
Particle Swarm Optimization Applied to Ascent Phase Launch Vehicle Trajectory Optimization Problem

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Abstract

The ascent phase trajectory optimization of a single stage liquid propellant hypersonic launch vehicle is considered in this paper. Trajectory optimization is done to achieve desired terminal conditions using angle of attack as a control variable. The formulation entails nonlinear 2-dimensional launch vehicle flight dynamics with mixed boundary conditions and multi-constraints. The burning time of the liquid rocket engine is fixed, leading to fixed time problem. By studying the behavior of the problem with boundary constraints, the non-linear problem is solved using the PSO method and the optimal trajectory is obtained. The effect of two types of dynamic inertia weight and constant inertia weight in PSO algorithm is examined. The influence of controlling parameters is also investigated.

Keywords: Linearly decreasing inertia weight; Trajectory optimization; Particle swarm optimization; Simulated annealing inertia weight.

1. Introduction

Trajectory optimization is the method of generating trajectory in which some measure of performance can be minimized or maximized. The major problem in space missions is to design a valid trajectory which allows some objectives (single or multiple) to be met by considering some constraints. Optimization process consists of two steps; mathematical formulation of the problem and solving the problem.

For space missions liquid propellants are widely used because of their constant and controllable thrust. The disadvantage of liquid propellants is its low reliability and high cost. Basically the missions are categorizing based on the payload type which the vehicle is carrying. Based on this payload the terminal conditions will vary. The two general categories of vehicle are the following. 1) For a multi stage vehicle, the first stage consists of the payload while considering from top to bottom. In such cases, the terminal condition of the first stage must guarantee good initial condition for the next stage. The subsequent stages can take care of dispersion in initial conditions. 2) For a single stage vehicle, the terminal stage of a vehicle is critical for the success of the mission. This kind of vehicle must possess highly accurate terminal conditions.

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There are many numerical methods to solve the trajectory optimization problem. Mainly, they are divided into direct and indirect methods. Betts\textsuperscript{1} took a survey on these two techniques, and described the relation between them. In early stages, indirect methods were very popular due to their accuracy; the only drawback of this method is that it will increase the size of the problem by introducing co-state variables. In recent years, evolutionary methods have become popular due to their simplicity and less time required for processing\textsuperscript{2,3}.

Today, evolutionary algorithms are widely used for finding global optima. Since they are direct methods, they do not require an initial value to perform the optimization, they will search the design space by selecting random values of design variables\textsuperscript{4}. These methods are based on the principle of Darwin’s theorem on survival of the fittest. Among all these techniques bio inspired are more popular especially the swarm intelligence based methods. These methods are developed my mimicking the natural social behaviour of group of birds, fish etc. Particle swarm optimization is one among them, and it is first introduced by Eberhart and Kennedy\textsuperscript{5} in 1995.

Trajectory optimization of an ascent phase hypersonic vehicle using Particle swarm optimization method is carried out in this paper. The following section describes the mathematical modelling and analysis.

2. Formulation

The problem formulation mainly consists of the mathematical modelling of launch vehicle and the propulsion system. Launch vehicle modelling is carried out using equations of motion.

2.1 Rocket equation of motion

The ascent phase launch vehicle modelling has been carried out in this paper by considering point mass equations of motion with non-rotating spherical earth, i.e.; Approximating the dynamics of the launch vehicle as a point mass model. It can be represented as,

\begin{align*}
\dot{r} &= V \sin \gamma \\
\dot{V} &= \frac{1}{mV}(T \cos \alpha - D - mg \sin \gamma) \\
\dot{\gamma} &= \frac{1}{mV}(T \cos \alpha + L) + \left(\frac{V}{r} - \frac{g}{V}\right) \cos \gamma \\
\dot{x} &= V \cos \gamma
\end{align*}

