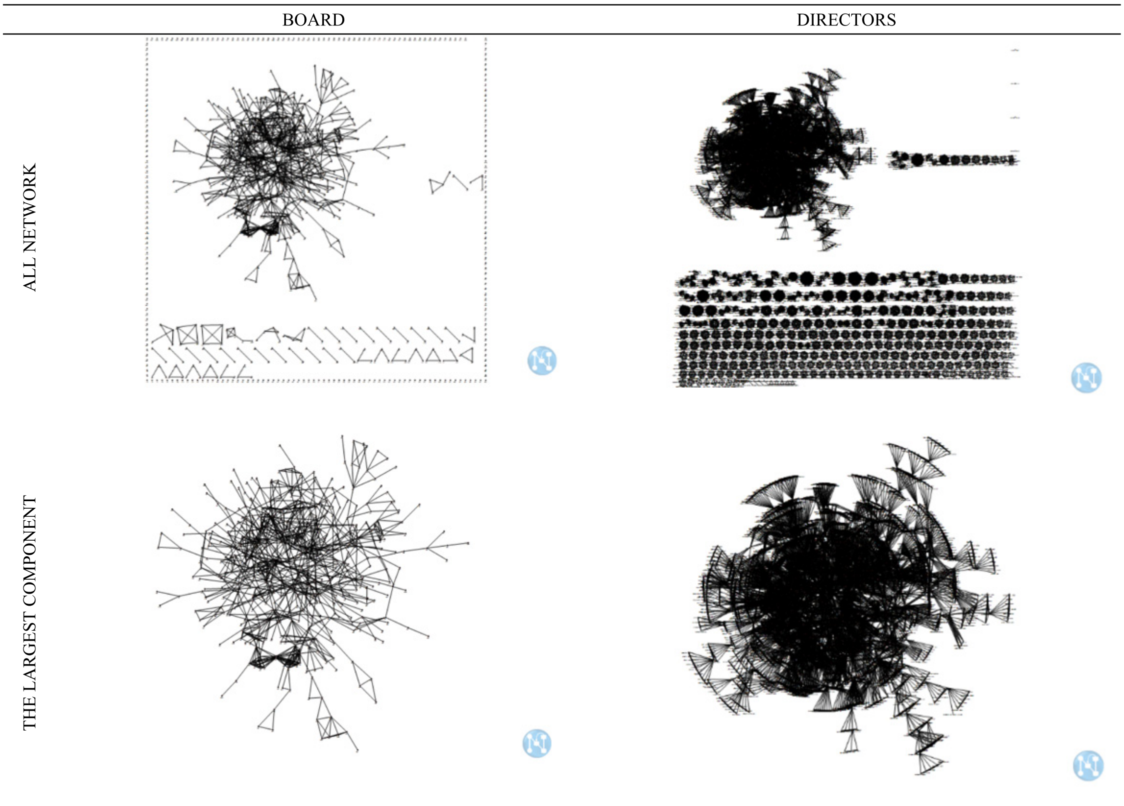

Fig. 1. The board and director networks.

also belong to 309 small components, but only 4 of them are isolated, which means that there are 4 boards composed of only one director. As a result, the director network is characterized by a small component ratio of 0.05, which can be compared with 0.34 for the board network. The component ratio reaches the maximum value of 1.0 when every vertex in the network is isolated [32]. These results indicate that the director network is more cohesive than the board network. This conclusion supports an analysis of the average degree of a network. The average number of edges that each vertex has – the mean degree – is 2.83 for the board network and 8.79 for the director network. For the largest component of each network, the mean degree is higher than it is for the entire network due to the exclusion of vertices of degree zero. Director network is more cohesive than the board network because the average board of directors is composed of almost 8 directors (as seen in Table 1, average board size is 7.8). In the bipartite graph isolated board usually composed more than one director and after projection into two one-mode networks there are much more isolated nodes in the board network than in the director network.

Fig. 1 shows a clearer picture of the networks considered.

4. Network structural indicators of small-worldness

The *network density* in an undirected graph is the proportion of the number of edges present (m) to the maximum possible

$$\rho = \frac{2m}{N\,(N-1)} \tag{1}$$

where N is the number of vertices in the network. Network density can be interpreted as the probability that an edge exists between any pair of randomly chosen nodes [32] in relation to quantities such as the mean distance, the transitivity measure and the clustering coefficient.

