ELSEVIER
Available at www.ComputerScienceWeb.com
POWERED BY SCIENCE DIRECT®
Artificial Intelligence 148 (2003) 1–9
Artificial Intelligence
www.elsevier.com/locate/artint

Editorial

Fuzzy set and possibility theory-based methods in artificial intelligence

Didier Dubois, Henri Prade *

Institut de Recherche en Informatique de Toulouse (IRIT), Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 4, France

1. Introduction

Fuzzy logic is almost 40 years old and was developed largely outside mainstream artificial intelligence (AI). As a consequence, fuzzy logic methods are not always well-understood by the AI community, in spite of some valuable efforts such as the organization of fuzzy logic workshops in AI conferences, e.g., [14,43,50,51]. Yet, an important part of the concern and research in fuzzy logic and possibility theory does focus on issues such as knowledge representation, approximate reasoning and reasoning under uncertainty, which are central to AI. These research concerns are indeed less widely known than the engineering applications of fuzzy if-then rules, where control laws are synthesized by means of interpolation techniques based on matching degrees [46,59,61]. Originally, fuzzy logic control was very close in spirit to expert, AI-based control. This field has however drifted away from main AI interests in the recent years, and the emphasis on knowledge representation issues in these works now tends to fade. Fuzzy control more and more becomes a subchapter of nonlinear control.

In contrast, this Special Issue is meant to bridge the gap between mainstream AI and current fuzzy set research, and to provide an organized view of recent works by gathering representative applications of fuzzy set and possibility theory-based methods to AI problems. Before providing a brief presentation of the contents of the special issue, basic facts and key issues in fuzzy set and possibility theories, which are relevant for AI applications, are first recalled and emphasized.

* Corresponding author.
E-mail address: henri.prade@irit.fr (H. Prade).

0004-3702/$ – see front matter
doi:10.1016/S0004-3702(03)00118-8

2. The modeling framework

2.1. What a fuzzy set can model

From a mathematical point of view, a fuzzy set is a mapping from a set of elements into an evaluation range that possesses a lattice structure at least, and is usually a linear order. This range is often taken in practice as the real interval $[0, 1]$ (or a subset of it), as originally proposed by Zadeh [65]. Such a basic idea of a set with gradual membership, or with weighted elements proved to be very fruitful in knowledge representation and information processing. Indeed grades of membership may convey different meanings, such as *similarity*, *preference*, *uncertainty*, or even permission. Thus, a class with borderline elements, represented by a fuzzy set, is modeled in terms of similarity or proximity to prototypical elements. A fuzzy set of more or less satisfactory or acceptable choices encodes a preference profile. An ill-known attribute value can be represented by a fuzzy set of more or less plausible values. See [20] for a discussion of the different interpretations that a fuzzy set can convey.

As generalized sets, fuzzy sets can be combined by extended set-theoretic operations, their images via mappings can be defined by means of an extension principle, as well as their composition with fuzzy relations. For instance, arithmetic operations can be extended to fuzzy subsets of the real line, in a way that agrees with interval calculus for each cut of the fuzzy sets (a cut is a classical set of elements whose membership grades are above some threshold).

2.2. Qualitative and quantitative possibility theories

Beside the above-mentioned operations that build a new fuzzy set from fuzzy set(s), set-functions can be associated with fuzzy sets. In particular, a possibility measure Π [66] can be defined from a fuzzy set viewed as a possibility distribution π (namely $\Pi(A) = \sup\{\pi(x) \mid x \in A\}$). This gives birth to a max-decomposable set-function Π, such that $\Pi(A \vee B) = \max(\Pi(A), \Pi(B))$. By duality, a necessity function is associated with Π, namely $N(A) = 1 - \Pi(\neg A)$, which thus satisfies $N(A \wedge B) = \min(N(A), N(B))$. It provides a graded account of the classical interrelationship between modalities expressing possibility and necessity respectively. $\Pi(A)$ evaluates the consistency of A with the graded information encoded by π. When π represents the uncertainty about the possible values of a variable, $\Pi(A)$ (respectively $N(A)$) expresses the plausibility (respectively certainty) that range A contains the value of the variable. Another noticeable set-function Δ, expressing *guaranteed* possibility, defined by $\Delta(A) = \inf\{\pi(x) \mid x \in A\}$, is such that $\Delta(A \vee B) = \min(\Delta(A), \Delta(B))$. The joint use of Π and Δ leads to a bipolar representation framework, where positive and negative information can be processed independently, yet in a coherent way [17]. In this setting, positive information describes what is possible for sure (e.g., because it has been observed), and negative information refers to what is impossible (e.g., because it is ruled out by some laws or constraints). Besides, scalar evaluations of sets such as cardinality can be also extended to fuzzy sets, as well as probabilities to fuzzy events.