where \(r, v\) and \(\gamma\) denote the instantaneous radial distance from earth centre, velocity and the flight path angle of the vehicle respectively. \(T, m\) and \(\alpha\) denotes thrust, instantaneous mass and angle of attack of the vehicle respectively. \(L, D\) and \(g\) denotes the lift, drag and gravitational pull by earth respectively. The lift \((L)\) and drag \((D)\) forces are defined by,

\begin{align*}
D &= 1/2 \rho S v^2 c_D \\
L &= 1/2 \rho S v^2 c_L
\end{align*}

where, \(\rho\) denotes the aerodynamic density and \(s\) is the launch vehicle’s aerodynamic reference area. Where \(r_e\), the radius of earth. The gravitational pull of earth is not constant towards the earth atmosphere.

The effect of the gravity is more at the height below 50 km from the sea level. To analyze the effect of gravity, it is modelled as function of height.

\[g = g_0 \left(\frac{r_e}{r}\right)^2\]

2.2 Propulsion system

The propulsion system considers the calculation of thrust variation in each time interval. In this paper constant thrust liquid propellant system is considered. Thus the mass flow rate will be constant.

\[T = \dot{m}v_e\]

where \(\dot{m}\) and \(v_e\) are denoted as exhaust velocity and mass flow rate of the vehicle respectively.
3. PSO Algorithm

PSO is one of the popular heuristic methods. It is basically inspired by the social behavior of animals in a group, birds flying and fish schooling etc. This method tries to take advantage of the information sharing mechanism from the group that affect the overall behavior of the swarm. Therefore, PSO works with population of potential solution rather than with a single individual. In this algorithm the each basic elements are called particles and each particle will represent the potential solutions in a hyper dimensional space. The PSO method is better for solving higher dimensional problems and it has been found that it is robust featuring non-linearity and non-differentiability.

Consider \( D \) the search space dimension, and \( i \)th particle of the entire swarm in \( D \)-dimension is denoted as \( x_i = (x_{i1}, x_{i2}, \ldots, x_{iD})^T \). The velocity of this particle is represented by \( v_i = (v_{i1}, v_{i2}, \ldots, v_{iD})^T \). The best experience of \( i \)th particle is notated as \( P-best \) and is denoted as, \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})^T \). Let \( g \) be the index of the best particle in the swarm (i.e., the \( g \)th is the best), and the superscripts denote the iteration number, then in global version the swarm is manipulated according to the following two equations.

\[
\begin{align*}
    v_{i}^{t+1}(d) & = \omega v_{i}^{t+1}(d) + c_1 \text{rand} (p_{i}^{t}(d) - x_{i}^{t}(d)) + c_2 \text{rand} (p_{g}^{t}(d) - x_{i}^{t}(d)) \quad (8) \\
    x_{i}^{t+1}(d) & = x_{i}^{t}(d) + v_{i}^{t+1}(d) \quad (9)
\end{align*}
\]

where \( x_{i}^{t}(d) \) and \( v_{i}^{t}(d) \) denoted as the current position and velocity of the \( d \)th dimension of the \( i \)th particle respectively. \( \text{rand} \) denotes the a random number which is in uniform distribution \( U \in [0, 1] \). \( c_1 \) and \( c_2 \) are called acceleration constants and both are positive quantities. \( \omega \) denoted as inertia weight.

Possible solution can get from each particle based on the fitness or objective function. Each particle remembers its best position in the hyper dimensional space and the corresponding best value (\( pbest \)). In a swarm each particle knows where the best value for the fitness function has occurred so far in the group (\( gbest \)).

A comparative study has been carried out by Bansal et al.\(^6\) which discusses different strategies implement inertia weight in PSO. Rajesh et al.\(^7\) are also done the similar work and they proposed a new approach to optimize the inertia weight variation. In this method the subtraction of fraction inertia weight \( \Delta \omega \) is subtracted from the maximum weight \( W_{\text{max}} \), thus effect of lower bound is zero. Liu et al.\(^8\) proposed a dynamic particle swarm optimization algorithm. In this each particle could obtain the best information of the local and global particle dynamically. Yang et al.\(^9\) proposed a new method to improve the search capability and avoid local minima search using two parameters called evolution speed and aggregation degree. Van den et al.\(^10\) have defined a condition which guarantees convergence.

