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Wrapping at four loops in $\mathcal{N}=4$ SYM

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We present the planar four-loop anomalous dimension of the composite operator $\mathrm{tr}(\phi[Z,\phi]Z)$ in the flavour $SU(2)$ sector of the $\mathcal{N}=4$ SYM theory. At this loop order wrapping interactions are present: they give rise to contributions proportional to $\zeta(5)$ increasing the level of transcendentality of the anomalous dimension. In a sequel of this Letter all the details of our calculation will be reported.

1. Introduction

After the advent of the AdS/CFT conjecture [1] there has been a renewed interest in $\mathcal{N}=4$ SYM theory, which represents one of the best playgrounds to test new ideas connected to non-perturbative results.

The first prediction that one would like to test is the matching of the spectra on the two sides of the conjecture. In fact we expect that the spectrum of the anomalous dimensions of gauge invariant operators of the planar $\mathcal{N}=4$ SYM theory matches the spectrum of strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

Thus it is very important to have tools for the computation of anomalous dimensions. A big progress in this direction has been made in the last five years after the realization [2,3] that the planar one-loop dilatation operator of $\mathcal{N}=4$ SYM maps into the Hamiltonian of an integrable spin chain. The spin chain picture revealed itself very fruitful in understanding the integrability properties of higher orders in perturbation theory [4,5]. In addition it suggested to compute anomalous dimensions while finding solutions of associated Bethe equations [6]. The form of these equations has been recently refined with the introduction of the so-called dressing phase [7–10].

Now the hope of a direct comparison between the spectra on the two sides of the AdS/CFT correspondence is definitely more concrete. However a major obstacle in pursuing this program is due to the fact that the Bethe ansatz is asymptotic, i.e. it applies only to long operators. Indeed the spin chain Hamiltonian is long range: at a given perturbative order K in the coupling constant

$$g = \frac{\sqrt{\lambda}}{4\pi} \tag{1.1}$$

(where $\lambda = g_{\mathrm{YM}}^2 N$ is the 't Hooft coupling) the range of the interactions between adjacent sites grows with the perturbative order as $K+1$. For an operator of length L we should expect new effects when the range exceeds L. The asymptotic Bethe ansatz breaks down at orders $K \geqslant L$ since the interaction is no longer localized in some limited region along the state and asymptotic states cannot be defined. This spreading of the interaction manifests itself with the insurgence of a new type of contributions, the so-called *wrapping interactions*.

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Several papers have addressed this issue. In [11] the properties of wrapping interactions have been worked out in terms of Feynman diagrams. In [12] it was proposed that the thermodynamic Bethe ansatz captures the wrapping interactions. This work has been extended in [13,14]. In [15] it was assumed that wrapping might be described by the Hubbard model. Finally by using restrictions from the BFKL equation [16–18], a quantitative proposal for the anomalous dimensions at four-loop order with wrapping effects was conjectured in [19].

The aim of this Letter is to shed light on the present situation: we perform an explicit field theoretical anomalous dimension computation for the length-four Konishi descendant $\mathrm{tr}(\phi[Z,\phi]Z)$ at four loops. This is the simplest case in which wrapping effects are present. Different conjectures on the value of this anomalous dimension were proposed in [10,15,19].

A complete four-loop Feynman graph calculation is terribly complicated, so we had to find a simplifying strategy in order to proceed. The diagrams are naturally divided in two classes: four-loop graphs with no wrapping and graphs with wrapping interactions. We obtain the contributions from the two classes as follows:

First we take advantage of the known form of the four-loop dilatation operator D_4 given in [20]: all the non-wrapping contributions can be obtained by subtracting from D_4 its range 5 part. The remaining terms contain all the contributions with range from 1 to 4, so they can be applied safely to our length four operator. In this way we avoid the explicit computation of this vast and difficult class of Feynman graphs. This will be done in Section 2.

We consider wrapping interactions in Section 3. Many diagrams need to be considered but $\mathcal{N}=1$ supergraph techniques allow us to drastically simplify the calculation. After D-algebra manipulations the diagrams are reduced to standard four-loop momentum integrals which we compute by means of uniqueness and the Gegenbauer polynomial x-space technique [21,22].

