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Selection principles and hyperspace topologies

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Abstract

In this paper we investigate relationships between closure-type properties of hyperspaces over a space X and covering properties of X.

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0. Introduction

Let X be a Hausdorff space. By 2^X we denote the family of closed subsets of X. If A is a subset of X and $\mathcal{A}$ a family of subsets of X, then we write

$$A^c = X \setminus A \quad \text{and} \quad \mathcal{A}^c = \{A^c\colon A \in \mathcal{A}\},$$
$$A^- = \{F \in 2^X\colon F \cap A \neq \emptyset\},$$
$$A^+ = \{F \in 2^X\colon F \subset A\}.$$

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The most known and popular among topologies on 2^X is the Vietoris topology $\mathsf{V} = \mathsf{V}^- \vee \mathsf{V}^+$, where the *lower Vietoris topology* V^- is generated by all sets A^-, $A \subset X$ open, and the *upper Vietoris topology* V^+ is generated by sets B^+, B open in X.

Let Δ be a subset of 2^X. Then the *upper Δ-topology*, denoted by Δ^+ [23] is the topology whose subbase is the collection

$$\{(D^c)^+ \colon D \in \Delta\} \cup \{2^X\}.$$

We consider here two important special cases:

(1) Δ is the family of all finite subsets of X, and
(2) Δ is the collection of compact subsets of X.

The corresponding Δ^+-topologies will be denoted by Z^+ and F^+, respectively and both have the collections of the above kind as basic sets. The F^+-topology is known as the *upper Fell topology* (or the *co-compact topology*) [6].

Let us fix some terminology and notation that we need.

Let $\mathcal{A}$ and $\mathcal{B}$ be sets whose elements are families of subsets of an infinite set X. Then (see [28,12]):

$\mathsf{S}_1(\mathcal{A}, \mathcal{B})$ denotes the selection principle:

For each sequence $(A_n \colon n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(b_n \colon n \in \mathbb{N})$ such that for each n $b_n \in A_n$ and $\{b_n \colon n \in \mathbb{N}\}$ is an element of $\mathcal{B}$.

$\mathsf{S}_{\mathrm{fin}}(\mathcal{A}, \mathcal{B})$ denotes the selection hypothesis:

For each sequence $(A_n \colon n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(B_n \colon n \in \mathbb{N})$ of finite sets such that for each n $B_n \subset A_n$ and $\bigcup_{n\in\mathbb{N}} B_n$ is an element of $\mathcal{B}$.

For a space (X, τ) and a point $x \in X$ we consider the following sets $\mathcal{A}$ and $\mathcal{B}$:

- $\mathcal{O}$: the collection of open covers of X;
- Ω: the collection of ω-covers of X;
- $\mathcal{K}$: the collection of k-covers of X;
- Ω_x^τ (or shortly Ω_x): the set $\{A \subset X \setminus \{x\} \colon x \in \overline{A}\}$.

Let us recall that an open cover $\mathcal{U}$ of a space X is called an *ω-cover* [7] (respectively a *k-cover* [19]) if every finite (respectively compact) subset of X is contained in a member of $\mathcal{U}$ and X is not a member of $\mathcal{U}$. (To avoid trivialities we suppose that considered spaces are infinite and non-compact.) Let us mention that $\mathcal{K} \subset \Omega$.

The property $\mathsf{S}_1(\mathcal{O}, \mathcal{O})$ was introduced by Rothberger in [24] and is called now the *Rothberger property* (see also [28,12,21]). The property $\mathsf{S}_{\mathrm{fin}}(\mathcal{O}, \mathcal{O})$ is known as the *Menger property* [20,9,21,28,12]. A space X has *countable fan tightness* [1,2] if for each $x \in X$ it satisfies $\mathsf{S}_{\mathrm{fin}}(\Omega_x, \Omega_x)$. X has *countable strong fan tightness* [25] if for each $x \in X$ the selection principle $\mathsf{S}_1(\Omega_x, \Omega_x)$ holds.

We consider only spaces having property that each ω-cover (k-cover) of X contains a countable ω-subcover (k-subcover), and so all covers are supposed to be *countable*.

A number of results in the literature show that for a Tychonoff space X closure properties of the function spaces $\mathsf{C}_p(X)$ and $\mathsf{C}_k(X)$ of continuous real-valued functions on X endowed with the topology of pointwise convergence or with the compact-open topology can be characterized by covering properties of X defined or characterized by using selection principles. We refer the reader to [1,2,7,25–27,16–19,29,22,13].

In this paper we show, in a similar spirit, that closure properties of spaces of closed subsets of a space X, endowed with appropriate topologies, can be characterized in terms of covering properties of X.

For similar investigation see [4,8,14,15].

In Section 1 we consider S_1 selection principles in hyperspaces and give complete proofs of the results, while in Section 2, in which we study $\mathsf{S}_{\mathrm{fin}}$ selection principles, the most of proofs are omitted being similar to the proofs of the corresponding results from Section 1. Section 3 contains results involving the notions of groupability and weak groupability introduced recently in the literature.

1. The Rothberger-like selection principles

In this section we study when hyperspaces $(2^X, \mathsf{Z}^+)$ and $(2^X, \mathsf{F}^+)$ have countable strong fan tightness. Related questions are also considered.

Theorem 1. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *has countable strong fan tightness*;
(2) *Each open set $Y \subset X$ satisfies* $\mathsf{S}_1(\Omega, \Omega)$.