The *geodesic distance* $d\,(i,j)$ is the *shortest path* between nodes i and j. The *network diameter* (s) is the length of the largest geodesic distance between any pair of nodes where a path actually exists. The *average shortest path length* (*mean geodesic distance* or simply *average distance—L*) is the average of the geodesic distance between nodes i and j in a connected network

$$L = \frac{\sum_{i\in V}\sum_{i\neq j\in V} d\,(i,j)}{N\,(N-1)}. \tag{2}$$

The *network clustering coefficient* in the sense of Watts and Strogatz (C_{WS}) [14] is defined as the average of the local clustering coefficients for all the single vertices and calculated as follows:

$$C = \frac{\sum_{i=1}^{N} C_i}{N}, \tag{3}$$

where C_i is the local clustering coefficient

$$C_i = \frac{2m_i}{k_i\,(k_i - 1)} \tag{4}$$

where m_i is the number of connections observed between neighbor nodes of i, and k_i is the degree of i.

In a transitivity triad each path of length two is closed, where a loop of length three is formed. Transitivity is defined as the fraction of all paths of length two that are connected, which can be expressed as

$$T = \frac{(\text{number of triangles}) \times 6}{\text{number of paths of length two}}. \tag{5}$$

Understood this way, the transitivity coefficient captures the average probability that two neighbors of a vertex are adjacent. A transitivity coefficient of 1 implies that a network includes edges connecting all three vertices in every triad. It describes the situation of perfect transitivity and expresses the cliquishness of a network.

The asymptotic approximations of the mean geodesic distance and clustering coefficient for a random graph with N vertices and k edges per vertex are [14,15]

$$L_{\mathrm{RANDOM}} \sim \frac{\ln(N)}{\ln(k)} \tag{6}$$

and

$$C_{\mathrm{RANDOM}} \sim \frac{k}{N}. \tag{7}$$

An indicator of small-world network in the sense of Watts–Strogatz for the corresponding E–R random graphs was proposed in Ref. [33]

$$S^{\mathrm{WS}} = \frac{\gamma^{\mathrm{WS}}}{\lambda} \tag{8}$$

where

$$\gamma^{\mathrm{WS}} = \frac{C_{\mathrm{ACTUAL}}}{C_{\mathrm{RANDOM}}} \tag{9}$$

$$\lambda = \frac{L_{\mathrm{ACTUAL}}}{L_{\mathrm{RANDOM}}}. \tag{10}$$

Using Eqs. (9), (10) Eq. (8) becomes

$$S^{\mathrm{WS}} = \frac{C_{\mathrm{ACTUAL}}}{C_{\mathrm{RANDOM}}} \frac{L_{\mathrm{RANDOM}}}{L_{\mathrm{ACTUAL}}}. \tag{11}$$

5. The small-world properties of the board and director networks

A global view of networks should be complemented with the investigation of the small-world properties of those networks. Small-world effects are recognized when, in a network with a large number of vertices, nearly any two vertices are connected with a small number of intermediary edges. In addition, a small-world network has the following characteristics: (1) a low network density; (2) a high clustering coefficient; and (3) a small mean distance [34,35]. Mathematical models of networks imply that the average shortest path length L in networks should typically scale with $\log(N)$, N being the total number of network vertices, and should tend to remain small even for large networks [35,36].

We start with an investigation of the network density. The value of the network density is very small for both networks: 0.003 for the board network and 0.001 for the director network (Table 3), which means that almost none of the potential edges in these networks should exist. This is a typical feature of large networks. At the same time, the network diameter, defined as the longest finite geodesic path anywhere in the connected network [35], is relatively small, especially for the largest component of the director network, which has a diameter of 17. However, the network diameter is less plausible as an indicator than the mean distance when analyzing real-world networks because it refers to one specific pair of vertices [35].

Table 3
Small-world quantities for the board and director networks.

Variable	Board		Directors	
	Entire network	Largest component	Entire network	Largest component
ρ	0.003	0.009	0.001	0.003
s	Na.	16	Na.	17
$(\mathbf{L_{ACTUAL}})$	5.819	5.823	6.513	6.526
L_{RANDOM}	Na.	7.55	Na.	5.04
$(\mathbf{C_{ACTUAL}})$	0.53	0.513	0.943	0.910
C_{RANDOM}	Na.	0.004	Na.	0.002
$(\mathbf{C_{ACTUAL}})/\rho$	176.7	57.0	943.0	303.3
T	0.496	0.493	0.753	0.68

ρ = Network density, s = Network diameter, $(\mathbf{L_{ACTUAL}})$ = Mean geodesic distance (average shortest path length), $(\mathbf{C_{ACTUAL}})$ = Clustering Coefficient, $(\mathbf{C_{ACTUAL}})/\rho$ = Clustering Coefficient/Network density, T = Transitivity.