Finally in Section 4 we collect all the terms and compute the four-loop planar anomalous dimension of the length four Konishi descendant. Our result shows that previous conjectures do not reproduce the correct anomalous dimension. In particular we find that at this loop order wrapping interactions give rise to contributions proportional to $\zeta(5)$ increasing the level of transcendentality of the anomalous dimension. In the following we describe the various steps that allowed us to reach the final result, while the details of the calculation will be reported in a separate publication [23].

2. Subtraction of range five interactions

In this section we compute the contributions to the anomalous dimension due to four-loop non-wrapping graphs. To this end we consider the four-loop planar dilatation operator in the $SU(2)$ subsector containing all operators made of two out of the three complex scalars of $\mathcal{N}=4$ SYM, denoted by ϕ and Z. It is given by [20]

$$\begin{aligned}
D_4 = {} & -(560+4\beta)\{\,\} \\
& + (1072+12\beta+8\epsilon_{3a})\{1\} \\
& - (84+6\beta+4\epsilon_{3a})\{1,3\} - 4\{1,4\} - (302+4\beta+8\epsilon_{3a})\big(\{1,2\}+\{2,1\}\big) \\
& + (4\beta+4\epsilon_{3a}+2i\epsilon_{3c}-4i\epsilon_{3d})\{1,3,2\} + (4\beta+4\epsilon_{3a}-2i\epsilon_{3c}+4i\epsilon_{3d})\{2,1,3\} \\
& + (4-2i\epsilon_{3c})\big(\{1,2,4\}+\{1,4,3\}\big) + (4+2i\epsilon_{3c})\big(\{1,3,4\}+\{2,1,4\}\big) + (96+4\epsilon_{3a})\big(\{1,2,3\}+\{3,2,1\}\big) \\
& - (12+2\beta+4\epsilon_{3a})\{2,1,3,2\} + (18+4\epsilon_{3a})\big(\{1,3,2,4\}+\{2,1,4,3\}\big) - (8+2\epsilon_{3a}+2i\epsilon_{3b})\big(\{1,2,4,3\}+\{1,4,3,2\}\big) \\
& - (8+2\epsilon_{3a}-2i\epsilon_{3b})\big(\{2,1,3,4\}+\{3,2,1,4\}\big) - 10\big(\{1,2,3,4\}+\{4,3,2,1\}\big),
\end{aligned} \tag{2.1}$$

where ϵ_{3a}, ϵ_{3b}, ϵ_{3c}, ϵ_{3d} parameterize the free choice of the renormalization scheme, and $\beta = 4\zeta(3)$ comes from the dressing phase. The permutation structures are defined as

$$\{a_1,\ldots,a_n\} = \sum_{r=0}^{L-1} \mathrm{P}_{a_1+r,a_1+r+1}\cdots \mathrm{P}_{a_n+r,a_n+r+1} \tag{2.2}$$

for the action on a cyclic state with L sites, where $\mathrm{P}_{a,a+1}$ permutes the flavours of the ath and $(a+1)$th site. Some rules for the manipulation of these structures can be found in [24].

In order to obtain the four-loop contributions we are interested in, we cannot use the expression (2.1) directly since it contains terms which describe the permutations among five neighbouring legs. Hence it can be applied only to a state in the asymptotic sense, i.e. the number of sites in the state has to be five or more. If we want to obtain the sum of all four-loop Feynman diagrams using D_4, we can correct it for the application on a length four state: the contributions from all the diagrams which describe the interactions of five neighbouring legs have to be replaced by the contributions from all four-loop wrapping interactions.

The flavour permutation structure of each Feynman diagram is completely determined by the scalar interactions. As will be explained in [23] the relevant flavour exchanges can be uniquely captured in terms of the four functions

$$\begin{aligned}
\chi(a,b,c,d) &= \{\,\} - 4\{1\} + \{a,b\} + \{a,c\} + \{a,d\} + \{b,c\} + \{b,d\} + \{c,d\} - \{a,b,c\} - \{a,b,d\} - \{a,c,d\} - \{b,c,d\} + \{a,b,c,d\}, \\
\chi(a,b,c) &= -\{\,\} + 3\{1\} - \{a,b\} - \{a,c\} - \{b,c\} + \{a,b,c\}, \\
\chi(a,b) &= \{\,\} - 2\{1\} + \{a,b\}, \\
\chi(1) &= -\{\,\} + \{1\},
\end{aligned} \tag{2.3}$$

where the number of arguments $a,b,c,d = 1,\ldots,4$ is given by the number of four-vertices. The independent flavour-exchange functions for the range five interactions are found by replacing a,b,c,d with the corresponding arguments of the range five permutation structures found in (2.1).