Proof. (1) ⇒ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of ω-covers of Y. Then $(\mathcal{U}_n^c\colon n \in \mathbb{N})$ is a sequence of subsets of 2^X and $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{U}_n^c)$ for each $n \in \mathbb{N}$. Indeed, given n, let $(F^c)^+$ be a Z^+-neighborhood of Y^c. Then F is a finite subset of Y so that there is a $U \in \mathcal{U}_n$ such that $F \subset U$. Thus $Y^c \subset U^c \subset F^c$, i.e. $U^c \in (F^c)^+ \cap \mathcal{U}_n^c$. Therefore, $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{U}_n^c)$. Since $(2^X, \mathsf{Z}^+)$ has countable strong fan tightness there is a sequence $(U_n^c\colon n \in \mathbb{N})$ such that for each n, $U_n \in \mathcal{U}_n$ and $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\{U_n^c\colon n \in \mathbb{N}\})$. We claim that $\{U_n\colon n \in \mathbb{N}\}$ is an ω-cover of Y. Indeed, let S be a finite subset of Y. Then $(S^c)^+$ is a Z^+-neighborhood of Y^c and thus $(S^c)^+ \cap \{U_n^c\colon n \in \mathbb{N}\} \neq \emptyset$; let U_k^c be a member of this intersection. Then from $U_k^c \subset S^c$ it follows $S \subset U_k$.

(2) ⇒ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of subsets of 2^X such that a point $S \in 2^X$ belongs to the Z^+-closure of $\mathcal{A}_n$ for each n. Then, as can be easily verified, $(\mathcal{A}_n^c\colon n \in \mathbb{N})$ is a sequence of ω-covers of S^c, and because S^c is an $\mathsf{S}_1(\Omega, \Omega)$-set, there is a sequence $(A_n^c\colon n \in \mathbb{N})$ such that for each n, $A_n^c \in \mathcal{A}_n^c$ and $\{A_n^c\colon n \in \mathbb{N}\}$ is an ω-cover of S^c. It is equivalent to the assertion that $S \in \mathrm{Cl}_{\mathsf{Z}^+}(\{A_n\colon n \in \mathbb{N}\})$, so that the sequence $(A_n\colon n \in \mathbb{N})$ witnesses for $(\mathcal{A}_n\colon n \in \mathbb{N})$ that $(2^X, \mathsf{Z}^+)$ has countable strong fan tightness. □

Theorem 2. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *has countable strong fan tightness*;
(2) *Each open set* $Y \subset X$ *satisfies* $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of k-covers of Y. Then $(\mathcal{U}_n^c\colon n \in \mathbb{N})$ is a sequence of subsets of 2^X and $Y^c \in \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{U}_n^c)$ for each $n \in \mathbb{N}$. Indeed, fix n. Let K be a compact subset of Y. There is a $U \in \mathcal{U}_n$ such that $K \subset U \subset Y$ and thus $Y^c \subset U^c \subset K^c$, i.e. $U^c \in (K^c)^+$ and $Y^c \in (K^c)^+$. So, $U^c \in (K^c)^+ \cap \mathcal{U}_n^c$, i.e. $Y^c \in \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{U}_n^c)$. Since $(2^X, \mathsf{F}^+)$ has countable strong fan tightness there is a sequence $(U_n^c\colon n \in \mathbb{N})$ such that for each n, $U_n \in \mathcal{U}_n$ and $Y^c \in \mathrm{Cl}_{\mathsf{F}^+}(\{U_n^c\colon n \in \mathbb{N}\})$. We prove that $\{U_n\colon n \in \mathbb{N}\}$ is a k-cover of Y. Let K be a compact subset of Y. Then $(K^c)^+$ is an F^+-neighborhood of Y^c and thus $(K^c)^+ \cap \{U_n^c\colon n \in \mathbb{N}\} \neq \emptyset$; let U_m^c belongs to this intersection. Then $U_m^c \in (K^c)^+$ implies $K \subset U_m$.

(2) $\Rightarrow$ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of subsets of 2^X such that a point $S \in 2^X$ belongs to the F^+-closure of $\mathcal{A}_n$ for each n. It is not hard to prove that $(\mathcal{A}_n^c\colon n \in \mathbb{N})$ is a sequence of k-covers of S^c and since S^c is an $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$-set, there is a sequence $(A_n^c\colon n \in \mathbb{N})$ such that for each n, $A_n^c \in \mathcal{A}_n^c$ and $\{A_n^c\colon n \in \mathbb{N}\}$ is a k-cover of S^c. It is equivalent to $S \in \mathrm{Cl}_{\mathsf{F}^+}(\{A_n\colon n \in \mathbb{N}\})$, so that the sequence $(A_n\colon n \in \mathbb{N})$ witnesses for $(\mathcal{A}_n\colon n \in \mathbb{N})$ that $(2^X, \mathsf{F}^+)$ has countable strong fan tightness. □

Theorem 3. *For a space* X *the following statements are equivalent*:

