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## Fatigue strength of HFMI-treated high-strength steel joints under constant and variable amplitude block loading

M. Leitner\*, S. Gerstbrein, M.J. Ottersböck, M. Stoschka

*Montanuniversität Leoben, Chair of Mechanical Engineering, Franz-Josef-Strasse 18, 8700 Leoben, Austria*

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### Abstract

Lightweight-design of welded high-strength steel structures in cyclic service necessitates the use of post-treatment methods like the high frequency mechanical impact treatment (HFMI). Service loads during operation mostly consist of variable amplitudes, whereat recommendations are only available for the as-welded condition. Therefore, this paper deals with the effect of variable amplitude block loading on the fatigue strength of HFMI-treated T-joints. An evaluation of the real damage sum exhibits characteristic distinctions to constant amplitude test results in regard to the base material strength. The application of an equivalent stress range method by nominal and effective notch stress approach is finally presented.

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*Keywords: Fatigue of welded joints; High-strength steel, HFMI-treatment, Variable amplitude block loading*

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### 1. Introduction

It is well known that the fatigue strength of welded structures is in general independent from the material base strength. Nevertheless, in case of high-strength steels, it is possible to increase the fatigue behavior by additional post treatment processes significantly. In the finite lifetime region, the fatigue behavior of welded high-strength steel joints is beneficial due to the increased yield limit. In regard to the high-cycle fatigue zone, the notch topography, the microstructure in the heat-affected-zone (HAZ), and finally the residual stress state influence the fatigue lifetime in a major way.

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\* Corresponding author. Tel.: +43-3842-402-1463; fax: +43-3842-402-1402.

*E-mail address:* [martin.leitner@unileoben.ac.at](mailto:martin.leitner@unileoben.ac.at)

According to the conservative IIW-recommendation [1], the fatigue life of welded structures is mostly independent of the base material yield strength. To assess the local fatigue strength of welded and HFMI post-treated high-strength steel joints, fatigue tests with thin-walled ( $t=5\text{ mm}$ ) welded T-joint specimens are investigated in this study. Fig. 1a illustrates the T-joint specimen geometry and a micrograph of the heat-affected-zone cross-section.

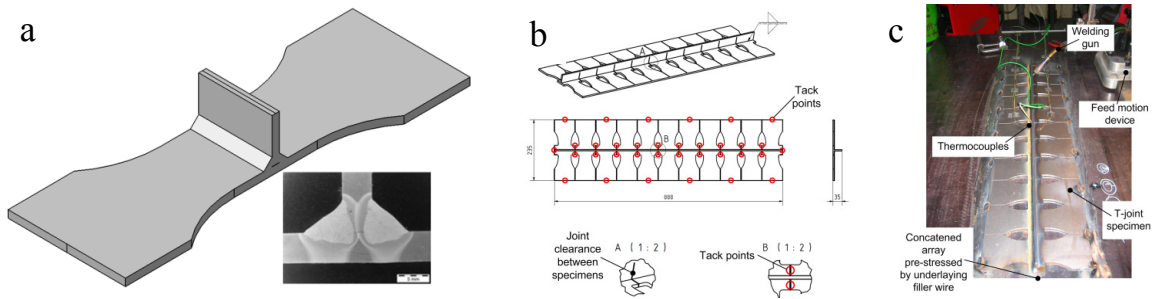


Fig. 1. (a) T-joint specimen for fatigue tests [2]; (b, c) Manufacturing process of T-joint specimen array [3].

