School Bus Problem and its Algorithm

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Abstract

This paper taking school bus routes running problem of Gansu Normal College for Nationalities as an example, discusses the school bus problem, given the mathematical model of route optimization, put forward an improved quantum-behaved particle swarm algorithm. Finally prove the model theory and effectiveness.

Keywords: School Bus Problem, Vehicle routing problem, Gansu Normal University for Nationalities, quantum-behaved particle swarm algorithm

1. Introduction

The school bus problem belongs to the vehicle routing problem, also affected by two factors: vehicle load constraint and time constraint. This kind of problem can be described as follows [1]: Given a directed graph \( G = (V, E) \), where \( V \) is the school bus stop collection, \( E \) is the set of edges, it is weighted edges, and each side has a service demand \( q_{ij} \geq 0 \ (i, j \) for two adjacent a stop, and \( i, j \in V \)), how to find a loop, all sides of demand of the loop are satisfied and the total service costs least? About the solution to the problem has accurate algorithm and heuristic algorithm, accurate algorithm has dynamic programming algorithm, nonlinear programming [2].

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heuristic algorithm: SFC\cite{3}, tabu search algorithm\cite{4}, genetic algorithm \cite{5}. In the solution of practical problems of large-scale general use heuristic algorithm.

2. The mathematical model of SCHOOL BUS problem

Mathematical models are as follows:

\[
\text{Min} \sum_{i \in V} \sum_{j \in V} \sum_{k \in S} x_{ijk} (c_{ij} + c_{ij}^{'}) + F
\]

s.t \[
\sum_{i \in V} \sum_{j \in V} \rho_i x_{ijk} \leq Q_k \quad \forall k \in S
\]

\[
\sum_{i \in V} x_{ijk} - \sum_{j \in V} x_{kj} = 0
\]

\[
\sum_{k \in S} \sum_{j \in V} x_{ijk} - z_i \geq 0
\]

\[
R_i - R_j + V \sum_{k \in S} x_{ijk} \leq V - 1 \quad \forall i, j \in V, i \neq j
\]

\[
x_{ijk} \in \{0,1\} \quad \forall i, j \in V, \forall k \in S, i \neq j
\]

\[
z_i \in \{0,1\} \quad \forall i \in V
\]

Among them: \(c_{ij}\) Express the average cost of transportation stop i to j, when the edge has Services; \(c_{ij}^{'}\) express the average cost of transportation stop i to j, when the edge hasn’t Services; \(x_{ijk}\) is the decision variables of service edge, when the vehicle K across stop i to stop j is 1, or it is 0; F is average fixed of driver; \(\rho_i\) is the service quantity of stop i; \(Q_k\) is the maximum capacity of vehicle K; \(z_i\) indicating whether or not stop at stop i, when stop is 1, or it is 0; V is the set of all docked points; S is the set of all vehicles.

Equation (1) meets the minimum required services; Equation (2) is transport capacity constraints, it meets each bus does not exceed its capacity in a line running; Equation (3) guarantees the route successive; Equation (4) ensures has at least one customer at each docking point; Equation (5) ensures that it has not internal closed loop on the vehicle routes; Equation (6) and Equation (7) satisfying the integer constraint.

3. Improved Algorithm of Quantum-behaved Particle Swarm

Quantum behaved particle swarm optimization algorithm is a heuristic algorithm, now it has been widely applied in solving the network routing, nonlinear equations \cite{6}. In this paper, according to the rule of school bus, based on the tradition of quantum-behaved particle swarm algorithm, join the loop scanning and analysis thought, this paper puts forward an improved quantum-behaved particle swarm algorithm to solve school bus problem. The algorithm can be expressed as:

Set parameters, including the maximum number of iterations, the population size \(P(t)=\{a_1(t), a_2(t),..., a_n(t)\}\), where t is the number of iterations, the article takes \(t = 2000\).

(1) Initialize the population of \(P(0)=\{a_1(0), a_2(0),..., a_n(0)\}\) and position vector of each particle;

(2) According to the fitness function calculation all the fitness of the particle \(P(0)=\{f(a_1(0)), f(a_2(0)),..., f(a_n(0))\}\). Adaptive value function as follow:
\[ f(i,j) = \sqrt{\sum_{k=1}^{n} (x_{ik} - x_{jk})^2} \]

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(3) According to the particle fitness, sequence the particle from high to low;

(4) According to the particle fitness, all current particle is divided into many groups, each subgroup of particles around with the best fitness of particles in this group;

(5) Using literature [7] mentioned loop point scanning method for scanning each subgroup;

\[ x(t+1) = p + \beta * (m_{best} - x_i) * \ln(\frac{1}{u}) \]

(6) According to the equation to change the position of the particle, where \( t \) is the number of iterations, \( \beta \) is the contraction of the expansion and coefficient, adjusting its value can control the speed of convergence, \( \beta = 0.5 * (Maxiter - t) / Maxiter + 0.5 \), where Maxiter is the maximum number of iterations, \( u = rand(0,1) \), \( p = \varphi * P_i + (1-\varphi) * P_g \), \( 0 < \varphi < 1 \), \( \varphi = rand(0,1) \), \( P \) is a bit extreme value, \( P_g \) is the global extreme value, \( m_{best} \) is local best position and the average value.

(7) Return to (2), until the condition is satisfied.

4. The test results

Data from Hezuo City (Gansu Normal College for Nationalities lies in Hezuo City), Bus running regional cooperation for the downtown of Hezuo City, docked points for 25, a unit distance transportation costs 1 yuan, the passenger volume of the bus is 80, maximum iterative algebra is 2000 in the algorithm, figure 1 is the final results, Figure 2 shows the algorithm of the paper and particle swarm algorithm optimization evolutionary comparison. From the above simulation data shows: this algorithm is very effective for school bus problem.

Figure 1 Gansu Normal College for Nationalities school bus running route map
5. Conclusions

This article proposed an improved quantum-behaved particle swarm algorithm based on school bus problem, this algorithm maintains the diversity of population, avoid the algorithm traps in local optimum effectively, and verified by examples, and proves that the algorithm is feasible and effective.

6. Acknowledgment

This work is supported by Gansu Provincial College graduate tutor of scientific research project (1112-09) and Dean Fund of Gansu Normal College for Nationalities (09-07).

References