(1) 2^X *satisfies* $\mathsf{S}_1(\Omega_A^{\mathsf{F}^+}, \Omega_A^{\mathsf{Z}^+})$ *for each* $A \in 2^X$;
(2) *Each open set* $Y \subset X$ *satisfies* $\mathsf{S}_1(\mathcal{K}, \Omega)$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of k-covers of Y. Then $(\mathcal{U}_n^c\colon n \in \mathbb{N})$ is a sequence of subsets of 2^X with $Y^c \in \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{U}_n^c)$ for each $n \in \mathbb{N}$ (see the proof of the previous theorem). Apply (1) to find a sequence $(U_n^c\colon n \in \mathbb{N})$ such that for each n, $U_n \in \mathcal{U}_n$ and $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\{U_n^c\colon n \in \mathbb{N}\})$. Then $\{U_n\colon n \in \mathbb{N}\}$ is an ω-cover of Y. Indeed, let S be a finite subset of Y. Then $(S^c)^+$ is a Z^+-neighborhood of Y^c and thus $(S^c)^+ \cap \{U_n^c\colon n \in \mathbb{N}\} \neq \emptyset$; let U_i^c be a member of this intersection. Then from $U_i^c \subset S^c$ it follows $S \subset U_i$.

(2) $\Rightarrow$ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of subsets of 2^X such that a point $S \in 2^X$ belongs to the F^+-closure of $\mathcal{A}_n$ for each n. Then $(\mathcal{A}_n^c\colon n \in \mathbb{N})$ is a sequence of k-covers of S^c and because S^c is an $\mathsf{S}_1(\mathcal{K}, \Omega)$-set, there is a sequence $(A_n^c\colon n \in \mathbb{N})$ such that for each n, $A_n^c \in \mathcal{A}_n^c$ and $\{A_n^c\colon n \in \mathbb{N}\}$ is an ω-cover of S^c. The last fact is equivalent with $S \in \mathrm{Cl}_{\mathsf{Z}^+}(\{A_n\colon n \in \mathbb{N}\})$, and thus the sequence $(A_n\colon n \in \mathbb{N})$ witnesses for $(\mathcal{A}_n\colon n \in \mathbb{N})$ that 2^X satisfies (1). □

After Theorems 1–3 it is natural to ask what happens if X satisfies $\mathsf{S}_1(\Omega, \Omega)$, or $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$, or $\mathsf{S}_1(\mathcal{K}, \Omega)$. The next three theorems answer these questions.

Let $\mathcal{D}$ denote the family of dense subsets of a space.

Theorem 4. *For a space* X *the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *satisfies* $\mathsf{S}_1(\mathcal{D}, \mathcal{D})$;
(2) X *satisfies* $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of open k-covers of X. For each n, $\mathcal{A}_n := \mathcal{U}_n^c$ is a dense subset of $(2^X, \mathsf{F}^+)$. Indeed, let n be fixed and let $(K^c)^+$ be a basic open subset of $(2^X, \mathsf{F}^+)$. There is a member $U_{K,n}$ of $\mathcal{U}_n$ containing K. Thus we have $U_{K,n}^c \in (K^c)^+$ and $U_{K,n}^c \in \mathcal{A}_n$, i.e. $\mathcal{A}_n$ is a dense set in $(2^X, \mathsf{F}^+)$. Applying (1) one finds a sequence $(A_n\colon n \in \mathbb{N})$ such that for each n, $A_n \in \mathcal{A}_n$ and $\mathrm{Cl}_{\mathsf{F}^+}(\{A_1, A_2, \ldots\}) = 2^X$. Let for each n, $A_n^c = U_n \in \mathcal{U}_n$. We claim that the sequence $(U_n\colon n \in \mathbb{N})$ witnesses for $(\mathcal{U}_n\colon n \in \mathbb{N})$ that X satisfies $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$. Let C be a compact subset of X. Then $(C^c)^+$ contains some A_i so that $U_i = A_i^c$ satisfies $C \subset U_i$, i.e. $\{U_n\colon n \in \mathbb{N}\}$ is a k-cover of X.

(2) $\Rightarrow$ (1): Let $(\mathcal{D}_n\colon n \in \mathbb{N})$ be a sequence of dense subsets of $(2^X, \mathsf{F}^+)$. Let for each n, $\mathcal{U}_n := \mathcal{D}_n^c$. Then $\mathcal{U}_n$ is an open k-cover of X. To prove the last statement, let K be a compact subset of X. Pick a set D in $(K^c)^+ \cap \mathcal{D}_n$. We have $D^c \in \mathcal{U}_n$ and $K \subset D^c$. Apply now (2) to $(\mathcal{U}_n\colon n \in \mathbb{N})$ and find a sequence $(D_n^c\colon n \in \mathbb{N})$ such that for each n, $D_n^c \in \mathcal{U}_n$ and $\{D_1^c, D_2^c, \ldots\}$ is a k-cover for X. It is easy to verify that the sequence $(D_n\colon n \in \mathbb{N})$ is a selector for the sequence $(\mathcal{D}_n\colon n \in \mathbb{N})$ which witnesses that $(2^X, \mathsf{F}^+)$ satisfies (1). □

In a similar way we prove:

Theorem 5. *For a space X the following are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *satisfies* $\mathsf{S}_1(\mathcal{D}, \mathcal{D})$;
(2) *All finite powers of X have the Rothberger property* (*i.e. X satisfies* $\mathsf{S}_1(\Omega, \Omega)$).

Theorem 6. *For a space X the following are equivalent*:

(1) 2^X *satisfies* $\mathsf{S}_1(\mathcal{D}_{\mathsf{F}^+}, \mathcal{D}_{\mathsf{Z}^+})$;
(2) *X satisfies* $\mathsf{S}_1(\mathcal{K}, \Omega)$.