Possibility and necessity measures can take their values on qualitative ranges (e.g., a totally ordered chain, finite or not), or on a numerical scale (such as $[0, 1]$). This leads to two different forms of conditioning for qualitative and quantitative possibility measures, namely $\Pi(A \wedge B) = \Pi(A|B) * \Pi(B)$ holds taking $*$ as the minimum and the product respectively. We may also use a reversed scale, $[0, +\infty)$ or the positive integer scale, where $+\infty$ expresses full impossibility, and 0 now means 'fully possible' (rather than 'impossible' as in a $[0, 1]$-graded possibility distribution). Then, taking $*$ as the sum (counterpart of the product in the scale transformation), we come very close to Spohn [58]'s ordinal conditional ("kappa") functions [19]. Williams [62] provides a qualitative counterpart to Spohn's conditioning, which turns out to be closely related to the possibilistic approach to belief revision. Qualitative necessity relations, which underlie necessity measures, are indeed epistemic entrenchment relations in the sense of Gardenförs' [32] revision operators. See [19,21] for details and developments. Qualitative possibility measures can be also closely related to the framework of so-called "plausibility functions" (valued on a partially ordered set) that has been proposed by Friedman and Halpern [30]. Moreover, there is a decision-theoretic foundation of qualitative possibility [27] that parallels Savage's [53] methodology justifying probabilities and expected utility. Quantitative possibility and necessity measures can be viewed as a simple non-trivial upper and lower probability system [60], or as consonant plausibility and belief functions in the sense of Shafer [56]. They also possess an operational semantics in the setting of belief functions [28]. See [22], [23, Chapter 7] for general introductions to possibility theory and its relation to probability theory.

2.3. Partial truth is not uncertainty

There has been a long-lasting misunderstanding in the literature of artificial intelligence and uncertainty modeling regarding the role of fuzzy set theory and many-valued logics. This is also related to the latent confusion between 'degrees of truth' and 'degrees of belief', although the former are usually compositional, while the latter cannot be so. Indeed 'degrees of truth' express the extent to which non-Boolean statements (involving gradual properties) hold in a given interpretation of the language, and their compositionality is allowed, but the underlying structures are for instance an MV-algebra, a Heyting algebra but no longer a Boolean algebra [35]. On the contrary, 'degrees of belief', be they probability degrees, necessity degrees or others, do apply to Boolean propositions and cannot be compositional with respect to all the connectives, since an ordered scale with more than two levels cannot be isomorphic to a Boolean structure [25].

In the fuzzy set framework, the distinction between 'degrees of truth' and 'degrees of belief', is captured in terms of degrees of membership to fuzzy classes (representing the extensions of fuzzy predicates) on the one hand, and necessity and possibility degrees on the other hand. This has given birth to two very different kinds of fuzzy set-based logics (see [26,48]). Namely, fuzzy logics in the "narrow sense" [35,36,47] are many-valued logics whose semantics can be expressed in terms of degrees of truth and fuzzy set connectives; see [13,29,31,44,55] for important developments on this type of logics. In contrast, possibilistic logic [1,18,39] handles pieces of information with their level of uncertainty, or with their level of priority if goals are represented. There is a major difference between possibilistic logic and weighted many-valued logics although they look alike syntactically. Namely, in the latter, a weight t attached to a (many-valued) formula p acts as a truth-value threshold, and (p, t) in a fuzzy knowledge base expresses the requirement that the truth-value of p should be at least equal to t, for (p, t) to be valid. So in such fuzzy logics, while truth is many-valued, the validity of a weighted formula is two-valued. On the contrary, in possibilistic logic, which in its basic form, handles classical logic formulas associated with lower bounds of necessity degrees, truth is two-valued (since p is Boolean), but the validity of a possibilistic formula (p, α) is many-valued (since the models of (p, α) form a fuzzy set, where counter-models of p have membership $1-\alpha$). In some sense, weights defuzzify many-valued logics, while they fuzzify Boolean formulas in possibilistic logic. The levels of certainty in possibilistic logic also provide a basis for a form of inconsistency handling by identifying a maximal level at which inconsistency takes place. Namely, formulas having a certainty level greater than this level are safe from any conflict.