Summing up, two low-alloyed high-strength steels *S690* and *S960*, and for comparative purposes a common construction steel *S355*, are used as different base materials. To guarantee an utmost manufacturing quality of each weld process variant under the observance of the welding process parameters, the introduced specimens are lined up to an array which is shown in Fig. 1b. This array is continuously welded on the metal base plate with two fillet welds (Fig. 1c) and the overlaying lateral attachments are subsequently truncated, machined and ground to plate. Further information in regard to the specimen manufacturing including detailed process data about the welding process are given in [3]. In the nominal stress concept, the fatigue strength enhancement for high-strength steel joints ( $f_y > 355\text{ MPa}$ ) improved by hammer or needle peening is expressed by a bonus factor of 1.5 [1]. An overview of the well-known post-weld treatment methods, their application and the recommended benefit in fatigue is given in [4]. In addition, recent research results [2, 5] lead to the observation that the fatigue strength of improved HFMI-treated welds increases with material yield strength. In [5], an extensive study based on constant amplitude fatigue tests exhibits that the benefit in fatigue can be expressed by an increase of 12.5 % for every 200 MPa material strength growth, choosing  $f_y = 355\text{ MPa}$  as base material strength reference.

## Nomenclature

$\Delta\sigma$	Stress range ( $\Delta\sigma_n$ based on nominal and $\Delta\sigma_k$ based on notch stress) [MPa]
D	Damage sum according to Palmgren-Miner-Rule [-]
k	Slope of Woehler- and Gassner-curve [-]
$L_S$	Spectrum size [-]
$N_k$	Transition knee point of S/N-curve [-]
R	Stress ratio [-]
$T_S$	Scatter band between 10 % and 90 % survival probability [-]

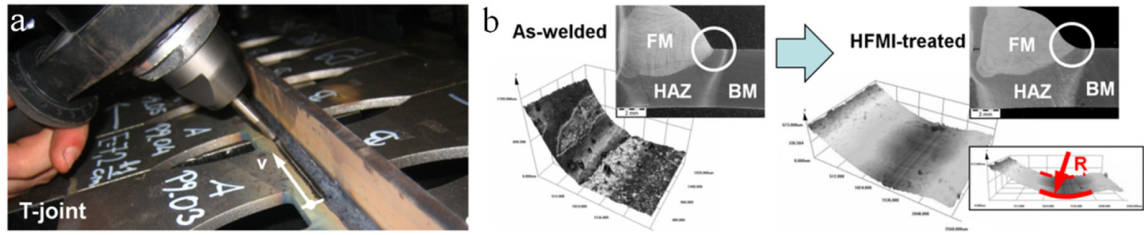
## 2. Effects of HFMI-treatment

### 2.1. Notch topography

Subsequent to the welding process the specimens are post-treated by the HFMI-technology, see Fig. 2a. The radius of the hardened pin applied in this study equals  $R=2\text{ mm}$  which fits to the weld seam size of the investigated specimens geometry. To achieve a comparison of the weld toe topography the surface of the weld toe region is both non-destructively inspected by laser-scanning-confocal (LSCM) and light-optical-microscopy (LOM). Geometric parameters like weld toe radius, undercut and notch depth are evaluated to gain topographical information.

In regard to the HFMI-treated joints, a significant increase of the weld toe radius is measurable which beneficially reduces the geometrical notch effect. Fig. 2b displays the local topography for both manufacturing conditions.

Fig. 2. (a) HFMI-treatment of T-joint specimens; (b) Improvement of notch topography at weld toe [6].



## 2.2. Residual stress condition

Another main effect explaining the increased fatigue behavior due to the HFMI-treatment is based on a change of the local residual stress condition. Therefore, X-ray diffraction measurements for as-welded and HFMI-treated joints are accomplished. Fig. 3 compares the residual stress change at the surface and in depth for the investigated high-strength *S960* T-joints. A significant beneficial reduction of the local residual stress is accomplished due to this post-treatment.