At the end of this section we consider when the spaces $(2^X, \mathsf{F}^+)$ and $(2^X, \mathsf{Z}^+)$ have the Rothberger property.

Recall that a family ξ of subsets of a space X is a *π-network* if for each open set $U \subset X$ there is a $M \in \xi$ such that $M \subset U$. For a space X let Π_k (respectively Π_ω) denote the family of π-networks of X consisting of compact (respectively finite) subsets of X. In this notation we have:

Theorem 7. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *has the Rothberger property*;
(2) *X satisfies* $\mathsf{S}_1(\Pi_k, \Pi_k)$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{K}_n\colon n \in \mathbb{N})$ be a sequence from Π_k. Then for each $n \in \mathbb{N}$, $(\mathcal{K}_n^c)^+ := \{(K^c)^+\colon K \in \mathcal{K}_n\}$ is an open cover of $(2^X, \mathsf{F}^+)$. To see this, fix n and let $A \in 2^X$. A^c is an open subset of X, hence there is a $K \in \mathcal{K}_n$ with $K \subset A^c$. Then $A \in (K^c)^+$ so that $(\mathcal{K}_n^c)^+$ is indeed an open cover of $(2^X, \mathsf{F}^+)$. By (1) pick for each n, $K_n \in \mathcal{K}_n$ such that the set $\{(K_n^c)^+\colon n \in \mathbb{N}\}$ is an open cover of $(2^X, \mathsf{F}^+)$. We show that $\{K_n\colon n \in \mathbb{N}\}$ is a π-network in X. Let U be an open subset of X. From $U^c \in 2^X$ it follows that there is some $i \in \mathbb{N}$ for which $U^c \in (K_i^c)^+$ holds, hence $K_i \subset U$.

(2) ⇒ (1): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of open covers of $(2^X, \mathsf{F}^+)$; without loss of generality one may suppose that for each n all elements of $\mathcal{U}_n$ are basic sets in $(2^X, \mathsf{F}^+)$: $\mathcal{U}_n = \{(K_{n,s}^c)^+\colon s \in S\}$. For each n, $\mathcal{K}_n := \{K_{n,s}\colon s \in S\}$ is a π-network in X. Indeed, if $G \subset X$ is open, then for some $(K_{n,s}^c)^+ \in \mathcal{U}_n$ we have $G^c \in (K_{n,s}^c)^+$ which implies $K_{n,s} \subset G$. Apply now (2) to the sequence $(\mathcal{K}_n\colon n \in \mathbb{N})$ to find sets $K_n \in \mathcal{K}_n$, $n \in \mathbb{N}$, such that $\{K_n\colon n \in \mathbb{N}\}$ is a π-network in X. Without difficulties one verifies that $\{(K_n^c)^+\colon n \in \mathbb{N}\}$ is an open cover of $(2^X, \mathsf{F}^+)$ showing that (1) is true. □

Similarly, we have

Theorem 8. *For a space X the following are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *has the Rothberger property*;
(2) *X satisfies* $\mathsf{S}_1(\Pi_\omega, \Pi_\omega)$.

2. The Menger-like selection principles

We investigate now the fan tightness of hyperspaces. As might expected the proofs of theorems of this section are similar to the proofs of the corresponding results concerning strong fan tightness. Thus we shall only give the proof of the following result.

Theorem 9. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *has countable fan tightness*;
(2) *Each open set $Y \subset X$ satisfies* $\mathsf{S}_{\mathrm{fin}}(\Omega, \Omega)$.

Proof. (1) ⇒ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of ω-covers of Y. Then $(\mathcal{U}_n^c\colon n \in \mathbb{N})$ is a sequence of subsets of 2^X and $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{U}_n^c)$ for each n (see the proof of Theorem 1). As $(2^X, \mathsf{Z}^+)$ has countable fan tightness there is a sequence $(\mathcal{V}_n^c\colon n \in \mathbb{N})$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\bigcup_{n\in\mathbb{N}} \mathcal{V}_n^c)$. We prove that $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an ω-cover of Y. Indeed, let S be a finite subset of Y. Then $(S^c)^+$ is a Z^+-neighborhood of Y^c, hence there is a U_k^c in $(S^c)^+ \cap \bigcup_{n\in\mathbb{N}} \mathcal{V}_n \neq \emptyset$. It follows from $U_k^c \subset S^c$ that $S \subset U_k$.

(2) ⇒ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of subsets of 2^X and let $S \in 2^X$ belong to the Z^+-closure of $\mathcal{A}_n$ for each n. Then $(\mathcal{A}_n^c\colon n \in \mathbb{N})$ is a sequence of ω-covers of S^c and because S^c satisfies $\mathsf{S}_{\mathrm{fin}}(\Omega, \Omega)$ there is a sequence $(\mathcal{B}_n^c\colon n \in \mathbb{N})$ such that for each n, $\mathcal{B}_n^c$ is a finite subset of $\mathcal{A}_n^c$ and $\bigcup_{n\in\mathbb{N}} \mathcal{B}_n^c$ is an ω-cover of S^c. It implies $S \in \mathrm{Cl}_{\mathsf{Z}^+}(\bigcup_{n\in\mathbb{N}} \mathcal{B}_n)$. Therefore, the sequence $(\mathcal{B}_n\colon n \in \mathbb{N})$ witnesses for $(\mathcal{A}_n\colon n \in \mathbb{N})$ that $(2^X, \mathsf{Z}^+)$ has countable fan tightness. □

The proofs of the following theorems are omitted.