3. Fuzzy set-based applications to artificial intelligence

An important feature of fuzzy sets is to provide a framework for interfacing in a non-rigid way classes, encoded by symbolic labels, and numerical values. This is in relation with the ability of fuzzy sets to model perceptions [68]. In classification problems the use of fuzzy classes obviates the need for arbitrarily classifying borderline cases, at the beginning of a reasoning stage. Numerical data can be summarized by means of linguistically labeled fuzzy sets so as to feed a symbolic reasoning machinery, or as a part of a learning process.

Main application areas of fuzzy set and possibility theory-based methods in AI include:

– approximate reasoning [38,67], [6, Chapter 1] where pieces of information are represented by possibility distributions expressing fuzzy restrictions on the values of variables, which are combined and projected on variables of interest;
– reasoning under uncertainty as in possibilistic logic, or in possibilistic Bayesian-like networks [4];
– nonmonotonic reasoning where default rules such as “if p then q, generally” can be represented by constraints of the form “p and q is more possible than p and not q” [5,33], in a way fully equivalent to approaches by Lehmann and Magidor [40], and by Pearl [49]; see also [54];
– case-based reasoning that can be modeled in terms of fuzzy rules involving graded similarity relations [10,15];
– qualitative reasoning on absolute or relative orders of magnitude [34,57];
– diagnosis for handling qualitative uncertainty or modeling the intensity of effects (e.g., [24,41,42,64]);
– soft constraint satisfaction problems (CSPs), where preference profiles on possible instantiations of variables, as well as prioritized constraints are handled in an egalitarist setting using a minimum-based aggregation [12,16], which is an important particular case of more general frameworks such as semi-ring-valued CSPs [8,9];

– qualitative decision or planning under uncertainty, and with a logical representation of preferences [3,27,52];
– pattern classification and computer vision, where the ideas of fuzzy clusters and fuzzy classes play a key role [2,7];
– learning of fuzzy rules [11,45];
– multiple source data fusion [6, Chapter 6];
– multiple agents systems [37,63].

Representative references provided above, complement the contents of this special issue which could not insure an equal coverage of all application areas.

4. Contents of the special issue

The contents of the special issue can be roughly divided into two main parts. The first five contributions can be viewed as applications of fuzzy sets only, while the six remaining articles involve possibility theory aspects.

The first three papers take advantage of the idea of a fuzzy constraint. Miguel and Shen discuss methods for solving dynamical constraint satisfaction problems (CSPs), viewed as a sequence of CSPs linked by restrictions and relaxations adding and removing constraints. The constraints are made flexible by the introduction of preferences and priorities. The approach is applied to flexible planning (where satisfaction degrees are associated with operators and goals) and illustrated on various examples and benchmarks. Luo, Jennings, Shadbolt, Leung and Lee propose a fuzzy constraint-based knowledge model for bilateral, multi-issue negotiations in semi-competitive environments (where the agents look for solutions maximizing their own payoff, each agent remaining fair with respect to the other agent). Prioritized fuzzy constraints are used, together with aggregation operators called ‘uninorms’ which have a unit element in the middle of the value scale and can express compromises, for representing trade-offs between the different possible values of the negotiation issues and for finding concessions to be made. Félix, Barro and Marín deal with networks defining temporal profiles expressed in terms of events referring to fuzzy dates and of episodes describing evolutions on fuzzy time intervals. The paper provides tools for checking local levels of consistency and discuss the application of their approach to signal pattern recognition.

Bloch, Géraud and Maître take advantage of fuzzy sets for representing heterogeneous pieces of knowledge about radiometry, space, morphology or structure, and for combining them using fuzzy set operators. A fuzzy extension of mathematical morphology is used for representing shapes and spatial relationships. The approach is applied to model-based structural recognition of 3D brain images.

Schwartz presents a formal logical system for agent-oriented epistemic reasoning where strengths of belief or disbelief are handled using the basic fuzzy set operators, in a way that reminds of the possibilistic logic handling of certainty levels. However, in the proposed logic language, “upper-level formulas” (which involve (dis)belief operators) obey the rules of classical propositional calculus.