In the devolving stage of PSO, inertia weight value was used to be fixed in every updates. But now it is changing dynamically to control exploration and exploitation of the search space. Due to this change inertia weight the following two problems can occur. 1) Premature convergence to the local minimum 2) particle can go out of the boundary of the search space. To avoid these two problems large inertia value can be given initially, which allows all particles to move freely in the search space. To optimize the global and local exploration rate, the inertia weight should adjust accordingly. So instead of giving constant inertia weight to the entire iteration, dynamic inertia weights are preferred. Two type of inertia weights are considered in this paper. The first one is the linearly decreasing inertia weight\(^11\) and the second one is the simulated annealing inertia weight\(^13\). They are denoted as,

\[
\begin{align*}
    \omega_{\text{iter}}^{t+1} & = \omega_{\text{max}} - \left( \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\text{max iter}} \right) \text{ iteration} \quad (10) \\
    \omega_{\text{iter}}^{t+1} & = \omega_{\text{max}} + (\omega_{\text{max}} - \omega_{\text{min}}) \lambda(t) \quad (11)
\end{align*}
\]

where \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) are usually fixed as 0.9 and 0.4, \( \lambda \) is 0.95.

4. Problem Statement

Ascent phase trajectory of launch vehicle is considered in this paper. Estimating the optimized angle of attack, \( \alpha \) to transfer the launch vehicle from a given initial conditions to the terminal conditions with minimum terminal error for a constant thrust (\( T \)) engine, operating for a fixed time \( t_f \). By analyzing the problem statement it is found that
this is fixed time two point boundary value problem. The trajectory should satisfy some boundary conditions in state variables. In this particular problem $r, v$ and $\gamma$ are considered as the state variables and angle of attack, $\alpha$ is called the control variable. The general ascent launch trajectory is shown in Fig. 1.

4.1 Objective function

Here, a special case of the objective function that depends only on the final state $x(t_f)$ is considered. The more general case with an integral term is not addressed here.

$$J = \sum s_X \| x_f - x_{t_f} \|^2$$  \hfill (12)

where $J$ is called the cost function; $x_f$ and $x_{t_f}$ denotes the state variables values at terminal stage and state variables values at final time respectively. $s_X$ denotes the weighting factor for the corresponding state variables. The state variables are radial distance between the earth center, velocity and the flight path angle of the launch vehicle.

4.2 Constraints

The main constraint of the state variables are velocity constraint and flight path angle constraints, they are given as,

$$V_{\text{min}} \leq V \leq V_{\text{max}}$$  \hfill (13)

$$\gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}}$$  \hfill (14)

$$\dot{m} = -m_c$$  \hfill (15)

where $V_{\text{min}}$ and $V_{\text{max}}$ are 5800 and 7700 respectively. $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ are $-6$ and 6 degree respectively. $m_c$ is the mass reduction in unit time.

4.3 Boundary constraints

The initial values of the state variables are, $r_0 = 6805854.5963$ m, $v_0 = 5890.1127$ m/s, $\gamma_0 = 5.2229$ deg and the initial mass, $m_0 = 5200$ kg. The terminal boundaries are, $r_f = 6862415.73$ m, $v_f = 5890.1127$ m/s, $\gamma_f = 0.002$ deg. Since this is affixed time problem, $t_f = 510$ sec. The launch vehicle is having the constant mass flow rate, $\dot{m} = 4.881$, which produce the constant thrust, $T = 14599.071$ N.
Table 1. Terminal errors with different inertia weight.