We have considered the contributions of all four-loop range five Feynman diagrams. As one important result we find that those diagrams, in which the first or the fifth line interacts with the rest of the graph only via flavour-neutral gauge bosons, cancel against each other. This means that the range five contributions can be extracted directly from D_4 given in (2.1). The terms we have to subtract from D_4 are uniquely given as a linear combination of the flavour exchange functions (2.3) which cancels all the range five permutation structures in (2.1). They read

$$\begin{aligned}\delta D_4 = {} & -10\big[\chi(1,2,3,4)+\chi(4,3,2,1)\big]+(18+4\epsilon_{3a})\big[\chi(1,3,2,4)+\chi(2,1,4,3)\big]-(8+2\epsilon_{3a}+2i\epsilon_{3b})\big[\chi(1,2,4,3)+\chi(1,4,3,2)\big]\\ & -(8+2\epsilon_{3a}-2i\epsilon_{3b})\big[\chi(2,1,3,4)+\chi(3,2,1,4)\big]-(4+4i\epsilon_{3b}+2i\epsilon_{3c})\big[\chi(1,2,4)+\chi(1,4,3)\big]\\ & -(4-4i\epsilon_{3b}-2i\epsilon_{3c})\big[\chi(1,3,4)+\chi(2,1,4)\big]-4\chi(1,4).\end{aligned} \tag{2.4}$$

Here we stress that the subtraction is not only a simple subtraction of all range five permutation structures as attempted in [25] for the BMN matrix model. The above subtraction modifies also the coefficients of the permutation structures of lower range in (2.1).

We have checked the above constructed range five contributions (2.4) with an explicit Feynman graph computation. In the used MS scheme we find

$$\epsilon_{3a}=-4, \qquad \epsilon_{3b}=-i\frac{4}{3}, \qquad \epsilon_{3c}=i\frac{4}{3}. \tag{2.5}$$

Now we can apply the subtracted dilatation operator to the states with $L=4$ sites. In the $SU(2)$ subsector there exist two composite operators of length $L=4$ and with two 'impurities' which mix under renormalization. The corresponding states are given by

$$\mathcal{O}_1=\operatorname{tr}(\phi Z\phi Z), \qquad \mathcal{O}_2=\operatorname{tr}(\phi\phi ZZ). \tag{2.6}$$

Defining a two-dimensional vector $\vec{\mathcal{O}}=(\mathcal{O}_1,\mathcal{O}_2)^{\mathrm{t}}$, the result for the subtracted dilatation operator becomes

$$D_4^{\mathrm{sub}} \equiv D_4-\delta D_4 \to 4\big(121+12\zeta(3)\big)M, \tag{2.7}$$

where the mixing matrix is given by

$$M=\begin{pmatrix}-4 & 4\\ 2 & -2\end{pmatrix}. \tag{2.8}$$

Now to the subtracted dilatation operator we add the wrapping interactions.

3. Wrapping interactions

In [11] a systematic Feynman-diagrammatic analysis of wrapping interactions has been performed. It was shown that the wrapping interactions have a genus changing effect. This means in particular that they are responsible for a non-trivial map between the planar part of the dilatation operator and the planar part of the 2-point functions of composite single trace operators. Also several unique properties of the wrapping interactions have been worked out, and a systematic method for projecting out them has been proposed. We want to make our calculation using $\mathcal{N}=1$ superspace techniques, hence we will adapt this direct approach to construct all wrapping supergraphs.

The $\mathcal{N}=4$ SYM action written in terms of $\mathcal{N}=1$ superfields is given by (we use notations and conventions of [26])

$$S=\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\operatorname{tr}\big(\mathrm{e}^{-g_{\mathrm{YM}}V}\,\bar{\phi}_i\,\mathrm{e}^{g_{\mathrm{YM}}V}\,\phi^i\big)+\frac{1}{2g_{\mathrm{YM}}^2}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\operatorname{tr}\big(W^\alpha W_\alpha\big)+\mathrm{i}g_{\mathrm{YM}}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\operatorname{tr}\big(\phi_1[\phi_2,\phi_3]\big)+\mathrm{h.c.} \tag{3.1}$$

where $W_\alpha=\mathrm{i}\bar{D}^2(e^{-g_{\mathrm{YM}}V}D_\alpha\,e^{g_{\mathrm{YM}}V})$, and $V=V^aT^a$, $\phi_i=\phi_i^aT^a$, $i=1,2,3$, T^a being $SU(N)$ matrices. We will usually denote the three chiral superfields ϕ^i as (ϕ,ψ,Z), using the same letters used before for their lowest components.

