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Soft nexuses

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ABSTRACT

In this paper, we define soft nexuses over a nexus N. After that we define the notions cyclic soft nexus, prime and maximal soft subnexuses and we obtain some results about the above definitions. Finally, we define the integer number induced by soft nexus (α, A) and denoted by $|(\alpha, A)|$ and show that for any $n \in \aleph$, there exists soft nexus (α, A), such that $|(\alpha, A)| = n$.

1. Introduction

Dealing with uncertainties is a major problem in many areas such as economics, engineering, environmental science, medical science and social science. These kinds of problem cannot be deal with classical methods, because classical methods have inherent difficulties. To overcome these kinds of difficulties, Molodtsov [1] proposed a completely new approach, which is called soft set theory, for modeling uncertainty. Then Maji [2] introduced several operations on soft sets. Aktas and Cagman [3] defined soft groups and obtained the main properties of these groups. Moreover, Feng [4–6] compared soft sets with fuzzy sets and rough sets. Beside, Jun [7] defined soft ideal on BCK/BCI-algebras and Feng [8] defined soft semirings, soft ideal on soft semirings and idealistic soft semirings. Sun [9] defined the concept of soft modules and studied their basic properties.

During the past several decades, a major interest in the Space Structures Research, was the development of convenient methods for the generation and processing of information about structural configurations. The work in this area eventually resulted in the creation of an algebra, called 'formex algebra'; see [10–12].

The concepts of formex algebra are general and can be used in many different fields. In particular, the ideas may be employed for the generation of geometric information about various aspects of structural systems such as element connectivity, nodal coordinates, local and support positions. The information generated may be used for different purposes such as graphic visualization and input data for structural analysis.

The practical use of formex algebra is through a programming language called 'Formian'. The origins of Formian date back to the late seventies and the current state of the language is described in [12].

Formex algebra has proved to be an elegant and convenient tool for generating and processing geometric information. However, there is another development that can be of great help in the systematic processing of all kinds of information. This development involves the use of a mathematical object called a 'plenix' (plural 'plenices').

The concept of a plenix allows complex processes on sets of mathematical objects to be formulated with ease and elegance. In particular, the use of plenices is ideally suited to processes involving digital computers. The analysis of large

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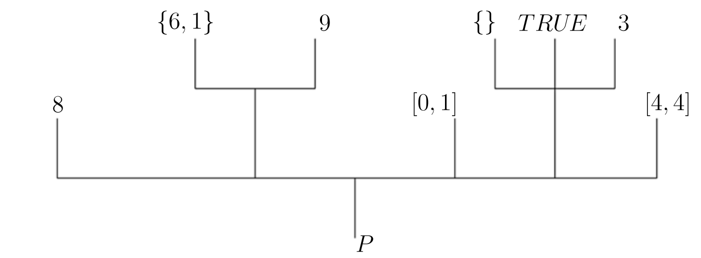

Fig. 1.1. A graphical representation of a plenix called *P*.

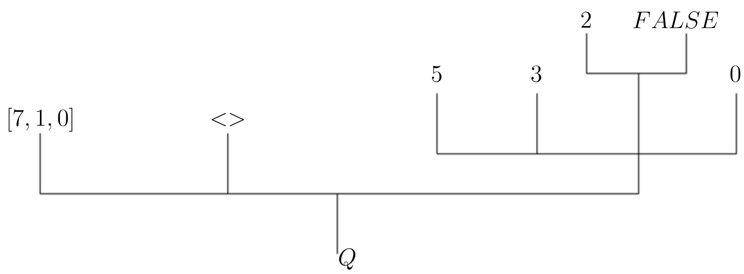

Fig. 1.2. Dendrogram of plenix *Q*.

and complex engineering structures involves an enormous amount of data and results which need to be handled efficiently. A powerful tool for organizing these pieces of information is provided by the theory of plenices. Detailed studies about the application of plenices as data organizers can be found in [11,12].

The basic idea of a plenix has been further developed as a mathematical object for general use as can be found in [12,13]. The aim of the study in [14] is to create a new abstract algebraic system and to investigate the properties of this system.

Fundamentally, a plenix is a mathematical object consisting of an arrangement of a mathematical object. A plenix is like a tree structure in which every branch is a mathematical object. For instance, Fig. 1.1 is a graphic representation of a plenix, consisting of a sequence of elements, each of which consists of a sequence of elements and so on. The graphical representation of plenix *P* in Fig. 1.1 is referred to as the 'dendrogram' of *P*.

The following construct is another way to represent a plenix *P*.

$$\langle 8, \langle \{6, 1\}, 9\rangle, [0, 1], \langle \{\}, \text{TRUE}, 3\rangle, [4, 4]\rangle.$$

Plenix *P* has five primary elements each of which is called a 'principal panel'.

That is,

$$8 \; \langle \{6, 1\}, 9\rangle \; [0, 1] \; \langle \{\}, \text{TRUE}, 3\rangle \; [4, 4]$$

are the principal panels of plenix *P* in Fig 1.1. Here, the second and the fourth principal panels themselves are plenices.

Two plenices are considered equal if they are identical. Also, any mathematical object can be a panel of a plenix provided that it has a clear definition of equality.

The term 'primion' is used to refer a mathematical object that is not a plenix and the term 'nonprimion panel' is used to refer a panel of a plenix that is not a plenix. In the example of Fig. 1.1

$$8 \; \{6, 1\} \; 9 \; [0, 1] \; \{\} \; \text{TRUE} \; 3 \; [4, 4]$$

are primion panels of plenix *P* of Fig. 1.1 and

$$\langle \{6, 1\}, 9\rangle \; \langle \{\}, \text{TRUE}, 3\rangle$$

are 'nonprimion' panels of *P*. Another example of a plenix is

$$Q = \langle [7, 1, 0], \langle\rangle, \langle 5, 3, \langle 2, \text{FALSE}\rangle, 0\rangle\rangle.$$

In this plenix the first principal panel is a vector and the second principal is an 'empty plenix', that is, a plenix that has no principal panels. A dendrogram of *Q* is shown in Fig. 1.2.