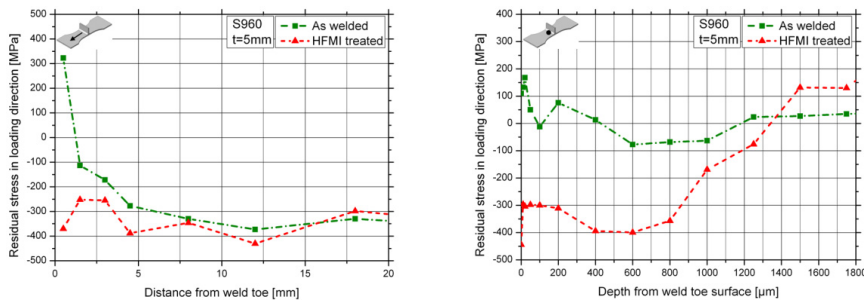


Fig. 3. Residual stress measurement results for high-strength *S960* T-joint specimen [7].

## 2.3. Hardness distribution

Finally, the third effect of the HFMI-treatment is based on a change of the local hardness within the cold-work hardened volume. Preliminary studies [3, 8] showed that different filler metals and shielding gases also strongly affect the local macro-hardness. But the subsequent HFMI-treatment additionally increases the local hardness by 5 % up to a value of  $360\text{ HV}_3$  in the area beneath the treated weld toe. Further investigations in [9] show similar tendencies within an edge layer depth of 0.2 to 0.3 mm.

## 3. Fatigue tests

### 3.1. Testing procedure and statistical methods

All specimens are tested until total burst rupture with a tumescent testing load involving a stress ratio of  $R=0.1$ . The fatigue assessment in the finite life regime is performed by using the evaluation procedure according to [10]. To determine the endurance stress range in the high-cycle fatigue region, the arcsin-sqrt transformation [11] is applied at constant number of fifty million load cycles to determine the fatigue limit. The fatigue assessment in the high cycle fatigue region is done assuming a second slope of  $k_2=22$  as suggested for constant-amplitude tests in [1]. For comparison purposes the recommended S/N-curve for the as-welded (nominal FAT-class of  $80\text{ MPa}$ ) and HFMI-treated condition, enhanced by a benign thinness factor discussed in [1, 3], are additionally displayed.

### 3.2. Constant amplitude tests

The evaluated nominal T-joint Woehler-curves for different steel grades are pictured in Fig. 4. At first, one can see that the as-welded fatigue test samples are clearly above the recommended curves which proof the applicability of the benign thinness factor for the examined high-quality manufactured joints within the nominal stress approach.

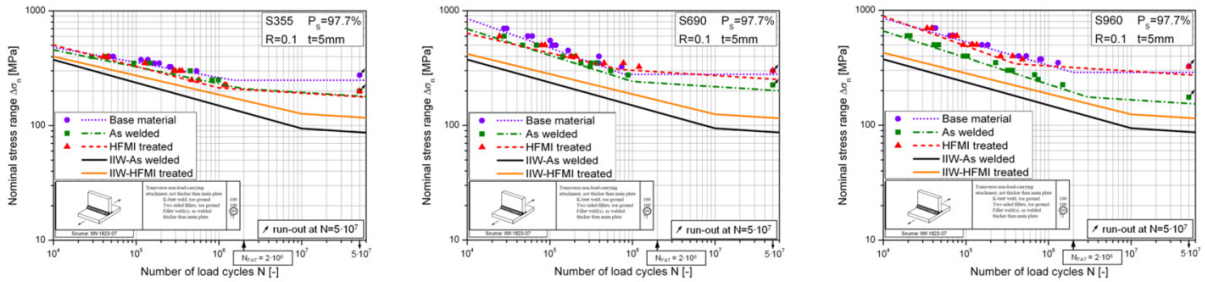


Fig. 4. Nominal S/N-curves for constant amplitude loading [2].

A comparison of the nominal stress fatigue test results show that in case of the common construction steel *S355* only a minor enhancement of the fatigue behavior due to the HFMI-treatment compared to the as-welded condition is achieved. For both conditions, the endurance limit is already about 85 % of the base material implying a high quality of the weld process which is caused by the use of optimized weld process parameters [3].