Theorem 10. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *has countable fan tightness*;
(2) *Each open set* $Y \subset X$ *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{K}, \mathcal{K})$.

Theorem 11. *For a space* X *the following statements are equivalent*:

(1) 2^X *satisfies* $\mathsf{S}_{\text{fin}}(\Omega_A^{\mathsf{F}^+}, \Omega_A^{\mathsf{Z}^+})$ *for each* $A \in 2^X$;
(2) *Each open set* $Y \subset X$ *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{K}, \Omega)$.

Theorem 12. *For a space* X *the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{D}, \mathcal{D})$;
(2) X *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{K}, \mathcal{K})$.

Theorem 13. *For a space* X *the following are equivalent*:

(1) $(2^X, Z^+)$ *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{D}, \mathcal{D})$;
(2) X *satisfies* $\mathsf{S}_{\text{fin}}(\Omega, \Omega)$, *i.e. each finite power of* X *has the Menger property.*

Theorem 14. *For a space* X *the following are equivalent*:

(1) 2^X *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{D}_{\mathsf{F}^+}, \mathcal{D}_{\mathsf{Z}^+})$;
(2) X *satisfies* $\mathsf{S}_{\text{fin}}(\mathcal{K}, \Omega)$.

3. Set-tightness

The *set-tightness* $t_s(X)$ of a space X is countable if for each A subset of X and each $x \in \overline{A}$ there is a sequence $(A_n: n \in \mathbb{N})$ of subsets of A such that $x \in \overline{\bigcup\{A_n: n \in \mathbb{N}\}}$ but $x \notin \overline{A_n}$ for each $n \in \mathbb{N}$ (see [11]). The set tightness of the function space $\mathsf{C}_p(X)$ was studied in [26].

Theorem 15. *For a space* X *the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *has countable set-tightness*;
(2) *For each open set* $Y \subset X$ *and each* k*-cover* $\mathcal{U}$ *of* Y *there is a countable collection* $\{\mathcal{U}_n: n \in \mathbb{N}\}$ *of subsets of* $\mathcal{U}$ *such that no* $\mathcal{U}_i$ *is a* k*-cover of* Y *and* $\bigcup_{n \in \mathbb{N}} \mathcal{U}_n$ *is a* k*-cover of* Y.

Proof. (1) $\Rightarrow$ (2): Let Y be an open subset of X and let $\mathcal{U}$ be a k-cover of Y. Then $\mathcal{A} = \mathcal{U}^c$ is a subset of 2^X and Y^c belongs to the F^+-closure of $\mathcal{A}$. Since $(2^X, \mathsf{F}^+)$ has countable set-tightness there is a family $\{\mathcal{A}_n: n \in \mathbb{N}\}$ of subsets of $\mathcal{A}$ such that for each n, $Y^c \notin \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{A}_n)$ but $Y^c \in \mathrm{Cl}_{\mathsf{F}^+}(\bigcup\{\mathcal{A}_n: n \in \mathbb{N}\})$. Let for each n, $\mathcal{U}_n = \mathcal{A}_n^c$. Then for each n, $\mathcal{U}_n \subset \mathcal{U}$ and the family $\{\mathcal{U}_n: n \in \mathbb{N}\}$ witnesses for $\mathcal{U}$ that (2) is true.

(2) $\Rightarrow$ (1): Let $\mathcal{A}$ be a subset of 2^X and let $S \in \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{A})$. Then S^c is an open subset of X and $\mathcal{U} = \mathcal{A}^c$ is its k-cover. Apply (2) to S^c and $\mathcal{U}$ and choose subsets $\mathcal{U}_1, \mathcal{U}_2, \ldots$ of

$\mathcal{U}$ such that $\bigcup_{n\in\mathbb{N}}\mathcal{U}_n$ is a k-cover of S^c but no $\mathcal{U}_n$ is such a cover. If we put for each n, $\mathcal{A}_n = \mathcal{U}_n^c$ we get a countable family of subsets of $\mathcal{A}$ such that for each n, $S \notin \mathrm{Cl}_{\mathsf{F}^+}(\mathcal{A}_n)$ but $S \in \mathrm{Cl}_{\mathsf{F}^+}(\bigcup_{n\in\mathbb{N}}\mathcal{A}_n)$. □

The following corresponding result on the space $(2^X, \mathsf{Z}^+)$ is proved similarly.

Theorem 16. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *has countable set-tightness*;
(2) *For each open set $Y \subset X$ and each ω-cover $\mathcal{U}$ of Y there is a countable collection $\{\mathcal{U}_n\colon n \in \mathbb{N}\}$ of subsets of $\mathcal{U}$ such that no $\mathcal{U}_i$ is an ω-cover of Y and $\bigcup_{n\in\mathbb{N}}\mathcal{U}_n$ is an ω-cover of Y.*

4. *T*-tightness

The *T-tightness* of a space X is countable if for each uncountable regular cardinal ρ and each increasing ρ-sequence $(F_\alpha\colon \alpha < \rho)$ of closed subsets of X the set $\bigcup\{F_\alpha\colon \alpha < \rho\}$ is closed (see [11]). In [26] the T-tightness of function spaces $\mathsf{C}_p(X)$ was studied, while in [13] the same question was considered for spaces $\mathsf{C}_k(X)$.