Every panel of a plenix may be associated with a sequence of positive integers that indicates the position of a panel in the plenix. Such a sequence of positive integers is referred to as the 'address' of the panel. For instance, referring to plenix *Q*, Fig. 1.2, the addresses of the panels are given in the following Table 1.1.

Table 1.1
Addresses the panels of Q.

Panel	Address
$[7, 1, 0]$	(1)
$\langle\rangle$	(2)
$\langle 5, 3, \langle 2, \text{FALSE}\rangle, 0\rangle$	(3)
5	$(3, 1)$
3	$(3, 2)$
$\langle 2, \text{FALSE}\rangle$	$(3, 3)$
0	$(3, 4)$
2	$(3, 3, 1)$
FALSE	$(3, 3, 2)$

An address (i, j, k) refers to the kth principal panel of the jth principal panel of the ith principal panel of the plenix. For example, the address of 0 is $(3, 4)$, indicating the 4th principal panel of the 3rd principal panel of the plenix.

The set of the addresses of all the panels of a plenix is called the 'address set' of that plenix. For instance, the set

$$\{(1), (2), (3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 3, 1), (3, 3, 2)\}$$

is the address set of plenix Q. The address set of a plenix P is denoted by A_P.

Let p and q be two panels of plenices P and Q, respectively. Then p and q are said to 'correspond' to each other provided that the address of p in P is identical to that of q in Q. For example, consider two plenices

$$P_1 = \langle\langle 1, 5\rangle, 4, \langle 3, 7, 2\rangle\rangle$$

and

$$P_2 = \langle\langle 0, 124\rangle, 8, \langle 9, 9, 9\rangle\rangle.$$

Every panel of plenix P_1 has a corresponding panel in plenix P_2 and vice versa. Two plenices that have such a relationship are said to have the same 'constitution'. The constitution of a plenix is the arrangement of its panels. This arrangement can be explicitly represented by the address set of the plenix. Therefore, two plenices have the same constitution provided that they have the same address set. For example, both plenices P_1 and P_2 have the same address set which is

$$A_{P_1} = A_{P_2} = \{(1), (2), (3), (1, 1), (1, 2), (3, 1), (3, 2), (3, 3)\}.$$

It is possible to define an equivalence relation on the set of all plenices, as follows:

$$P \sim Q \Leftrightarrow P \quad \text{and} \quad Q \text{ have the same constitution.}$$

The equivalence class of P is defined as the set of all plenices which have the same constitution as P. The equivalence class of P is denoted by $[P]$. For example, the above two plenices, P_1 and P_2 are in the same equivalence class. The equivalence class of P, that is $[P]$, may be viewed in two different ways. First, it may be viewed as a set that contains all the plenices whose constitution is the same as that of P, as discussed above. Alternatively, $[P]$ may be viewed as a mathematical object that represents the constitution of P. Such a mathematical object is called a 'nexus'. The nexus of P represents the constitution of P as well as that of all the plenices that have the same constitution as P. On the other hand, the most direct representation of the constitution of a plenix is the address set of the plenix. Bolourian in [13], defined nexus and properties of this structure algebra such as subnexuses, cyclic nexuses and homomorphism of nexuses are studied in detail. Moreover, Afkhami et al. [15], defined the notion of fraction over a nexuses and studied its basic properties.

The main purpose of this paper is to introduce basic notion of soft nexuses. Moreover, the prime and maximal soft subnexuses are introduced and illustrated with a related example.

2. Basic results on nexuses

Definition 2.1 (*[13]*). An address, is a sequence of $\aleph \cup \{0\}$, such that $a_i = 0$ implies $a_{i+k} = 0$ for all $k \geq 0$. The sequence of zero is called the empty address and is denoted by $()$. If $(a_1, a_2, \ldots, a_n, 0, 0, 0)$ is a nonempty address, we denote this address by $(a_1, a_2, \ldots, a_n)$.

Definition 2.2 (*[13]*). A nexus N is a set of addresses with the following properties.

(i) If $(a_1, a_2, \ldots, a_n) \in N$, then $(a_1, a_2, \ldots, a_{n-1}, t) \in N$, for all $0 \leq t \leq a_n$,
(ii) If $(a_1, a_2, \ldots) \in N$, then $(a_1, a_2, \ldots, a_n) \in N$ for all $n \geq 1$.

Example 2.3. Consider the set $N = \{(), (1), (2), (3), (1, 1), (1, 2), (3, 1), (3, 2)\}$; this set has the above two properties and so it is a nexus. However, the set $A = \{(), (1), (2), (2, 2)\}$ is not a nexus, because $(2, 2)$ is an element of A, but $(2, 1)$ is not included in A.

Definition 2.4 ([13]). Let w be an element of N. The level of w is said to be:

(i) n, if $w = (a_1, a_2, \dots, a_{n-1}, a_n)$.
(ii) ∞, if w is an infinite sequence of N.
(iii) 0, if $w = ()$.

The level of w is denoted by $l(w)$.

If $l(w) < \infty$, we say that w is finite length. Nexus N is called finite length, whenever every element of N be finite length.

Definition 2.5 ([13]). Let $w = \{a_i\}_{i\in\aleph}$, $v = \{b_i\}_{i\in\aleph}$ be two elements of nexus N. Then we said that $w \leq v$, if $l(w) = 0$ or one of the following cases satisfies.