<table>
<thead>
<tr>
<th>Inertia weight</th>
<th>Error in radial distance from earth center</th>
<th>Error in velocity</th>
<th>Error in flight path angle</th>
<th>Final objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearly Decreasing</td>
<td>0.732</td>
<td>0.521</td>
<td>0.030</td>
<td>0.154</td>
</tr>
<tr>
<td>Simulated annealing</td>
<td>1.030</td>
<td>0.751</td>
<td>0.042</td>
<td>0.289</td>
</tr>
<tr>
<td>0.375</td>
<td>2.732</td>
<td>1.018</td>
<td>0.075</td>
<td>0.634</td>
</tr>
<tr>
<td>0.575</td>
<td>1.324</td>
<td>0.839</td>
<td>0.059</td>
<td>0.431</td>
</tr>
<tr>
<td>0.775</td>
<td>3.893</td>
<td>0.936</td>
<td>0.052</td>
<td>0.547</td>
</tr>
</tbody>
</table>

Fig. 2. (a) Angle of attack with time; (b) Radial distance from center of earth with respect to time.

5. Result and Discussions

In this section, the results of simulation done on Matlab environment is explained. Two different type of inertia weight is considered in solving procedure of this optimization problem. The first one is the linearly decreasing inertia weight and the other is the simulated annealing inertia weight which will encourage more convergence to the problem. Apart from these two the problem is analysed with fixed inertia weight to analyse the effect. Trajectory is designed for a single stage liquid launch vehicle and it is optimized for minimize the terminal error using the angle of attack as control variable. Proper care is taken in the trajectory optimization formulation to ensure that the structural load doesn’t exceed the maximum allowable limit. This is achieved by forcing angle of attack to a maximum allowable value at that instant.

In this problem minimizing the terminal error is the objective and all others are considered as constraints. The objective function can be written as,
\[ J = (r_{tf} - r_f)^2 s_r + (v_{tf} - v_f)^2 s_v + (\gamma_{tf} - \gamma_f)^2 s_\gamma \]
and the parameters, \( s_r, s_v \) and \( s_\gamma \) are used to allocating the importance of each variable. The PSO parameters are assigned by the basis on reference. The particles are selected randomly with in the boundary. The particles update will be done using equation (8) & (9) based on the fitness evaluation. The results are presented using different inertia weight. The fixed inertia values are chosen from the interval [0.35 0.8].

Table 1 shows the terminal error occurred with respect change in the inertia weight and shows that the desired terminal conditions are achieved with an error of less than 1%. The variation of the control variable with respect to time is shown in Fig. 2(a). From the figure it can be seen that the usage of linearly decreasing inertia weight reduces the control effort as compared to other method and fixed inertia weight. Also the upper and lower boundaries of the control parameter are less in the linearly decreasing inertia weight.

Figure 2(b) shows the radial distance from the center of earth with respect to time for different values of inertia weight. Figure 3(a), 3(b) and 4 represents the variation of velocity, flight path angle and mass of the vehicle with respect to time respectively. In these figures the star mark indicates the terminal conditions of each variable. From the results it is clear that the PSO with linearly decreasing inertia weight is having better performance than the others.

By analysing Fig. 2(b) it is clear that the launch vehicle is achieving the target position (Final radial distance between the earth centre) at around 180 seconds and after that the launch vehicle is moving further to the higher altitude and
coming down. This is due to that at the time 180 sec the target velocity is not achieved so the vehicle went to higher altitude and achieved the target velocity on the final time.

6. Conclusion

Considering the liquid propellant constant thrust launch vehicle, the ascent phase trajectory optimization has been carried out. PSO is used to solve the problem to minimize the terminal condition error by using angle of attack as the control variable. It is shown that the PSO with adaptive inertia weight scheme used to develop the continuous control history is having better efficiency than others. The results show that the vehicle achieved the terminal conditions with higher accuracy. This approach helps to achieve the near-optimal solution without calculating derivatives of the Hamiltonian function.

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References