Denote by $T(\Omega)$ the following statement on a space X:

For each regular infinite cardinal ρ and each increasing ρ-sequence $(\mathcal{U}_\alpha\colon \alpha < \rho)$ of families of open subsets of X such that $\mathcal{U} := \bigcup\{\mathcal{U}_\alpha\colon \alpha < \rho\}$ is an ω-cover there is a $\beta < \rho$ so that $\mathcal{U}_\beta$ is an ω-cover of X.

$T(\mathcal{K})$ is defined in the similar way when "ω-cover" is replaced by "k-cover".

Theorem 17. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *has countable T-tightness*;
(2) *Each open set $Y \subset X$ satisfies $T(\Omega)$.*

Proof. (1) ⇒ (2): Let ρ be a regular uncountable cardinal and let $\mathcal{U} = \bigcup\{\mathcal{U}_\alpha\colon \alpha \in \rho\}$, with $\mathcal{U}_\alpha \subset \mathcal{U}_\beta$ whenever $\alpha < \beta < \rho$, be an ω-cover of Y. For each $\alpha < \rho$ let $\mathcal{A}_\alpha = \mathcal{U}_\alpha^c := \{U^c\colon U \in \mathcal{U}_\alpha\}$ and $\mathcal{A} = \bigcup\{\mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{A}_\alpha)\colon \alpha < \rho\}$. Since the T-tightness of $(2^X, \mathsf{Z}^+)$ is countable, the set $\mathcal{A}$ is a closed subset of $(2^X, \mathsf{Z}^+)$. We claim $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{A}) = \mathcal{A}$. Let $(F^c)^+$, F a finite subset of Y, be a standard basic Z^+-neighborhood of Y^c. Since $\mathcal{U}$ is an ω-cover of Y and $F \subset Y$ there is a $\xi < \rho$ and an element $U \in \mathcal{U}_\xi$ such that $F \subset U$. It implies $U^c \in (F^c)^+$, hence $(F^c)^+ \cap \mathcal{A}_\xi \neq \emptyset$, so that $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{A}_\xi) \subset \mathcal{A}$. Further, from $Y^c \in \mathcal{A}$ it follows $Y^c \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{A}_\beta)$ for some $\beta < \rho$. We prove that $\mathcal{U}_\beta$ is an ω-cover of Y. Let S be a finite subset of Y. Then $Y^c \in (S^c)^+$ so that there is a member $A \in \mathcal{A}_\beta \cap (S^c)^+$. This means $S \subset A^c \in \mathcal{U}_\beta$.

(2) ⇒ (1): Let $(\mathcal{A}_\alpha\colon \alpha < \rho)$, ρ a regular uncountable cardinal, be an increasing ρ-sequence of closed subsets of 2^X. We have to prove that $\mathcal{A} = \bigcup\{\mathcal{A}_\alpha\colon \alpha < \rho\}$ is a

closed set in $(2^X, \mathsf{Z}^+)$. Let S be an element in the Z^+-closure of $\mathcal{A}$. For each $\alpha < \rho$ let $\mathcal{U}_\alpha = \mathcal{A}_\alpha^c$ and put $\mathcal{U} = \bigcup\{\mathcal{U}_\alpha\colon \alpha < \rho\}$. Then $\mathcal{U}$ is an ω-cover of S^c. Apply (2) to S^c and the ρ-sequence $(\mathcal{U}_\alpha\colon \alpha < \rho)$. There is a $\beta < \rho$ such that $\mathcal{U}_\beta$ is an ω-cover of S^c. It implies $S \in \mathrm{Cl}_{\mathsf{Z}^+}(\mathcal{A}_\beta) = \mathcal{A}_\beta \subset \mathcal{A}$. □

As before, similarly one proves:

Theorem 18. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *has countable T-tightness*;
(2) *Each open set $Y \subset X$ satisfies $T(\mathcal{K})$*.

5. The Hurewicz-like selection principles

In this section we need the notion of groupability [17].

An ω-cover (a k-cover) $\mathcal{U}$ of a space X is *groupable* [16,17,13] if it can be represented as a union of infinitely many finite, pairwise disjoint subfamilies $\mathcal{U}_n$ such that each finite (compact) set $A \subset X$ is contained in a member of $\mathcal{U}_n$ for all but finitely many n. Ω^{gp} ($\mathcal{K}^{\mathrm{gp}}$) will denote the family of groupable ω-covers (k-covers) of a space.

In [9] (see also [10]) Hurewicz introduced a covering property, nowadays known as the *Hurewicz property*, in the following way. A space X is said to have the Hurewicz property if for each sequence $(\mathcal{U}_n\colon n \in \mathbb{N})$ of open covers of X there is a sequence $(\mathcal{V}_n\colon n \in \mathbb{N})$ of finite sets such that for each n, $\mathcal{V}_n \subset \mathcal{U}_n$ and for each $x \in X$, for all but finitely many n, $x \in \bigcup \mathcal{V}_n$. In [17] this property was described by an $\mathsf{S}_{\mathrm{fin}}$-type property and it was shown that all finite powers of a space X have the Hurewicz property if and only if X satisfies $\mathsf{S}_{\mathrm{fin}}(\Omega, \Omega^{\mathrm{gp}})$. Similarly, the property $\mathsf{S}_1(\Omega, \Omega^{\mathrm{gp}})$ was characterized: X has this property if and only if each finite power of X has the Gerlits–Nagy property $(*)$ [7] ($\equiv$ the Hurewicz property as well as the Rothberger property).