Case 1: If $l(w) = 1$, then $l(v) \geq 1$ and $w = (a_1)$ that $a_1 \leq b_1$.
Case 2: If $1 < l(w) \leq n$, then $l(v) \geq l(w)$ and $w = (b_1, b_2, \dots, b_{l(w)-1}, a_{l(w)})$ that $a_{l(w)} \leq b_{l(w)}$.
Case 3: If $l(w) = \infty$, then $v = w$.

Theorem 2.6 ([13]). *For a nexus N, the following conditions are satisfied.*

(i) *$(N, \leq)$ is a lower semilattice.*
(ii) *For $a = \{a_i\}_{i=1}^{i=\infty}$ and $b = \{b_i\}_{i=1}^{i=\infty}$,*[1] *$a \wedge b = (a_1, a_2, \dots, a_{i-1}, a_i \wedge b_i)$, where i is the smallest index such that $a_i \neq b_i$ and $a_i \wedge b_i = \min\{a_i, b_i\}$.*

Example 2.7. Consider the nexus $N = \{(), (1), (2), (3), (1, 1), (1, 2), (2, 1)\}$. Then $() \wedge a = ()$ for all $a \in N$, $(1) \wedge a = (1)$ for all $a \neq ()$, $(2) \wedge (3) = (2) \wedge (2, 1) = (2)$, $(2) \wedge (1, 1) = (2) \wedge (1, 2) = (1)$, $(3) \wedge (1, 1) = (3) \wedge (1, 2) = (1)$, $(3) \wedge (2, 1) = (2)$, $(1, 1) \wedge (1, 2) = (1, 1)$, $(1, 1) \wedge (2, 1) = (1)$.

Example 2.8. Let N be a nexus and $a = (2, 3, 4, 5, 8, 10)$, $b = (2, 3, 4, 6, 12, 16)$ be two elements of N, then $a \wedge b = (2, 3, 4, 5)$.

Definition 2.9 ([13]). Let N be a nexus. A nonempty subset S of N is called a subnexus of N provided that S itself is a nexus. The set of all subnexuses of N is denoted by SUB(N). $\{()\}$ is said to be the trivial subnexus of N.

Example 2.10. Consider the nexus

$$N = \{(), (1), (2), (3), (1, 1), (1, 2), (3, 1), (3, 2), (3, 3)\}.$$

The following sets are some subnexuses of N.

$$\{()\} \qquad \{(), (1)\} \qquad \{(), (1), (2), (3), (3, 1), (3, 2)\}.$$

Theorem 2.11 ([13]). *Let S be a subset of nexus N. S is a subnexus of N if and only if $a \in S$, $b \in N$ and $b \leq a$ imply that $b \in S$.*

Definition 2.12 ([13]). Let $\emptyset \neq X \subseteq N$. The smallest subnexus of N containing X is called the subnexus of N generated by X and denoted by $\langle X\rangle$. If $|X| = 1$, then $\langle X\rangle$ is called a cyclic subnexus of N.

Example 2.13. Consider the nexus

$$N = \{(), (1), (2), (3), (2, 1), (2, 2), (3, 1)\},$$

then the subnexus of N generated by two addresses (3) and (2, 1) is

$$\langle (2, 1), (3)\rangle = \{(), (1), (2), (3), (2, 1)\}$$

and cyclic subnexus of N generated by $\{(2, 2)\}$ is

$$\langle (2, 2)\rangle = \{(), (1), (2), (2, 1), (2, 2)\}.$$

Theorem 2.14 ([13]). *Let $\emptyset \neq X \subseteq N$. Then*

$$\langle X\rangle = \{x \in N | \exists a \in X, x \leq a\}.$$

In particular, if $a = \{a_i\}_{i\in\aleph}$ then

$$\langle a\rangle = \{(), (t_1), (a_1, t_2), (a_1, a_2, t_3), \dots, (a_1, a_2, \dots, a_{n-1}, t_n), \dots, a | 0 \leq t_n \leq a_n, \forall n \in \aleph\}.$$

[1] $a \wedge b$ is the greatest lower bound of a and b.

Theorem 2.15 (*[13]*)**.** *Let M and K be two subnexuses of nexus N. Then $M \cap K$ and $M \cup K$ are subnexuses of N.*

Theorem 2.16 (*[13]*)**.** *For a nexus N, the following assertions are satisfied.*

(i) *Every two elements of N are comparable if and only if N is a cyclic nexus.*
(ii) *If nexus N is cyclic and $M \in \mathrm{SUB}(N)$, then M is cyclic. In particular, $(\mathrm{SUB}(N), \subseteq)$ is a chain.*

Theorem 2.17 (*[13]*)**.** *Let $N = \langle (a_1, a_2, a_3, \ldots, a_n) \rangle$. Then*

$$|N| = \left(\sum_{i=1}^{i=n} a_i \right) + 1.$$

Definition 2.18 (*[13]*)**.** Let M and N be two nexuses. Then the product of nexuses M and N, is defined as

$$M \times N = \{(a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m) | (a_1, a_2, \ldots, a_n) \in M, (b_1, b_2, \ldots, b_m) \in N\}.$$

We contradict that, $()a = ()$ and $b() = b$ for all $a \in N$ and $b \in M$.

We can see that, $M \times N$ is a nexus.

Example 2.19. If $M = \{(), (1), (2)\}$ and $N = \{(), (1), (2), (2, 1)\}$, Then $M \times N = \{(), (1), (1, 1), (1, 2), (1, 2, 1), (2), (2, 1), (2, 2), (2, 2, 1)\}$.