For the high-strength steel *S690* T-joint specimen, the nominal S/N-curves depict a slight increase of about 25 % in the fatigue strength due to the HFMI-treatment. With a further rise of the base material yield strength up to *960 MPa*, the potential of the HFMI-treatment increases further on. Hence, an increase of about 60 % compared to the as-welded condition can be evaluated. After the HFMI-treatment, the *S960* T-joint specimens show a fatigue resistance almost similar to that of the untreated base material in the low- and even in the high-cycle fatigue regime.

Summing up, the applied post-treatment leads to a significant fatigue enhancement, especially in the high-cycle fatigue region due to a shift of the transition knee point  $N_k$  to lower load-cycles. In case of the common constructions steel *S355*, the HFMI-treated specimens exhibits a fatigue strength of about 85 % and the two analyzed high-strength steels *S690* and *S960* are almost equal to the base material. All test series exceed both the recommended, and by a benign thinness factor enhanced, allowable stress ranges significantly and the evaluated scatter bands for the stress level  $T_S$  are within the limits of  $T_S=1:1.5$ . Further details about the fatigue test results are given in [2] and the S/N-curve parameters are summarized in Table 1.

Table 1. S/N-curve parameters for HFMI-treated condition at constant amplitude load

Base material	$\Delta\sigma_n$ (2e6) [MPa]	$k_1$ [-]	$N_k$ [-]	$1/T_S$ [-]
S355	210	5.3	$1 \cdot 10^6$	1.12
S690	300	5.9	$7 \cdot 10^5$	1.13
S960	315	3.9	$4 \cdot 10^5$	1.12

### 3.3. Variable amplitude block tests

For welded joints in the as-welded condition recommendations are available in regard to variable amplitude loadings, see [1]. Thereby, a cumulative damage sum of  $D=0.5$  is advised but due to insufficient test results even this value may be sometimes non-conservative. Based on investigations in [12] involving fatigue tests on butt joints and transverse stiffeners ranging from mild to high-strength steel, a damage sum of  $D=0.3$  for fatigue life calculations of welded joints is proposed. It has been also observed that in case of mean stress fluctuations [13, 14], the damage sum may be even lower, possibly down to  $D=0.2$  [1]. In addition, overload effects may influence the fatigue life significantly. Comprehensive fatigue test results in [15] including different weld geometries and steel grades showed that overloads harmed only the low-strength joints under pulsating bending, but in contrary all other investigated test series exhibited a significant increase in fatigue life.

For HFMI post-treated joints only a few variable amplitude test results currently exist in literature. The influence of variable amplitude loading is examined in [16] and the effect of high stress-ratio is investigated in [17] for HFMI-treated high-strength steel joints. It is indicated that all of the HFMI-improved welds satisfy the drafted fatigue assessment guidelines [18] based on both the yield strength and specimen geometry.

An extensive overview and practical engineering information about performing variable amplitude tests is provided in [19]. A procedure to evaluate the real damage sum by comparing Woehler- and Gassner-line is shown in Fig. 5a. Thereby, the damage content of a certain load-spectrum  $D_{spec.}$  with the size  $L_s$  can be simply determined based on the Palmgren-Miner-Rule modified by Haibach [1, 20]. Further on, the real damage sum  $D_{real}$  is calculated from the experimental fatigue life  $N_{exper.}$ . The applied load-spectra for the variable amplitude block tests within this paper is pictured in Fig. 5b whereat the spectrum size  $L_s=125.000$  load-cycles.