The six remaining papers of the special issue exploit the possibility theory framework for dealing with uncertainty. First, the paper by Raufaste, da Silva Neves and Mariné reports on psychological experiments. They suggest that the way human judgment handles uncertainty is qualitative in essence and close to the normative framework of possibility theory. Dubois, Fargier and Perny investigate a purely qualitative approach to decision under uncertainty, where utility and uncertainty scales are not comparable. The paper identifies possibilistic structures as the only ones that are compatible with decision rules obeying an ordinal invariance axiom and preserving the transitivity of the strict preference.

The next two articles present applications of possibilistic logic. Grabisch proposes an approach to the modeling and the recognition of temporal scenarios. At the representation level, the logical dependencies between the elements of a scenario, which can be pervaded with uncertainty, are described by possibilistic logic expressions, while temporal constraints on durations are handled separately. Then, a graded matching procedure assesses to what extent a given scenario fits with observations. Benferhat and Kaci use propositional logic formulas weighted in terms of guaranteed possibility measures for representing prioritized pieces of information. This particular encoding is proved to be efficient for computing distance-based fusion of classical logic bases. They also show how to translate the particular representation they use into a "classical" possibilistic logic base (where formulas are weighted in terms of necessity measures) and conversely.

The two last papers deal with two different learning problems. First, Hüllermeier studies a method for instance-based learning, based on an extrapolation principle expressed in terms of (guaranteed) possibility, and similarity between stored cases and the considered situation. It allows for the handling of incomplete information. A comparison with classical instance-based learning methods, as well as experimental results show the interest of the approach. Borgelt and Kruse use possibilistic graphical models similar to Bayes nets, for the decomposition of possibility distributions. This approach can also account for imprecise information. The authors develop counterparts to probabilistic algorithms for learning such possibilistic graphical models from imprecise data sets. Appropriate evaluation measures for this learning task are thoroughly discussed.

Hopefully, this special issue will contribute to help understanding the main ideas underlying fuzzy set and possibility theory-based approaches, and to show their merits from representational and computational points of view, for a better handling of qualitative information. It is also meant to be instrumental in favoring a dialogue between researchers in the fuzzy set community and in artificial intelligence after decades of mutual ignorance or misunderstanding.

Acknowledgements

The guest-editors are deeply indebted to many people for making this special issue possible. First, thanks are due to the previous editors-in-chief of the journal *Artificial Intelligence*, Dany Bobrow and Mike Brady for their encouragements to prepare this special issue, and to the present editors-in-chief, Ray Perrault and Erik Sandewall, for their kind support. Special thanks are also due to Mike Brady for the separate handling of the submissions coming from the guest-editors' research group.

Jennet Batten deserves a very special mention for her constant assistance during the long preparation process of this special issue.

Deep thanks are also due to the reviewers of the papers submitted to the special issue, who greatly helped us select the papers, and made useful recommendations for preparing revised versions. Lastly, the guest-editors wish to thank all the authors of submitted papers and contributors.

References

[1] T. Alsinet, L. Godo, Towards an automated deduction system for first-order possibilistic logic programming with fuzzy constants, in: T. Martin (Ed.), Internat. J. Intelligent Systems (Special Issue on Logic Programming and Soft Computing) 17 (2002) 887–924.

[2] O. Al-Jarrah, A. Halawani, Recognition of gestures in Arabic sign language using neuro-fuzzy systems, Artificial Intelligence 133 (2001) 117–138.

[3] S. Benferhat, D. Dubois, H. Fargier, H. Prade, R. Sabbadin, Decision nonmonotonic reasoning and possibilistic logic, in: J. Minker (Ed.), Logic-Based Artificial Intelligence, Kluwer Academic, Dordrecht, 2000, pp. 333–358.

[4] S. Benferhat, D. Dubois, L. Garcia, H. Prade, On the transformation between possibilistic logic bases and possibilistic causal networks, Internat. J. Approx. Reason. 29 (2) (2002) 135–173.

[5] S. Benferhat, D. Dubois, H. Prade, Nonmonotonic reasoning, conditional objects and possibility theory, Artificial Intelligence 92 (1997) 259–276.