Definition 2.20. A proper subnexus P of N is said to be a prime subnexus of N, if $a \wedge b \in P$ implies that $a \in P$ or $b \in P$ for any $a, b \in P$.

Example 2.21. Consider the nexus

$$N = \{(), (1), (2), (1, 1), (1, 2), (1, 3), (2, 1), (2, 2)\}.$$

Then subnexus $P = \{(), (1), (2), (2, 1), (2, 2)\}$ is a prime subnexus of N, but $K = \{(), (1), (2), (1, 1), (2, 1)\}$ is not a prime subnexus, because $(1) = (1, 2) \wedge (2, 2) \in K$, but $(1, 2) \notin K$ and $(2, 2) \notin K$.

Theorem 2.22. *Let $\{P_i\}_{i \in I}$ be a family of prime subnexuses of N and let $P = \bigcup_{i \in I} P_i$ be a proper subset of N. Then P is a prime subnexus of N.*

Proof. The proof is trivial. $\square$

Definition 2.23. For $a \in N$, the set $\{b \in N \mid a \leq b\}$ is called an upper bound for $\{a\}$ and is denoted by R_a.

It is easy to check that for all $a \in N$, $N - R_a$ is a prime subnexus of N.

Theorem 2.24. *A proper subnexus P of a nexus N is prime if and only if $P = N - R_a$ for some $a \in N$.*

Proof. Let P be a prime subnexus of N. We show that there is $a \in N$ such that $N - P = R_a$. Since $N \neq P$, then there is $b \in N$ such that $b \notin P$. Put $a = \min\{x \in \langle b \rangle | x \notin P\}$. We show that $R_a = N - P$. Let $d \in R_a$. Then $a \leq d$, if $d \in P$, which is not true, so $R_a \subseteq N - P$. Now let $d \in N - P$, we show that $d \in R_a$. If $d \notin R_a$ then $a \leq d$ and so $d \wedge a \neq a$. Thus $d \wedge a < a$. On the other hand, $a \in \langle b \rangle$ implies that $d \wedge a \in \langle b \rangle$. Since $d \wedge a < a$, then by hypothesis we get that $d \wedge a \in P$. Hence $a \in P$ or $d \in P$, which is not true, because $a \notin P$ and $d \in N - P$. Therefore, $N - P = R_a$ and so $P = N - R_a$. $\square$

Definition 2.25. A proper subnexus M of nexus N is a maximal subnexus, if there exist no subnexus K of N, such that $M \subset K \subset N$.

Example 2.26. If $N = \{(), (1), (2), (1, 1), (1, 2), (2, 1), (2, 2), (2, 3)\}$, then subnexus $U = \{(), (1), (2), (1, 1), (1, 2), (2, 1), (2, 2)\}$ is a maximal subnexus, but $T = \{(), (1), (2), (2, 1), (2, 2), (2, 3)\}$ is not maximal subnexus, since $K = \{(), (1), (2), (1, 1), (2, 1), (2, 2), (2, 3)\}$ is a subnexus such that $T \subseteq K$.

Theorem 2.27. *If a nexus N does not have any maximal elements, then N does not have any maximal subnexus.*

Proof. Let M be a subnexus of N. If $M = N$, then M is not a maximal subnexus. If $M \neq N$, then there is $a \in N$ such that $a \notin M$. Since a is not a maximal element, then there is $b \in N$ such that $a < b$. So $a \notin M$ implies that $b \notin M$ and also $b \notin \langle a \rangle$. Hence $b \notin M \cup \langle a \rangle$ and so $M \subset M \cup \langle a \rangle \subset N$. Therefore M is not a maximal subnexus of N, i.e. any subnexus of N is not maximal. $\square$

Theorem 2.28. *Let N be a finite length nexus and T be the set of all maximal elements of a nexus N. Then every maximal subnexus of N is of the form $M = N - \{m\}$ for some $m \in T$.*

Proof. First, since N is of finite length, then $T \neq \emptyset$ and $N = \cup_{m\in T}\langle m\rangle$. Now let $m \in T$. We can see that $N - \{m\}$ is a maximal subnexus. Suppose that $M \neq N$ be a maximal subnexus of N. M does not contain all maximal addresses of N. Now we show that M must contain all of the maximal addresses of N unless one of them. Because if m_1 and m_2 are maximal addresses of N such that $m_1, m_2 \notin M$, then $M \subset M \cup \langle m_1\rangle \subset N$. So M is not a maximal subnexus. Let m' be the only maximal element such that $m' \notin M$. We show that $M = N - \{m'\}$. We have $M \subseteq N - \{m'\}$. Now let $a \in N - \{m\}$, so there is $m'' \in T$ such that $a \leq m''$, if $a \notin M$, then $m'' \notin M$ and so $M \subset M \cup \langle a\rangle \subset N$. Thus M is not the maximal subnexus of N, which is not true. Consequently M contains all addresses of N unless one of them. □

Definition 2.29. Let $a = (a_1, a_2, \dots, a_n) \in N$. Then $\sum_{i=1}^{n} a_i$ is called the integer number induced by a and denoted by $|a|$. If $a \in N$ and $l(a) = \infty$, then $|a| = \infty$. If $M \in \mathrm{SUB}(N)$, then we define $|M| = \sup\{|a| \mid a \in M\}$.

Example 2.30. In Example 2.26, $|N| = \sup\{0, 1, 2, 3, 4, 5\} = 5$, $|U| = \sup\{0, 1, 2, 3, 4\} = 4$ and $|T| = 5$.

3. Basic results on soft sets

Definition 3.1 (*[2]*)**.** A pair (α, A) is called a soft set over U, where α is a mapping given by $\alpha : A \to P(U)$.