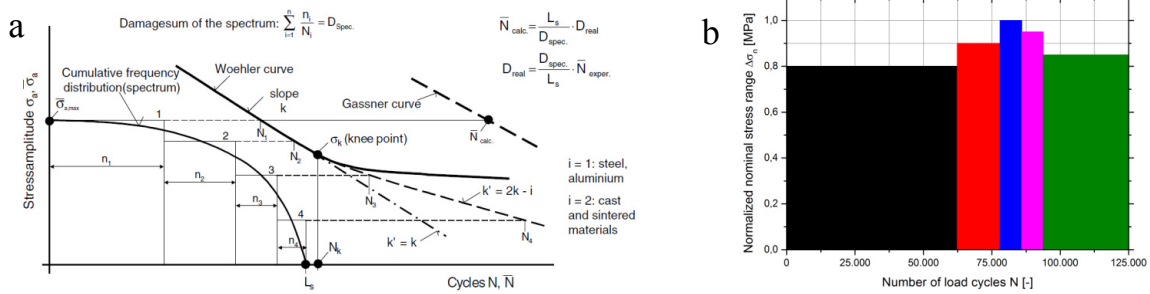


Fig. 5. (a) Calculation of fatigue life for variable amplitude loading [19]; (b) Applied load-spectra for variable amplitude block tests.

The results of the investigated variable amplitude block tests for the HFMI-treated T-joints with different base material strengths are illustrated in Fig. 6 by an assessment of the maximum (peak) nominal stress range  $\Delta\sigma_{n,max}$ .

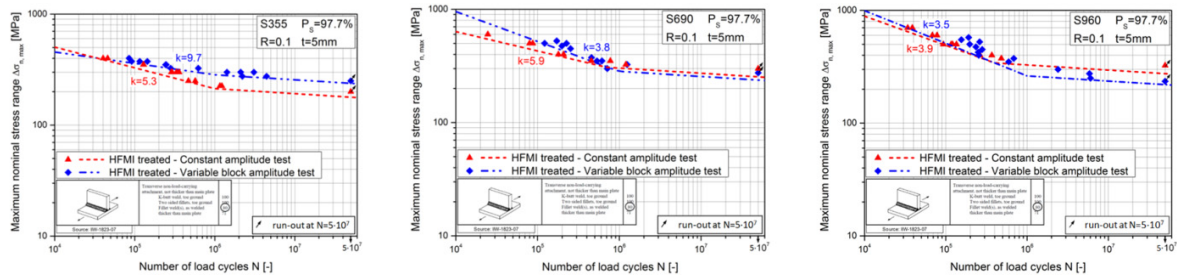


Fig. 6. Nominal S/N-curves for variable amplitude block loading - assessed with maximum (peak) nominal stress range  $\Delta\sigma_{n,max}$ .

An overview of the S/N-curve parameters for variable amplitude block tests is given in Table 2.

Table 2. S/N-curve parameters for HFMI-treated condition at variable amplitude block load

Base material	$\Delta\sigma_n (2\epsilon6)$ [MPa]	$k_1$ [-]	$N_k$ [-]	$1/T_s$ [-]
S355	275	9.7	$1 \cdot 10^6$	1.08
S690	275	3.8	$1 \cdot 10^6$	1.14
S960	250	3.5	$1 \cdot 10^6$	1.31

A comparison of constant and variable amplitude block tests shows significant differences in regard to the base material strength of the investigated T-joints. For the mild steel *S355*, the fatigue behavior of the variable amplitude block loading is clearly above the constant amplitude tests, especially in the high-cycle fatigue region. But in case of the high-strength steel *S690* only a slight increase in the finite life regime can be observed. For *S960* the fatigue strength in the finite lifetime region is almost identical but for higher load-cycles above the transition knee point the

fatigue strength due to variable amplitude block loading is even beneath the constant test results. A calculation of the real damage sum at one specific load-level in the finite lifetime region for each steel grade clearly presents the influence of the base material strength, see Table 3.

Table 3. Evaluation of real damage sum in finite lifetime regime.