[6] J.C. Bezdek, D. Dubois, H. Prade (Eds.), Fuzzy Sets in Approximate Reasoning Information Systems, The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 1999.

[7] J.C. Bezdek, J. Keller, R. Krishnapuram, N.R. Pal, Fuzzy Models and Algorithms for Pattern Recognition and Image Processing, in: The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 1999.

[8] S. Bistarelli, U. Montanari, F. Rossi, T. Schiex, G. Verfaillie, H. Fargier, Semiring-based CSPs and valued CSPs: Frameworks, properties, and comparison, Constraints 4 (3) (1999) 199–240.

[9] S. Bistarelli, P. Codognet, F. Rossi, Abstracting soft constraints: Framework, properties, examples, Artificial Intelligence 139 (2002) 175–211.

[10] P. Bonissone, W. Cheetham, Financial applications of fuzzy case-based reasoning to residential property valuation, in: Proc. 6th IEEE Conf. on Fuzzy Systems (FUZZ-IEEE'97), Barcelona, Spain, 1997, pp. 37–44.

[11] B. Bouchon-Meunier, C. Marsala, Learning fuzzy decision rules, in: J.C. Bezdek, et al. (Eds.), Fuzzy Sets in Approximate Reasoning and Information Systems, in: The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 1999, pp. 279–304.

[12] J. Bowen, R. Lai, D. Bahler, Lexical imprecision in fuzzy constraints networks, in: Proc. AAAI-92, San Jose, CA, 1992, pp. 616–620.

[13] K. Broda, D.M. Gabbay, A. Russo, A unified compilation style labelled deductive system for modal, substructural and fuzzy logic, in: V. Novak, I. Perfilieva (Eds.), Discovering the World with Fuzzy Logic, Springer, Berlin, 2000, pp. 495–548.

[14] D. Driankov, P.W. Eklund, A.L. Ralescu (Eds.), Fuzzy Logic and Fuzzy Control, Proc. IJCAI-91, Sydney, Australia, 1991, Lecture Notes in Artificial Intelligence, Vol. 833, Springer, Berlin, 1994.

[15] D. Dubois, F. Esteva, P. Garcia, L. Godo, R. Lopez de Mantaras, H. Prade, Fuzzy set modelling in cased-based reasoning, Internat. J. Intelligent Systems 13 (4) (1998) 345–373.

[16] D. Dubois, H. Fargier, H. Prade, Possibility theory in constraint satisfaction problems: Handling priority, preference and uncertainty, Appl. Intelligence 6 (1996) 287–309.

[17] D. Dubois, P. Hajek, H. Prade, Knowledge-driven versus data-driven logics, J. Logic Language Inform. 9 (2000) 65–89.

[18] D. Dubois, J. Lang, H. Prade, Possibilistic logic, in: D.M. Gabbay, C.J. Hogger, J.A. Robinson, D. Nute (Eds.), Handbook of Logic in Artificial Intelligence Logic Programming, Vol. 3, Oxford University Press, Oxford, 1994, pp. 439–513.

[19] D. Dubois, H. Prade, Epistemic entrenchment and possibilistic logic, Artificial Intelligence 50 (1991) 223–239.

[20] D. Dubois, H. Prade, The three semantics of fuzzy sets, Fuzzy Sets and Systems 90 (1997) 141–150.

[21] D. Dubois, H. Prade (Eds.), Belief Change, D.M. Gabbay, Ph. Smets (Eds.), Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 3, Kluwer Academic, Dordrecht, 1998.

[22] D. Dubois, H. Prade, Possibility theory: Qualitative and quantitative aspects, in: Quantified Representation of Uncertainty and Imprecision, in: D.M. Gabbay, Ph. Smets (Eds.), Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 1, Kluwer Academic, Dordrecht, 1998, pp. 169–226.

[23] D. Dubois, H. Prade (Eds.), Fundamentals of Fuzzy Sets, The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 2000.

[24] D. Dubois, H. Prade, An overview of ordinal and numerical approaches to causal diagnostic problem solving, in: J. Gebhardt, R. Kruse (Eds.), Abductive Reasoning and Learning, in: D.M. Gabbay, P. Smets (Eds.), Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 4, 2000, pp. 231–280.

[25] D. Dubois, H. Prade, Possibility theory, probability theory and multiple-valued logics: A clarification, Ann. Math. Artificial Intelligence 32 (2001) 35–66.