Definition 3.2 (*[2]*)**.** Let (α, A) and (β, B) be two soft sets over a common universe U. The bi-intersection of (α, A) and (β, B) is defined as the soft set (ξ, C), satisfying the following conditions.

(1) $C = A \cap B$.
(2) For all $x \in A \cap B$, $\xi(x) = \alpha(x) \cap \beta(x)$.

This is denoted by $(\alpha, A)\tilde{\sqcap}(\beta, B) = (\xi, C)$.

Definition 3.3 (*[2]*)**.** Let (α, A) and (β, B) be two soft sets over a common universe U. The union of (α, A) and (β, B) is defined as the soft set $(\delta, A \cup B)$, satisfying the following conditions.

(1) $C = A \cup B$.
(2) For all $x \in A \cup B$

$$\delta(x) = \begin{cases} \alpha(x) & \text{if } x \in A \setminus B \\ \beta(x) & \text{if } x \in B \setminus A \\ \alpha(x) \cup \beta(x) & \text{if } x \in A \cap B. \end{cases}$$

This is denoted by $(\delta, A \cup B) = (\alpha, A)\tilde{\cup}(\beta, B)$.

Definition 3.4 (*[2]*)**.** Let (α, A) and (β, B) be two soft sets over a common universe U. '(α, A) AND (β, B)', denoted by $(\alpha, A)\tilde{\wedge}(\beta, B)$ and defined as $(\alpha, A)\tilde{\wedge}(\beta, B) = (\gamma, A \times B)$, that is $\gamma(x, y) = \alpha(x) \cap \beta(y)$, for all $(x, y) \in A \times B$.

Definition 3.5 (*[2]*)**.** Let (α, A) and (β, B) be two soft sets over a common universe U. '(α, A) OR (β, B)', denoted by $(\alpha, A)\tilde{\vee}(\beta, B)$ and defined as $(\alpha, A)\tilde{\vee}(\beta, B) = (\eta, A \times B)$ that is $\eta(x, y) = \alpha(x) \cup \beta(y)$, for all $(x, y) \in A \times B$.

Definition 3.6 (*[8]*)**.** For a soft set (α, A), the set

$$\mathrm{Sup}(\alpha, A) = \{x \in A \mid \alpha(x) \neq \emptyset\}$$

is called the support of the soft set (α, A). Soft set (α, A) is called non-null if $\mathrm{Sup}(\alpha, A) \neq \emptyset$.

4. Soft nexuses

Definition 4.1. Let (α, A) be a non-null soft set over a nexus N. Then (α, A) is called a soft nexus on N if $\alpha(x) \in \mathrm{SUB}(N)$, for all $x \in \mathrm{Sup}(\alpha, A)$.

Example 4.2. Let $N = \langle(1, 2), (2, 3)\rangle$, $A = Z_3$ and $\alpha : A \to P(N)$, given by $\alpha(\bar{0}) = \langle(1, 1)\rangle$, $\alpha(\bar{1}) = \langle(1, 2), (2, 1)\rangle$, $\alpha(\bar{2}) = \langle(1, 2), (2, 1)\rangle$, then (α, A) is a soft nexus over N.

Remark 4.3. $\coprod \aleph = \{(a_1, a_2, \dots, a_m) \mid a_1, a_2, \dots, a_m, m \in \aleph\}$.

Example 4.4. Let N be a nexus, $\alpha : \amalg\aleph \to P(N)$ given by

$$\alpha(a) = \begin{cases} \langle a\rangle & \text{If } a \in N \\ \{()\} & \text{o.w.} \end{cases}$$

Theorem 4.5. *Let (α, A) and (β, B) be two soft nexuses over a nexus N. Then $\tilde{\sqcap}$, $\tilde{\cup}$, $\tilde{\wedge}$, $\tilde{\vee}$ of (α, A) and (β, B) are soft nexuses.*

Proof. $(\alpha, A)\tilde{\sqcap}(\beta, B) = (\xi, A \cap B)$, that $\xi(x) = \alpha(x) \cap \beta(x)$ for all $x \in A \cap B$. Since for all $x \in A \cap B$, $\alpha(x)$ and $\beta(x) \in \mathrm{SUB}(N)$, then by Theorem 2.15, $\xi(x) \in \mathrm{SUB}(N)$.

Since the union of two subnexuses is a subnexus, therefore by Theorem 2.15, $(\alpha, A)\tilde{\cup}(\beta, B) = (\delta, A \cup B)$ is a soft nexus. For $\tilde{\wedge}$, $\tilde{\vee}$, by similarity we can see that $(\alpha, A)\tilde{\wedge}(\beta, B)$, $(\alpha, A)\tilde{\vee}(\beta, B)$ are soft nexuses. □

Now if we consider a partial ordered relation $\subseteq$ on the set of all soft nexuses on N, then this set with $\subseteq$ is a lattice and for all soft sets (α, A) and (β, B) on N, we have

$$(\alpha, A) \wedge (\beta, B) = (\alpha, A)\tilde{\sqcap}(\beta, B)$$

and

$$(\alpha, A) \vee (\beta, B) = (\alpha, A)\tilde{\cup}(\beta, B).$$

Remark 4.6. In general, the set of all soft groups and soft rings (see [3,8]) over a group G or ring R with $\tilde{\sqcap}$ and $\tilde{\cup}$ are not lattices. But, the set of all soft nexuses over a nexus N with $\tilde{\sqcap}$ and $\tilde{\cup}$ is a lattice.