Real damage sum	S355 ( $\Delta\sigma_{n,max}=300$ MPa)	S690 ( $\Delta\sigma_{n,max}=400$ MPa)	S960 ( $\Delta\sigma_{n,max}=400$ MPa)
$D_{real}$ [-]	1.60	0.74	0.63

Mild steel S355 exhibiting a more ductile material behavior shows a damage sum which is clearly above one and hence variable amplitude block loading investigated within this work does not reduce the fatigue strength. In contrary, for high-strength steels S690 and S960 the real damage sums are substantially beneath one, close to the recommended value of  $D_{real}=0.5$  for welded joints [1]. Fatigue test results in [12] showed that for more than 90 % of the analyzed data base the real damage sum is between  $1/3 < D_{real} < 3$  and based on these results the conservative value of  $D_{real}=0.3$  is proposed. To compare constant and variable amplitude tests more directly an evaluation of an equivalent constant amplitude stress range  $\Delta\sigma_{equ}$  is necessary. According to [1], the equivalent stress range  $\Delta\sigma_{equ}$  is calculated as shown in equation (1), at which  $k_1$  represents the slope in the finite life and  $k_2$  in the high-cycle fatigue region,  $n_i$  the number of load-cycles above and  $n_j$  beneath the transition knee point,  $\Delta\sigma_i$  the stress range above and  $\Delta\sigma_j$  beneath the transition knee point stress range  $\Delta\sigma_L$  and  $D$  the specified damage sum.

$$\Delta\sigma_{equ} = k_1 \sqrt{\frac{1}{D} \cdot \frac{\sum (n_i \cdot \Delta\sigma_i^{k_1}) + \Delta\sigma_L^{(k_1-k_2)} \cdot \sum (n_j \cdot \Delta\sigma_j^{k_2})}{\sum n_i + \sum n_j}} \quad (1)$$

Adapted from the variable amplitude block results in Fig. 6, the influence of the specified damage sum  $D$  ranging from  $D=1.0$  to  $0.3$  in case of an assessment based on the nominal equivalent stress range  $\Delta\sigma_{n, equ}$  for the high-strength S960 T-joint specimens is depicted in Fig. 7.

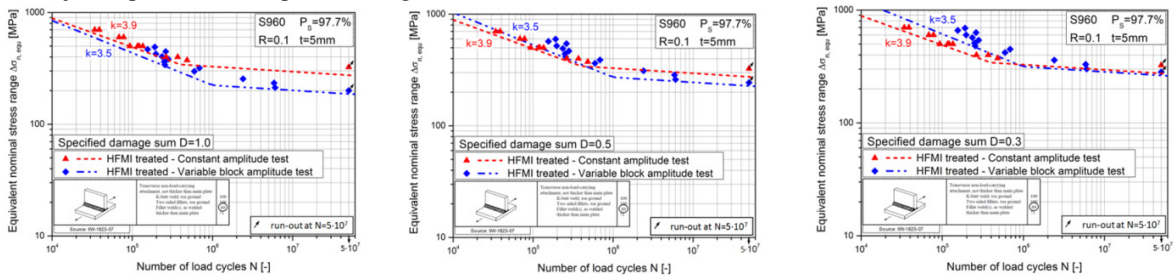


Fig. 7. Nominal S/N-curves for S960 T-joints under variable amplitude block loading - assessed with equivalent nominal stress range  $\Delta\sigma_{n, equ}$  assuming a damage sum of  $D=1.0, 0.5$  and  $0.3$ .

An assumption of damage sum  $D=1.0$  shows that a distinctive difference between constant and variable amplitude block results can be observed, especially on the high-cycle fatigue region. A reduction down to  $D=0.5$  as recommended in [1] increases the curve for variable amplitude block loading and the results are in good accordance to the examined data for constant amplitude testing, particularly in the finite lifetime regime. Finally, a value of  $D=0.3$  as proposed in [12] leads to an improved agreement in the high-cycle fatigue region but in the finite-life region the assessment is a bit conservative. Summing up, a damage sum of  $0.3 < D < 0.5$  between the proposed and recommended values for welded joints seems to fit best for the investigated test series.