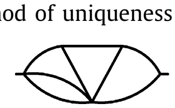
Following [11] we then proceed to construct all wrapping supergraphs contributing to the renormalization of the chiral operators (2.6). To do this, we rely on the following findings, specialized to the length $L=4$ diagrams

1. Wrapping diagrams which differ by an application of the cyclic rotations $C_4\times C_4$, acting respectively on the incoming and outgoing four legs, give the same contribution to the dilatation operator. They are just different graphical representations of a single diagram in the two-dimensional plane.
2. After removing the composite operator(s), the three-loop range four diagrams are simply-connected tree level graphs.
3. Wrapping diagrams containing at least one gauge boson line can be constructed by adding a single (wrapping) vector line to three-loop range four diagrams.
4. The remaining few fully chiral wrapping diagrams can be directly built by inspection of the Feynman rules.

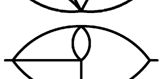
After constructing all diagrams of this type, we have to perform standard superspace D-algebra. This reduces the various contributions to ordinary massless four-loop momentum integrals. The anomalous dimension is then given in terms of the divergent part of these integrals. In a dimensional regularization approach it is given directly by the $\frac{1}{\varepsilon}$ pole.

We list here only the results of the calculation, leaving a detailed discussion for the forthcoming paper [23]. First we give a description of the needed integrals which have been computed using dimensional regularization with $D=4-2\varepsilon$ in the MS scheme.

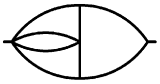
After completion of the D-algebra we produce both integrals without and with derivatives. The first ones are computed with the method of uniqueness [22]. We find for the overall poles, i.e. after subtraction of subdivergencies

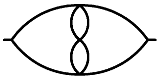


$$=\frac{1}{(4\pi)^8}\left(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{5}{4\varepsilon}\right),$$



$$=\frac{1}{(4\pi)^8}\left(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{5}{4}-\zeta(3)\right)\right),$$


$$= \frac{1}{(4\pi)^8}\left(-\frac{1}{12\varepsilon^4}+\frac{1}{3\varepsilon^3}-\frac{5}{12\varepsilon^2}-\frac{1}{\varepsilon}\left(\frac{1}{2}-\zeta(3)\right)\right),$$


$$= \frac{1}{(4\pi)^8}\left(-\frac{1}{6\varepsilon^4}+\frac{1}{3\varepsilon^3}+\frac{1}{3\varepsilon^2}-\frac{1}{\varepsilon}\big(1-\zeta(3)\big)\right). \tag{3.2}$$

The integrals with derivatives (i.e. with non-trivial numerators) which cannot be rephrased in terms of integrals of the previous type, are calculated with the Gegenbauer polynomial x-space technique (GPXT) [21], and are independently checked with the help of MINCER [27], a computer program to compute 3-loop integrals. Furthermore we have used the GPXT and MINCER to check once again the above results for the integrals without derivatives.

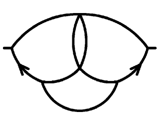
In particular the GPXT turns out to be very useful in the following situation:

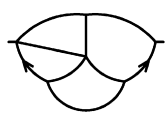
1. The integral contains a vertex at which a large number of propagators end (in our case provided by the composite operator) which is chosen as the 'root vertex'.
2. The corresponding angular graph is planar, and it only contains loops which can be resolved by a sequential use of the Clebsch–Gordan series for the Gegenbauer polynomials.
3. The integral can be rearranged such that the linear combinations of momenta in the numerator become the momenta of propagators which end at the root vertex.
4. Only the pole structures of the integrals are of interest.

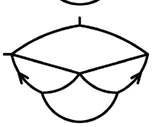
With the GPXT we can then find analytic expressions for the pole structure of the required four-loop integrals. In particular, the fact that we are only interested in the pole structure allows us to considerably simplify the procedure by introducing an alternative IR regularization procedure into the x-space integrals.