Remark 4.7. On every countable set $A = \{a_i\}_{i=1}^{i=\infty}$, we can consider the following ordered

$$a_i \leq a_j \Leftrightarrow i \leq j.$$

Theorem 4.8. *Let $A = \{a_i\}_{i=1}^{i=\infty}$ be a countable set and $\{b_i\}_{i=1}^{\infty} \in N$. Then there exists a one to one soft nexus (α, A) over a nexus N such that*

$$a_i \leq a_j \Leftrightarrow \alpha(a_i) \leq \alpha(a_j)$$

for all $a_i, a_j \in A$.

Proof. We define, $\alpha : A \to P(N)$ given by $\alpha(a_i) = \langle (b_1, b_2, \ldots, b_i) \rangle$. It is trivial that (α, A) is a soft nexus over N. □

Definition 4.9. Let (α, A) and (β, B) be two soft nexuses over a nexus N. Then (β, B) is called a soft subnexus of (α, A), if it satisfies the following.

(i) $B \subseteq A$
(ii) $\beta(x) \in \mathrm{SUB}(\alpha(x))$, for all $x \in \mathrm{Sup}(\beta, B)$.

Theorem 4.10. *Every soft nexus (α, A) over a nexus N can be written as the union of soft subnexuses of (α, A).*

Proof. Let N be a nexus and $\{N_i\}_{i=1}^{\infty}$ be a subnexus of N. We take $A_i = \{a \in A | \alpha(x) = N_i\}$. We define $\alpha_i : A_i \to P(N_i)$ given by $\alpha_i(a) = N_i$. Then (α_i, A_i) is a soft nexus over N_i and for all $i = 1, 2, \ldots, (\alpha_i, A_i)$ are soft subnexuses of (α, A) and we have $\tilde{\bigcup}_{i=1}^{i=\infty} (\alpha_i, A_i) = (\alpha, A)$. □

Lemma 4.1. *Let $N = \langle (a_1, a_2, \ldots, a_n) \rangle$ be a cyclic nexus and (α, A) be a soft nexus over a nexus N. Then*

(i) *$\{\alpha(x) | x \in A\}$ is a chain.*
(ii) *$|\mathrm{Im}\alpha| \leq 1 + (\sum_{i=1}^{n} a_i)$.*

Proof. By Theorems 2.16 and 2.17, the proof is trivial. □

Definition 4.11. Let (α, A) and (β, B) be two soft nexuses over M and N, respectively. The product of soft nexuses (α, A) and (β, B) is defined as $(\alpha, A) \times (\beta, B) = (\rho, A \times B)$, where $\rho(x, y) = \alpha(x) \times \beta(y)$ for all $(x, y) \in A \times B$. By Definition 2.18, it is clear that $(\alpha, A) \times (\beta, B)$ is a soft nexus over $M \times N$.

Definition 4.12. Let (α, A) be a soft nexus over a nexus N. Then (α, A) is called a cyclic soft nexus, if for all $x \in \mathrm{Sup}(\alpha, A)$, $\alpha(x)$ be a cyclic subnexus of N.

Lemma 4.2. *Nexus N is a cyclic nexus if and only if every soft nexus (α, A) over a nexus N be cyclic.*

Proof. Since every subnexus of a cyclic nexus N is cyclic, then every soft nexus (α, A) on N is cyclic. Now suppose that every soft nexus (α, A) on N be cyclic. If $N = \langle a, b \rangle$ where $a \not\leq b$, $b \not\leq a$ then define $\alpha : Z_3 \to P(N)$ given by $\alpha(\bar{0}) = \langle a \rangle$, $\alpha(\bar{1}) = \langle a \rangle$ and $\alpha(\bar{2}) = \langle a, b \rangle$. We can see that (α, Z_3) is a soft nexus over a nexus N, but is not cyclic, which is the contradiction. Thus N is cyclic. □

Lemma 4.3. *Every soft subnexus (β, B) of cyclic soft nexus (α, A) over a nexus N is cyclic.*

Proof. Suppose $x \in B$ and $x \in \mathrm{Sup}(\beta, B)$. Since $(\beta, B) \subseteq (\alpha, A)$, thus $B \subseteq A$ and $\beta(x) \subseteq \alpha(x)$ for all $x \in B$. Since (α, A) is cyclic, hence $\alpha(x) = \langle a \rangle$ for some $x \in \alpha(x)$. Thus by Theorem 2.14, $\beta(x) = \langle b \rangle$ for some $b \in \beta(x)$. □

Lemma 4.4. *Let (α, A) and (β, B) be two cyclic soft nexuses over a nexus N. Then $(\alpha, A)\tilde{\sqcap}(\beta, B)$ and $(\alpha, A)\tilde{\wedge}(\beta, B)$ are cyclic soft nexuses. Also if $A \cap B = \emptyset$, then $(\alpha, A)\tilde{\cup}(\beta, B)$ is a cyclic soft nexus.*

Proof. Suppose $(\alpha, A)\tilde{\sqcap}(\beta, B) = (\xi, A \cap B)$. Thus $\xi(x) = \alpha(x) \cap \beta(x)$, for all $x \in A \cap B$. Now let $x \in \mathrm{Sup}(\xi, A \cap B)$, $\alpha(x) = \langle a \rangle$ and $\beta(x) = \langle b \rangle$. Thus by Theorem 2.14, $\xi(x)$ is cyclic and so $(\xi, A \cup B)$ is cyclic.