[26] D. Dubois, H. Prade, E.P. Klement (Eds.), Fuzzy Sets, Logics and Reasoning about Knowledge, Applied Logic Series, Vol. 15, Kluwer Academic, Dordrecht, 1999.

[27] D. Dubois, H. Prade, R. Sabbadin, Decision theoretic foundations of qualitative possibility theory, Eur. J. Oper. Res. 128 (2001) 459–478.

[28] D. Dubois, H. Prade, Ph. Smets, New semantics for quantitative possibility theory, in: Symbolic and Quantitative Approach to Reasoning under Uncertainty, Proc. 6th Europ. Conf. (ECSQARU 2001), Toulouse, in: Lecture Notes in Artificial Intelligence, Vol. 2143, Springer, Berlin, 2001, pp. 410–421.

[29] F. Esteva, L. Godo, Monoidal t-norm based logic: Towards a logic for left-continuous t-norms, Fuzzy Sets and Systems 124 (3) (2001) 271–288.

[30] N. Friedman, J.Y. Halpern, Plausibility measures and default reasoning, J. ACM 48 (4) (2001) 648–685.

[31] D.M. Gabbay, How to make your logic fuzzy: Fibred semantics and the weaving of logics, in: D. Dubois, H. Prade, E.P. Klement (Eds.), Fuzzy Sets, Logics and Reasoning about Knowledge, Kluwer Academic, Dordrecht, 1999, pp. 51–84.

[32] P. Gardenförs, Knowledge in Flux. Modeling the Dynamics of Epistemic States, MIT Press, Cambridge, MA, 1988.

[33] P. Gardenförs, D. Makinson, Nonmonotonic inference based on expectations, Artificial Intelligence 65 (1994) 197–245.

[34] A. Hadj-Ali, D. Dubois, H. Prade, Qualitative reasoning based on fuzzy relative orders of magnitude, IEEE Trans. Fuzzy Systems 11 (1) (2003) 9–23.

[35] P. Hajek, Metamathematics of Fuzzy Logic, Kluwer, Dordrecht, 1998.

[36] P. Hajek, Basic fuzzy logic and BL-algebras, Soft Comput. 2 (3) (1998) 124–128.

[37] R. Kowalczyk, Fuzzy e-negotiation agents, Soft Comput. 6 (2002) 337–347.

[38] R. Kruse, J. Gebhardt, F. Klawonn, Foundations of Fuzzy Systems, Wiley, Chichester, 1994.

[39] C.-J. Liau, B.I.-P. Lin, Possibilistic reasoning—A mini-survey and uniform semantics, Artificial Intelligence 88 (1–2) (1996) 163–193.

[40] D. Lehmann, M. Magidor, What does a conditional knowledge base entail? Artificial Intelligence 55 (1992) 1–60.

[41] H. Leitich, K.P. Adlassnig, G. Kolarz, Evaluation of two different models of semi-automatic knowledge acquisition for the medical consultant system CADIAG-II/RHEUMA, Artificial Intelligence in Medicine 25 (3) (2002) 215–225.

[42] H. Leitich, K.P. Adlassnig, G. Kolarz, Development and evaluation of fuzzy criteria for the diagnosis of rheumathoid arthritis, Methods Inf. Med. 35 (1996) 334–342.

[43] T.P. Martin, A.L. Ralescu (Eds.), Fuzzy Logic in Artificial Intelligence (Proc. of selected papers of the IJCAI'95 Workshop, Montréal, Quebec, 1995), Lecture Notes in Artificial Intelligence, Vol. 1188, Springer, Berlin, 1997.

[44] D. Mundici, Reasoning on imprecisely defined functions, in: V. Novak, I. Perfilieva (Eds.), Discovering the World with Fuzzy Logic, in: Studies in Fuzziness and Soft Computing, Vol. 57, Physica-Verlag, Springer, 2000, pp. 331–366.