#### 4. Local fatigue assessment

For the calculation of the fatigue behavior of welded structures different design concepts are available [1, 21]. Global approaches like the nominal stress approach use the nominal stress range with the corresponding notch cases.

The notch stress approach, which is common in industrial applications, takes the notch effect of the weld toe or root into account and is recommended for the estimation of the fatigue strength of complex structures [22]. For the local fatigue assessment a numerical model of the investigated T-joint with a recommended element type and mesh size [23] is used to determine the stress concentration factor by applying a reference radius of  $r_{ref}=1\text{ mm}$  at the weld toe [24, 25]. The numerical result is shown in Fig. 8a whereat a stress concentration of about  $K_{t,r=1mm}=1.7$  is evaluated.

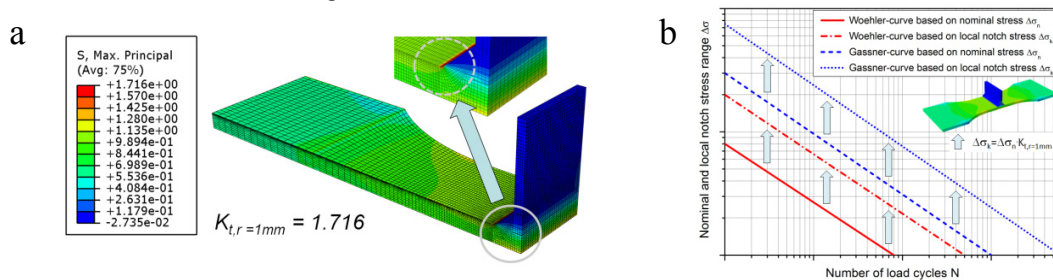


Fig. 8. (a) Numerical calculation of stress concentration factor; (b) Translation from nominal to local notch stress assessment for variable amplitude loading, according to [12].

Based on the evaluated stress concentration at the weld toe of the T-joint specimen a translation of the nominal to local notch fatigue strength is simply possible, see Fig. 8b. Thereby, the nominal Woehler- and Gassner-curves are multiplied by  $K_{t,r=1mm}$  and further on a local assessment of constant and variable amplitude loaded welded structures based on the effective notch stress concept is feasible. Especially in case of welded joints, the superposition of residual and load based stresses leads to a local yielding of the material and therefore plastic deformations occur, whereat the application of the notch strain approach may be more suitable. In [26] a comparison of the notch stress and strain approach by a numerical assessment based on a manufacturing process simulation is presented. Thereby, the application of the notch strain method, as defined by [27, 28, 29], shows a good accordance to experimental results and proves the applicability of this concept. To assess the lifetime of welded joints more precisely, an additional focus has to be laid on the crack propagation stage by the aid of fracture mechanical calculations. For welded joints, complex residual stress conditions at the crack tip influence the crack growth rate during fatigue life significantly. Based on investigations in [30], a definition of the local stress ratio at the crack tip [31] enables the consideration of the residual stress state on the applied range of the stress intensity factor and therefore improves the fatigue assessment of welded and post-treated joints.

## 5. Conclusion

Constant fatigue tests for welded T-joint specimens present a high potential of the HFMI-treatment to enhance the fatigue behavior of welded steel structures. Additional variable amplitude block tests show a significant difference in the evaluated fatigue strength. A calculation of the real damage sum based on the procedure presented in [19] illustrates that for the investigated high-strength steel joints the assessed values are clearly beneath  $D_{real}=1.0$  and closer to the recommendation of  $D=0.5$  as given in [1]. A detailed equivalent stress range assessment studies the effect of three different assumed damage sums  $D=1.0$ ,  $0.5$  and  $0.3$  on the fatigue behavior. Thereby,  $D=0.5$  shows a good accordance in the finite life and  $D=0.3$  in the high-cycle fatigue region to the constant amplitude test results.

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