Here we study the mentioned properties in the context of hyperspaces. For similar investigation see [14].

Call a countable element D from $\mathcal{D}$ *groupable* if there is a partition $D = \bigcup_{n\in\mathbb{N}} D_n$ into finite sets such that each open set of the space intersects D_n for all but finitely many n.

Let $\mathcal{D}^{\mathrm{gp}}$ denote the family of groupable elements of $\mathcal{D}$.

Theorem 19. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *satisfies* $\mathsf{S}_1(\mathcal{D}, \mathcal{D}^{\mathrm{gp}})$;
(2) X *satisfies* $\mathsf{S}_1(\mathcal{K}, \mathcal{K}^{\mathrm{gp}})$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of open k-covers of X. For each n, $\mathcal{A}_n := \mathcal{U}_n^c$ is a dense subset of $(2^X, \mathsf{F}^+)$. Use (1) and for each n, pick an $A_n \in \mathcal{A}_n$ such that $\mathcal{A} = \{A_n\colon n \in \mathbb{N}\}$ is a groupable dense subset of $(2^X, \mathsf{F}^+)$; let the partition $\mathcal{A} = \bigcup_{n\in\mathbb{N}} \mathcal{B}_n$ witnesses that fact, i.e. each open subset of $(2^X, \mathsf{F}^+)$ meets $\mathcal{B}_n$ for all but finitely many n. Put $\mathcal{V} = \mathcal{A}^c$ and for each n, $\mathcal{W}_n = \mathcal{B}_n^c$. We claim that $\mathcal{V}$ is a groupable k-cover of X and that

$\mathcal{W}_n$'s witness that fact. Let K be a compact subset of X. Then $(K^c)^+$ is open in $(2^X, \mathsf{F}^+)$ and so there is n_0 such that for each $n \geqslant n_0$ one finds a member $B_n \in \mathcal{B}_n$ with $B_n \in (K^c)^+$. Then B_n^c belongs to $\mathcal{W}_n$ and $K \subset B_n^c$.

(2) $\Rightarrow$ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of dense subsets of $(2^X, \mathsf{F}^+)$. Let for each n, $\mathcal{U}_n := \mathcal{A}_n^c$. Then $\mathcal{U}_n$ is an open k-cover of X. Applying (2) we find a sequence $(U_n\colon n \in \mathbb{N})$ such that for each n, $U_n = A_n^c \in \mathcal{U}_n$ and $\mathcal{U} := \{U_1, U_2, \ldots\}$ is a groupable k-cover for X: there is a partition $\mathcal{U} = \bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ into finite sets such that each compact subset of X is contained in a member of $\mathcal{V}_n$ for all but finitely many n. Then one can easily check that letting for each n, $\mathcal{B}_n = \mathcal{V}_n^c$, the partition $\mathcal{A} = \bigcup_{n\in\mathbb{N}} \mathcal{B}_n$ witnesses that for the sequence $(\mathcal{A}_n\colon n \in \mathbb{N})$ there is a selector showing that $(2^X, \mathsf{F}^+)$ satisfies (1). □

Similarly one proves:

Theorem 20. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *satisfies* $\mathsf{S}_{\mathrm{fin}}(\mathcal{D}, \mathcal{D}^{\mathrm{gp}})$;
(2) X *satisfies* $\mathsf{S}_{\mathrm{fin}}(\mathcal{K}, \mathcal{K}^{\mathrm{gp}})$.

Theorem 21. *For a space X the following are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *satisfies* $\mathsf{S}_1(\mathcal{D}, \mathcal{D}^{\mathrm{gp}})$;
(2) X *satisfies* $\mathsf{S}_1(\Omega, \Omega^{\mathrm{gp}})$.

Theorem 22. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *satisfies* $\mathsf{S}_{\mathrm{fin}}(\mathcal{D}, \mathcal{D}^{\mathrm{gp}})$;
(2) X *satisfies* $\mathsf{S}_{\mathrm{fin}}(\Omega, \Omega^{\mathrm{gp}})$*, i.e. each finite power of X has the Hurewicz property.*

For the next consideration we need the notion of weak groupability.

A countable open cover $\mathcal{U}$ of a space X is *weakly groupable* [17] (respectively *k-weakly groupable* [5]) if there is a partition $\mathcal{U} = \bigcup\{\mathcal{U}_n\colon n \in \mathbb{N}\}$ such that each $\mathcal{U}_n$ is finite, $\mathcal{U}_m \cap \mathcal{U}_n = \emptyset$ whenever $m \neq n$, and each finite (respectively compact) subset of X is covered by $\mathcal{U}_n$ for some $n \in \mathbb{N}$.

$\mathcal{O}^{\mathrm{wgp}}$ (respectively $\mathcal{O}^{k\text{-wgp}}$) denotes the family of open weakly groupable (respectively k-weakly groupable) covers of a space.

In [3] a covering property of a space ($\mathsf{U}_{\mathrm{fin}}(\Gamma, \Omega)$ in the notation from [12]) which is intermediate between the Hurewicz property and the Menger property was characterized as an $\mathsf{S}_{\mathrm{fin}}$ property, namely as $\mathsf{S}_{\mathrm{fin}}(\Omega, \mathcal{O}^{\mathrm{wgp}})$. In [5] the property $\mathsf{S}_{\mathrm{fin}}(\mathcal{K}, \mathcal{O}^{k\text{-wgp}})$ was considered. We characterize these two properties of a space X by the corresponding properties of hyperspaces $(2^X, \mathsf{Z}^+)$ and $(2^X, \mathsf{F}^+)$, respectively.