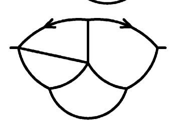
We will describe the GPXT, and its modifications in detail in [23].

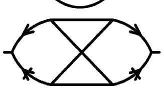
The overall poles of the integrals with a non-trivial numerator, which are required to compute the wrapping contributions, are given by

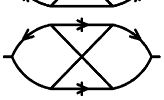

$$= \frac{1}{(4\pi)^8}\left(\frac{1}{12\varepsilon^2}-\frac{7}{12\varepsilon}\right),$$


$$= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\big(-\zeta(3)\big),$$


$$= \frac{1}{(4\pi)^8}\left(\frac{1}{4\varepsilon^2}-\frac{11}{12\varepsilon}\right),$$


$$= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\left(\frac{1}{2}\zeta(3)-\frac{5}{2}\zeta(5)\right),$$


$$= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\left(-\frac{1}{2}-\frac{1}{2}\zeta(3)+\frac{5}{2}\zeta(5)\right),$$


$$= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\left(-\frac{1}{4}-\frac{3}{2}\zeta(3)+\frac{5}{2}\zeta(5)\right),$$


$$= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\left(-\frac{1}{8}-\frac{1}{4}\zeta(3)+\frac{5}{4}\zeta(5)\right), \tag{3.3}$$

where two arrows of the same type indicate a scalar product of the corresponding momenta in the numerator.

We now go on and list the supergraphs. Among the potentially contributing diagrams a lot of cancellations already take place at intermediate steps of the D-algebra. We give in Fig. 1 a list of the supergraph structures with the corresponding integrals that are obtained after D-algebra. The shown results contain also the factors coming from combinatorics and colour.

We have distinguished purely chiral wrapping graphs (denoted by C) from the graphs W, which contain a number of gauge boson propagators (indicated by the number in front of the wiggled line) which have to be added to the corresponding chiral structure, such that the corresponding graph becomes a four-loop wrapping diagram. The complete list of all contributing diagrams will be given in [23]. The equalities in Fig. 1 present the explicit results in the $L=4$ basis (2.6) after inserting the values for the integrals.

Thus we find that the sum of all wrapping terms contributes to D_4 as

$$D_4^{\mathrm{w}} \to -8\left(\frac{17}{2}+18\zeta(3)-30\zeta(5)\right)M. \tag{3.4}$$

4. The final result

We now collect our results from the subtracted dilatation operator (2.7) and from the wrapping part (3.4). The total contribution is given by

$$D_4^{\mathrm{sub}}+D_4^{\mathrm{w}} \to \big(416-96\zeta(3)+240\zeta(5)\big)M. \tag{4.1}$$