By the definition of $(\alpha, A)\tilde{\cup}(\beta, B)$, if $A \cap B = \emptyset$, then since (α, A) and (β, B) are cyclic, the proof is trivial. □

Lemma 4.5. *Let $M \in Mat_{m\times n}(\aleph)$ and $A = \{A^1, A^2, A^3, \ldots, A^n\}$, where A^i is the ith column of M. Then there exist nexus N and mapping $\alpha : A \to P(N)$, such that (α, A) is a cyclic soft nexus over a nexus N.*

Proof. we define the nexus $N = \langle A^1, A^2, A^3, \ldots, A^n \rangle$ and $\alpha : A \to P(N)$ given by $\alpha(A^i) = \langle A^i \rangle$. □

Definition 4.13. Let (α, A), (β, B) be two soft nexuses over a nexus N and $(\beta, B) \subseteq (\alpha, A)$. Then (β, B) is called a prime soft subnexus of (α, A) if $\beta(x)$ is a prime subnexus of $\alpha(x)$ for all elements in $\mathrm{Sup}(\beta, B)$.

Example 4.14. Let N be a nexus, $M \neq N \in \mathrm{SUB}(N)$, $\alpha : N \to P(N)$ given by $\alpha(a) = N - R_a$ and $\beta : M \to P(N)$ given by $\beta(a) = M - R_a$. Then (β, B) is a prime soft subnexus of (α, A).

Example 4.15. Consider the nexus

$$N = \{(), (1), (2), (1, 1), (1, 2), (1, 3), (2, 1), (2, 2)\}.$$

If $\alpha : A = \{a, b, c\} \to P(N)$ given by

$$\alpha(a) = \langle (1, 2), (2, 2) \rangle, \qquad \alpha(b) = \langle (1, 1), (2, 2) \rangle, \qquad \alpha(c) = \langle (1, 2), (2, 2) \rangle,$$

and $\beta : A = \{a, b, c\} \to P(N)$ given by

$$\beta(a) = \langle (2, 2) \rangle, \qquad \beta(b) = \langle (2, 2) \rangle, \qquad \beta(c) = \langle (1, 2), (2, 1) \rangle,$$

then (β, A) is a prime soft subnexus of (α, A).

Lemma 4.6. *Let (β_1, B_1) and (β_2, B_2) be two prime soft subnexuses of soft nexus (α, A) over a nexus N. Then $(\beta_1, B_1)\tilde{\cup}(\beta_2, B_2)$ and $(\beta_1, B_1)\tilde{\vee}(\beta_2, B_2)$ are prime soft subnexuses.*

Proof. By the definitions of $(\beta_1, B_1)\tilde{\cup}(\beta_2, B_2)$, $(\beta_1, B_1)\tilde{\vee}(\beta_2, B_2)$ and the definition of prime subnexus, the proof is trivial. □

Note that $(\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2)(\beta_1, B_1)\tilde{\wedge}(\beta_2, B_2)$ are not prime soft subnexuses.

Example 4.16. Consider the nexus $N = \langle (1, 2), (2, 3) \rangle$, $A = \{1, 2\}$, $\alpha : A \to P(N)$ given by

$$\alpha(1) = N, \qquad \alpha(2) = \langle (1, 2), (2, 1) \rangle$$

$\beta_1 : A \to P(N)$ given by

$$\beta_1(1) = N - \{(2, 3)\}, \qquad \beta_1(2) = \langle (1, 2), (2, 1) \rangle$$

and $\beta_2 : A \to P(N)$ given by

$$\beta_2(1) = \beta_1(1), \qquad \beta_2(2) = \langle (1, 1), (2, 2) \rangle.$$

It is clear that (β_1, A) and (β_2, A) are prime soft subnexuses of (α, A), but $(\xi, B_1 \cap B_2) = (\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2)$ is not a prime soft subnexus, because $\xi(2) = \langle (1, 1), (2, 1) \rangle$ is not a prime subnexus of $\alpha(2)$. Since $(1, 2) \wedge (2, 2) = (1) \in \xi(2)$, but neither $(1, 2)$ nor $(2, 2)$ belong to $\xi(2)$.

Theorem 4.17. *Let (β_1, B_1) and (β_2, B_2) be two prime soft subnexuses of soft nexus (α, A) over a nexus N. Thus for all $x \in B_1 \cap B_2$, $\beta_1(x) = \alpha(x) - R_a$ and $\beta_2(x) = \alpha(x) - R_b$, for some $a, b \in \alpha(x)$.*

(i) *If for all $x \in B_1 \cap B_2$, a and b are two comparable addresses, Then $(\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2)$ is a prime soft subnexus of (α, A).*
(ii) *If there exists $x \in B_1 \cap B_2$, such that $\beta_1(x) = \alpha(x) - R_a$ and $\beta_2(x) = \alpha(x) - R_b$, a and b are not comparable addresses, then $(\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2)$ is not a prime soft subnexus of (α, A).*
(iii) *Suppose (ii) does hold and $B_1 = B_2 = A$, then $(\beta_1, A)\tilde{\cup}(\beta_2, A) = (\alpha, A)$.*

Proof. (i) Without loss of generality, suppose that $x \in B_1 \cap B_2$, $\beta_1(x) = \alpha(x) - R_a$, $\beta_2(x) = \alpha(x) - R_b$ and $a \leq b$. So $R_b \subseteq R_a$ and $\alpha(x) - R_a \subseteq \alpha(x) - R_b$. Thus if $(\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2) = (\xi, B_1 \cap B_2)$, then $\xi(x) = \alpha(x) - R_a$. Hence $(\xi, B_1 \cap B_2)$ is a prime soft subnexus of (α, A).

(ii) Let $x \in B_1 \cap B_2$, $\beta_1(x) = \alpha(x) - R_a$, $\beta_2(x) = \alpha(x) - R_b$, a and b be incomparable. Thus $a \wedge b < a$ and $a \wedge b < b$. So that $a \wedge b \notin R_a = \alpha(x) - \beta_1(x)$ and $a \wedge b \notin R_b = \alpha(x) - \beta_2(x)$. Therefore $a \wedge b \in \beta_1(x) \cap \beta_2(x)$. Consequently $(\beta_1, B_1)\tilde{\sqcap}(\beta_2, B_2)$ is not prime.