Theorem 23. *For a space X the following statements are equivalent*:

(1) $(2^X, \mathsf{Z}^+)$ *satisfies*: *for each sequence $(\mathcal{A}_n\colon n \in \mathbb{N})$ of dense subsets of $(2^X, \mathsf{Z}^+)$ there are finite sets $\mathcal{B}_n \subset \mathcal{A}_n$ such that $\bigcup_{n\in\mathbb{N}} \mathcal{B}_n$ can be partitioned into finite sets $\mathcal{C}_n$, $n \in \mathbb{N}$,*

such that $\{\bigcap \mathcal{C}_n\colon n \in \mathbb{N}\}$ *is dense in* $(2^X, \mathsf{Z}^+)$;

(2) X *satisfies* $\mathsf{S}_{\mathrm{fin}}(\Omega, \mathcal{O}^{\mathrm{wgp}})$.

Proof. (1) $\Rightarrow$ (2): Let $(\mathcal{U}_n\colon n \in \mathbb{N})$ be a sequence of open ω-covers of X. Then $(\mathcal{A}_n \equiv \mathcal{U}_n^c\colon n \in \mathbb{N})$ is a sequence of dense subsets of $(2^X, \mathsf{Z}^+)$, so that there is a sequence $(\mathcal{B}_n\colon n \in \mathbb{N})$ such that for each n, $\mathcal{B}_n$ is a finite subset of $\mathcal{A}_n$ and $\mathcal{B} = \bigcup_{n\in\mathbb{N}} \mathcal{B}_n$ is a union $\bigcup_{n\in\mathbb{N}} \mathcal{C}_n$ of finite, pairwise disjoint sets such that $\{\bigcap \mathcal{C}_n\colon n \in \mathbb{N}\}$ is dense in $(2^X, \mathsf{Z}^+)$. Let $\mathcal{V} = \mathcal{B}^c$, $\mathcal{W}_n = \mathcal{C}_n^c$, $n \in \mathbb{N}$. Then the sets $\mathcal{V}$ and $\mathcal{W}_n$'s witness for $(\mathcal{U}_n\colon n \in \mathbb{N})$ that X belongs to the class $\mathsf{S}_{\mathrm{fin}}(\Omega, \mathcal{O}^{\mathrm{wgp}})$.

Let F be a finite subset of X. The open subset $(F^c)^+$ of $(2^X, \mathsf{Z}^+)$ meets $\{\bigcap \mathcal{C}_n\colon n \in \mathbb{N}\}$, i.e. there exists $m \in \mathbb{N}$ such that $\bigcap \mathcal{C}_m \in (F^c)^+$. This means that $F \subset (\bigcap \mathcal{C}_m)^c = \bigcup \mathcal{W}_m$, i.e. $\mathcal{V}$ is an open weakly groupable cover of X.

(2) $\Rightarrow$ (1): Let $(\mathcal{A}_n\colon n \in \mathbb{N})$ be a sequence of dense subsets of $(2^X, \mathsf{Z}^+)$. For each n, $\mathcal{U}_n := \mathcal{A}_n^c$ is an ω-cover of X. By (2) there is a sequence $(\mathcal{V}_n\colon n \in \mathbb{N})$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\mathcal{V} = \bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is a weakly groupable open cover of X. We have that $\mathcal{V}$ is a union of countably many finite, pairwise disjoint sets $\mathcal{W}_n$ satisfying: for each finite set $F \subset X$ there is an n with $F \subset \bigcup \mathcal{W}_n$. Let us check that the sequence $(\mathcal{B}_n\colon n \in \mathbb{N})$, where for each n, $\mathcal{B}_n = \mathcal{W}_n^c$, shows that $(2^X, \mathsf{Z}^+)$ satisfies (1). Indeed, pick a basic open set $(S^c)^+$ of $(2^X, \mathsf{Z}^+)$. Then there is m for which $S \subset \bigcup \mathcal{W}_m$ holds. It follows $\bigcap \mathcal{W}_m^c \equiv \bigcap \mathcal{B}_m \in (S^c)^+$, i.e. the set $\{\bigcap \mathcal{B}_n\colon n \in \mathbb{N}\}$ is dense in $(2^X, \mathsf{Z}^+)$. □

In a similar way one can prove

Theorem 24. *For a space* X *the following are equivalent*:

(1) $(2^X, \mathsf{F}^+)$ *satisfies*: *for each sequence* $(\mathcal{A}_n\colon n \in \mathbb{N})$ *of dense subsets of* $(2^X, \mathsf{F}^+)$ *there are finite sets* $\mathcal{B}_n \subset \mathcal{A}_n$ *such that* $\bigcup_{n\in\mathbb{N}} \mathcal{B}_n$ *can be partitioned into finite sets* $\mathcal{C}_n$, $n \in \mathbb{N}$, *such that* $\{\bigcap \mathcal{C}_n\colon n \in \mathbb{N}\}$ *is dense in* $(2^X, \mathsf{F}^+)$;

(2) X *satisfies* $\mathsf{S}_{\mathrm{fin}}(\mathcal{K}, \mathcal{O}^{k\text{-wgp}})$.

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