Table 4
Additional quantities of small-world phenomenon for the board and director networks (the largest component).

Variable	Board network	Director network
γ^{WS}	128.25	455.00
λ	0.771	1.295
$S^{WS} = \gamma^{WS}/\lambda$	166.342	351.351

Therefore, the most commonly-used small-world statistics are the average shortest path length, the median distance, and the clustering coefficient [14,15]. The average distance between two vertices is small (5.82 for the board network and 6.51 for director network) in comparison with the number of vertices in both networks, particularly in the director network, which has 5943 vertices.

When each director holds a position on only one board (each director's directorship is 1, which implies that no corporations have interlocking directorates), the clustering coefficient of a director network is 1. In other words, all of the board network's vertices are isolated and the clustering coefficient of the board network is 0. Therefore, the reason that the value of the board network's clustering coefficient is higher than 0 is the interlocking directorates. The clustering coefficient expressing the tendency for a neighbor's vertices to be connected is above 0.5 for the board network and above 0.9 for the director network.

Hypothetically, if director A is connected with director B, and director B is connected with director C, the assumption that the network can be represented by the simple random Erdös & Renyi (E–R) model [37] implies that the probability of a link between A and C is close to the network density, which is 0.001 (for the largest component, it is 0.003). The clustering coefficient of the real world director network shows that this probability is actually 0.943 (0.91 for the largest component), which suggests that the real network is not random because randomly added edges do not tend to cluster. Therefore, the chance of directors A and C being connected in a given triad is 943 times as high as it is in a hypothetical random network with the same number of vertices and edges (303 times for the largest component). For the board network this probability is 177 times as high (57 times for the largest component). This presents a topic for further research.

Now we turn to the question of whether the networks exhibit small-world properties in the sense of Watts and Strogatz [14]. We focus on the largest component of the board network as this meets the four basic criteria of the small-world phenomenon [15], namely: (1) the board network is numerically large ($N \gg 1$); (2) the board network is sparse (its density is 0.009); (3) the board network is decentralized, as shown by its average degree of 4.58, which is much smaller than the number of vertices $N = 518$ (this network has a low degree centralization index of 1,80%, 2,80% for its largest component); (4) the maximal degree of all vertices is 19, which is also much less than N (see Table 2); and (5) the network is highly clustered, as discussed previously. Moreover, the board network is connected in the sense that any vertex can be reached from any other vertex in the component through a finite number of edges. We performed asymptotic approximations of the average shortest path length L and clustering coefficient C for a random graph with N vertices and k edges per vertex and found $L_{RANDOM} \sim \ln(N)/\ln(k) = 7.55$ and $C_{RANDOM} \sim k/N = 0.004$. These results were compared with $L_{ACTUAL} = 5.823$ and $C_{ACTUAL} = 0.513$ for our board network (see Table 3). Because L_{ACTUAL} is almost equal to L_{RANDOM} but clustering is prominent ($C_{ACTUAL} \gg C_{RANDOM}$), the largest component in the board network exhibits the small-world phenomenon.

A similar conclusion can be drawn from additional quantity of small-world phenomenon for both networks. The value of $S^{WS} > 1$ indicates the small-world property [33]. The value of the quantitative categorical definition of small-world network is much higher than one ($S^{WS} \gg 1$) for both networks: 166.34 for the board network and 351.35 for the director network (Table 4), which means that both real-world networks exemplify the small-world phenomenon for the corresponding E–R random graph. The relationship between γ^{WS} and λ is shown in Fig. 2.

Additionally, because the average distance between board members is approximately 6 in the actual board network and approximately 8 in the classical random graph (Table 3), the board network is more compact than the classical random graph.

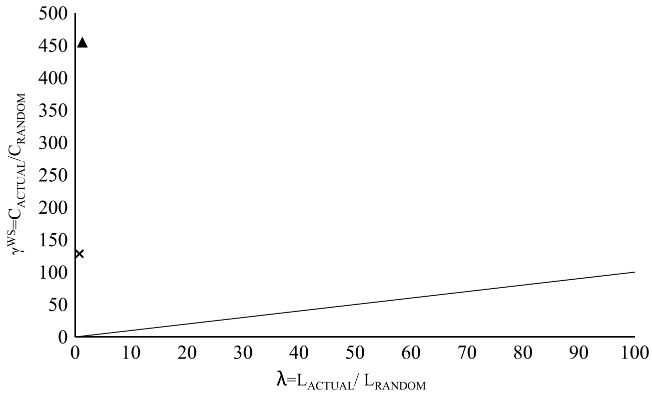

Fig. 2. Relationship between γ^{WS} and λ for the board and director networks.