[45] D. Nauck, R. Kruse, Neuro-fuzzy methods in fuzzy rule generation, in: J.C. Bezdek, et al. (Eds.), Fuzzy Sets in Approximate Reasoning and Information Systems, in: The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 1999, pp. 305–334.
[46] H.T. Nguyen, M. Sugeno (Eds.), Fuzzy Systems. Modeling and Control, The Handbooks of Fuzzy Sets Series, Kluwer Academic, Dordrecht, 1998.
[47] V. Novak, I. Perfilieva, J. Mockor, Mathematical Principles of Fuzzy Logic, Kluwer Academic, Dordrecht, 1999.
[48] V. Novak, I. Perfilieva (Eds.), Discovering the World with Fuzzy Logic, Studies in Fuzziness and Soft Computing, Vol. 57, Physica-Verlag, Springer, 2000.
[49] J. Pearl, System Z: A natural ordering of defaults with tractable applications to default reasoning, in: Proc. of the 3rd Conf. on Theoretical Aspects of Reasoning about Knowledge (TARK'90), Morgan Kaufmann, San Mateo, CA, 1990, pp. 121–135.
[50] A.L. Ralescu (Ed.), Fuzzy Logic in Artificial Intelligence. Towards Intelligent Systems, Proc. IJCAI-93, Chambéry, France, 1993, Lecture Notes in Artificial Intelligence, Vol. 847, Springer, Berlin, 1994.
[51] A.L. Ralescu, J.G. Shanahan (Eds.), Fuzzy Logic in Artificial Intelligence (Proc. of selected papers of the IJCAI-97 Workshop), Lecture Notes in Artificial Intelligence, Vol. 1566, Springer, Berlin, 1999.
[52] R. Sabbadin, H. Fargier, J. Lang, Towards qualitative approaches to multi-stage decision making, Internat. J. Approx. Reason. 19 (3–4) (1998) 441–471.
[53] L.J. Savage, The Foundations of Statistics, Wiley, 1954. Reprinted by Dover, New York, 1972.
[54] D.G. Schwartz, Dynamic reasoning with qualified syllogisms, Artificial Intelligence 93 (1–2) (1997) 103–167.
[55] M.I. Sessa, Approximate reasoning by similarity-based SLD resolution, Theoret. Comput. Sci. 275 (1–2) (2002) 389–426.
[56] G. Shafer, A Mathematical Theory of Evidence, Princeton Univ. Press, Princeton, NJ, 1976.
[57] Q. Shen, R. Leitch, Fuzzy qualitative simulation, IEEE Trans. Syst. Man Cybernet. 23 (1993) 1038–1061.
[58] W. Spohn, Ordinal conditional functions: A dynamic theory of epistemic states, in: W. Harper, B. Skyrms (Eds.), Causation in Decision, Belief Change and Statistics, Vol. 2, D. Reidel, Dordrecht, 1988, pp. 105–134.
[59] H.B. Verbruggen, H.-J. Zimmermann, R. Babuska (Eds.), Fuzzy Algorithms for Control, Kluwer Academic, Dordrecht, 1999.
[60] P. Walley, Measures of uncertainty in expert systems, Artificial Intelligence 83 (1996) 1–58.
[61] R.R. Yager, D.P. Filev, Essentials of Fuzzy Modeling and Control, Wiley, New York, 1994.
[62] M.-A. Williams, Transmutations of knowledge systems, in: J. Doyle, E. Sandewall, P. Torasso (Eds.), Proc. 4th Internat. Conf. on Principles of Knowledge Representation and Reasoning (KR-94), Bonn, Germany, Morgan Kaufmann, San Mateo, CA, 1994, pp. 619–629.
[63] O. Wong, R. Lau , Possibilistic reasoning for intelligent payment agents, in: R. Kowalczyk, M. Lee (Eds.), Proc. 2d Workshop on Artificial Intelligence in Electronic Commerce (AIEC 2000), Melbourne, Australia, in: Lecture Notes in Artificial Intelligence, Vol. 2112, Springer, Berlin, 2000, pp. 170–180.
[64] K. Yamada, Possibilistic causality consistency problem based on asymmetrically-valued causal model, Fuzzy Sets and Systems 132 (2002) 33–48.
[65] L.A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965) 338–353.
[66] L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1978) 3–28.
[67] L.A. Zadeh , A theory of approximate reasoning, in: J.E. Hayes, D. Michie, L.I. Mikulich (Eds.), Machine Intelligence, Vol. 9, Elsevier, New York, 1979, pp. 149–194.
[68] L.A. Zadeh, A new direction in AI: Toward a computational theory of perceptions, AI Magazine 22 (1) (2001) 73–84.