The non-vanishing eigenvalue of this matrix is

$$\gamma_4 = -2496+576\zeta(3)-1440\zeta(5). \tag{4.2}$$

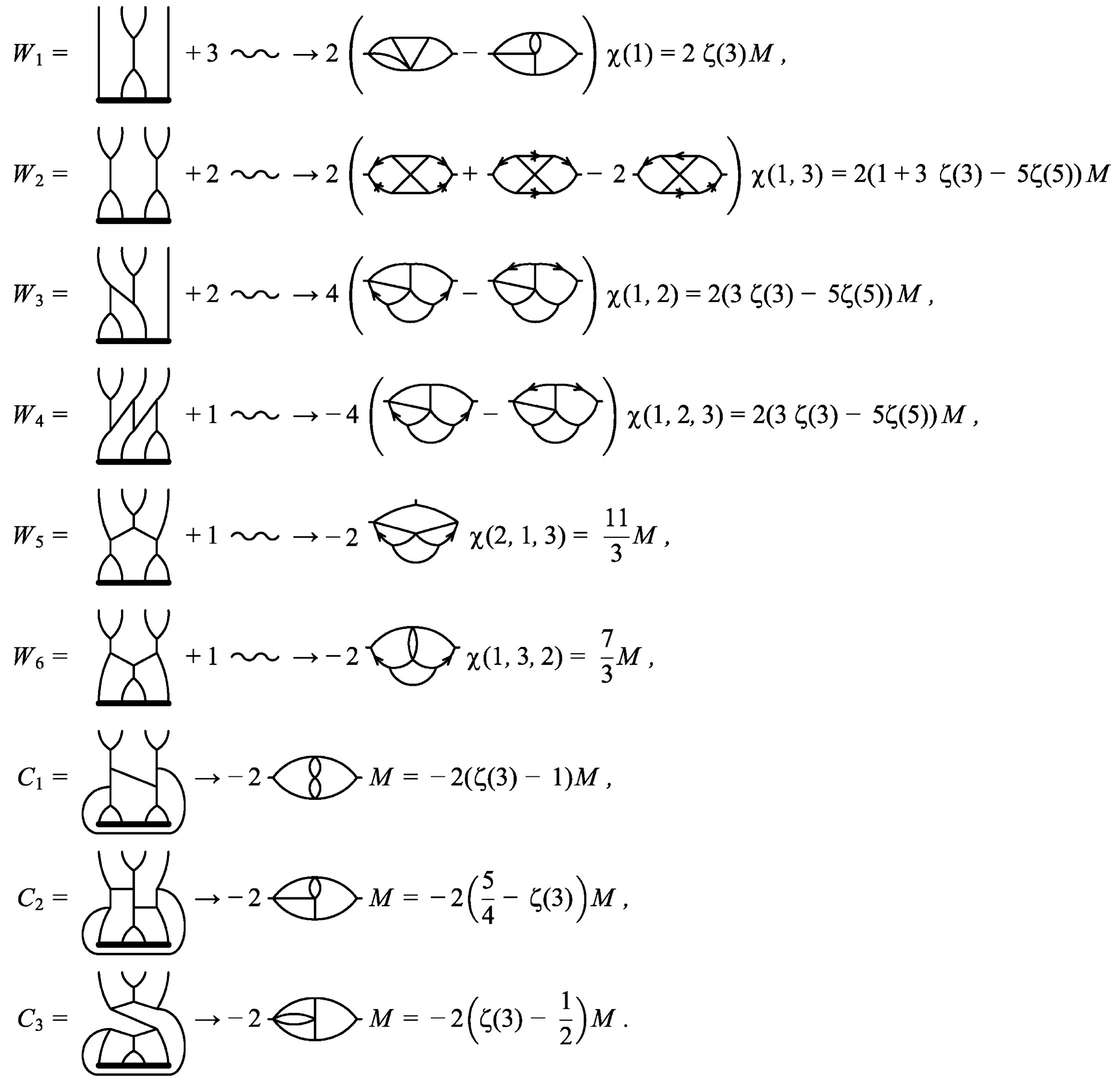


Fig. 1. Classes of four-loop wrapping graphs and their quantitative contributions. For the classes W_i, $i = 1, \ldots, 6$, only the underlying chiral structure is shown. It has to be completed to the wrapping graphs by appropriately adding the indicated number of gauge boson lines. For the chiral classes C_j, $j = 1, \ldots, 3$, all interactions are present.

Restoring the dependence on the coupling constant (1.1), and including also the contributions at lower orders [10], our final result for the planar anomalous dimension of the length four Konishi-descendant up to four loops reads

$$\gamma = 4 + 12g^2 - 48g^4 + 336g^6 + g^8\big(-2496 + 576\zeta(3) - 1440\zeta(5)\big). \tag{4.3}$$

We conclude with some comments. In our calculation we could partially save the knowledge of the asymptotic dilatation operator at four loops by suitably subtracting all range five contributions. The task was simplified by the cancellation of those range five Feynman graphs with the first or the last line interacting with the rest of the graph only via flavour-neutral gauge bosons. This makes the explicit evaluation of all range five Feynman graphs not necessary. We believe that the absence of these contributions persists also to higher orders. This would be very important to check, since it would allow us to directly determine the necessary subtractions in the case of D_K when applied to a length $L = K$ operator.

The use of $\mathcal{N} = 1$ supergraph techniques was very powerful for the explicit Feynman graph evaluation of the wrapping part of the calculation. We found the cancellation of the overwhelming majority of the potentially contributing supergraphs.

In the literature two different results for the Konishi anomalous dimension are conjectured on the basis of the Hubbard model [15] and on an analysis of the BFKL equation [19]. Our result obtained from explicit calculation differs from them. In particular, the presence of a term proportional to $\zeta(5)$ is new. The compatibility of this term with transcendentality principles deserves further investigation.

Note added in proof

We have corrected a factor of 2 missing in W_5 of Fig. 1 after the appearance of [28].

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