Table 5
Assortativity coefficients.

Network	Number of iterations QAP	Observed r	Expected r	Std. Dev.	P (expected $r \geq$ Observed r)	P (expected $r =$ Observed r)	P (expected $r \leq$ Observed r)
Boards—full network	100 000	**0.438**	−0.003	0.028	0	0	1
Boards—largest component	100 000	**0.373**	−0.003	0.029	0	0	1
Directors—full network	10 000	**0.254**	0	0.006	0	0	1
Directors—largest component	10 000	**0.128**	0	0.008	0	0	1

6. Assortative mixing of the board and director networks

We now move to the assortativity, which is the tendency for edges in a social network to attach preferentially to others similar to them [11,38]. In other words, high-degree vertices associate preferentially with other high-degree vertices, while low-degree vertices tend to be adjacent to other low-degree vertices. In particular, the assortativity of a network is measured by the assortativity coefficient r, which is the Pearson correlation coefficient of the degrees between connected nodes [17]. In a random network, the coefficient is 0 [17]. If a network has a positive correlation coefficient, then the network has assortative mixing. The value of the assortativity coefficient is positive for the board and director networks (see Table 5); positive values have also been observed in social networks [35]. This coefficient's especially high value of 0.438 for the board network can be interpreted as a sign of very strong assortative mixing [39]. Therefore, corporations of high degree tend to link with other cooperation of high degree. Such assortativity adds to a core–periphery pattern [40].

Removing high-degree vertices from an assortativity mixed network is unlikely to destroy the network's connectivity [17]. The assortativity of the director network is strong, with $r = 0.254$, which can be interpreted as saying that directors who sit on many boards are prone to sit on them with others who also sit on many boards. The directors who sit on only one board, who comprise 88% of all of the directors, have the same degree as other board members who also hold only one directorship. Low value of the average number of directorship held increases the assortativity for the director network. When each director holds only one board position, the assortativity achieves a maximum value of 1 for the director network and is undefined for the board network. In that case, the two-mode network consists of components equal number of boards and after projection into one-mode network, the director network consists of separate components connecting directors all with the same degree. However, the one-mode board network consists of only isolated vertices (network without any edges).

However, in our data, a small number of directors (12%) sit on more than one board and are less likely to be of the same degree as other directors. To illustrate, if a certain director holds a position on board A, when this director joins board B, his/her degree increases nonlinearly by $q - k$, when $q > 0$ stands for the number of other directors on board B and k stands for the number of directors in board B who the director in question knows from other boards. This effect significantly decreases the value of the assortativity coefficient for the director network. The director network's assortativity coefficient is low compared with the board network's one; however, it cannot be explained solely by the fact that the value of the assortativity coefficient for the director network is decreasing. To estimate whether the value of the assortativity coefficient is due to chance, we perform a Quadratic Assignment Procedure (QAP) [41–43], the results of which are reported in Table 5. QAP is a bootstrapping approach, designed to make inferences comparing various networks on the same set of vertices [43]. QAP makes new networks by randomly rearranging rows and columns from the original matrix. The new networks are independent of the original network, and the properties (with autocorrelation) of the new matrices are preserved. The expected value in those networks is computed and then compared with the actual distribution of assortativity coefficient in the networks observed. As seen in Table 5, the original observed value of the assortativity, r, is much higher than the expected value, and the probability of such an occurrence is estimated to be 1. Therefore, we are in a strong position to state that the original values of the assortativity coefficient are statistically significant for the networks under consideration.

7. Conclusions

From the above analyses, we can state that connections between boards and directors are not formed independently of their other ties. This finding suggests that social processes that depend on node attributes generate network edges. The network data's positive degree–degree correlation (assortative mixing by degree) implies that these networks are prone to outbreaks of diseases, as in the ''poison pill'' phenomenon. Moreover, due to their high transitivity, these networks are likely to convey redundant information. At the same time, the networks in question are robust to strategic attacks [17]. In addition, the sizes of the largest components of the board and director networks are significantly larger than they are in randomized networks because of the high clustering tendency combined with a low propensity to form edges (low network density), suggesting that the networks exhibit the small-world phenomenon. Therefore, the speed of information transfer is higher than it is in a randomized network.

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