(iii) Let (ii) does hold and $B_1 = B_2 = A$. It is clear that $(\beta_1, B_1)\tilde{\sqcup}(\beta_2, B_2) \subseteq (\alpha, A)$. Now suppose that $x \in A$ and $c \in \alpha(x) - \beta_1(x)$, so $c \in R_a$. Hence $a \leq c$. If $c \in R_b$ then $b \leq c$, so $a, b \in \langle c \rangle$ and it means that a and b are comparable, that is not true. Therefore $c \notin R_b$ and so $c \in \alpha(x) - R_b = \beta_2(x)$. Thus $c \in (\beta_1, B_1)\tilde{\sqcup}(\beta_2, B_2)$. Consequently $(\beta_1, B_1)\tilde{\sqcup}(\beta_2, B_2) = (\alpha, A)$. □

Theorem 4.18. *A soft nexus (α, A) over a nexus N is cyclic if and only if every proper soft subnexus (β, B) of (α, A) is prime.*

Proof. Let (β, B) be a proper soft subnexus of (α, A), $x \in B$ and $a \wedge b \in \beta(x)$. Thus $a \wedge b \in \alpha(x)$. Since $\alpha(x)$ is cyclic and $a, b \in \alpha(x)$, then $a \leq b$ or $b \leq a$. Hence $a \in \beta(x)$ or $b \in \beta(x)$. Conversely, suppose that every proper soft subnexus (β, B) of (α, A) be prime, $x \in A$ and $a, b \in \alpha(x)$. Consider $\beta_1 : A \to P(N)$ given by $\beta_1(x) = \alpha(x) - R_a$, for all $x \in A$ and $\beta_2 : A \to P(N)$ given by $\beta_2(x) = \alpha(x) - R_b$, for all $x \in A$. It is clear that (β_1, A) and (β_2, A) are prime soft subnexuses of (α, A). If a and b are not comparable, by (ii) of Theorem 4.17, $(\beta_1, A)\tilde{\sqcap}(\beta_2, A)$, it is a proper soft subnexus of (α, A), which is not prime, which is a contradiction. Thus a and b are comparable and so by Theorem 2.16, $\alpha(x)$ is cyclic for all $x \in A$. □

Definition 4.19. A soft subnexus (β, B) of (α, A) is called a maximal soft subnexus, if it satisfies the following conditions.

(i) $B = A$.
(ii) $\beta(x)$ be a maximal subnexus of $\alpha(x)$, for all $x \in \mathrm{Sup}(\beta, B)$.

Example 4.20. Consider the nexus $N = \langle(1, 1), (2, 2), (3, 3)\rangle$. Now let $\alpha : Z_4 \to P(N)$ given by $\alpha(\bar{0}) = \langle(1, 1), (2)\rangle$, $\alpha(\bar{1}) = \langle(1, 1), (2, 1)\rangle$, $\alpha(\bar{2}) = \langle(1, 1), (3)\rangle$, $\alpha(\bar{3}) = \langle(2, 2), (3, 1)\rangle$, $\alpha(\bar{4}) = \langle(2, 2), (3, 2)\rangle$ and $\beta : Z_4 \to P(N)$ given by $\beta(\bar{0}) = \langle(1, 1)\rangle$, $\beta(\bar{1}) = \langle(2, 1)\rangle$, $\beta(\bar{2}) = \langle(1, 1), (2)\rangle$, $\beta(\bar{3}) = \langle(2, 2), (3)\rangle$, $\beta(\bar{4}) = \langle(2, 2), (3, 1)\rangle$. Since for all $x \in Z_4$, by Theorem 2.28, $\beta(x)$ is a maximal subnexus of $\alpha(x)$. So (β, Z_4) is a maximal soft subnexus of (α, Z_4).

Lemma 4.7. *Every maximal soft subnexus of (α, A) is a prime soft subnexus.*

Proof. Let (β, A) be a maximal soft subnexus of the soft nexus (α, A). Thus for all $x \in \mathrm{Sup}(\beta, A)$, $\beta(x)$ is a maximal subnexus of $\alpha(x)$. Hence by Theorem 2.28, for some maximal element $m \in \alpha(x)$, we have $\beta(x) = \alpha(x) - \{m\}$. Since m is a maximal element, then $R_m = \{m\}$ and so $\alpha(x) - \{m\} = \alpha(x) - R_m$. So by Theorem 2.28, $\beta(x)$ is a prime soft subnexus of (α, A). □

Theorem 4.21. *A soft nexus (α, A) over a nexus N is cyclic, if and only if it has just one maximal soft subnexus.*

Proof. Suppose that the soft nexus (α, A) over a nexus N is cyclic. Then for all $x \in A$, $\alpha(x)$ is a cyclic subnexus of N. Hence $\alpha(x) = \langle a \rangle$, for some $a \in \alpha(x)$. So $\alpha(x)$ has just one maximal address, that is a. Thus $M = \alpha(x) - \{a\}$ is the only maximal subnexus of $\alpha(x)$. If for all $x \in A$, generator of $\alpha(x)$, denoted by $a_{\alpha(x)}$ and $\beta : A \to P(N)$ defined by $\beta(x) = \alpha(x) - \{a_{\alpha(x)}\}$, then (β, A) is the only maximal subnexus (α, A). Conversely, let (α, A) has just one maximal soft nexus (β, A) and $x \in A$. So $\beta(x) = \alpha(x) - \{a\}$, where a is the maximal address of $\alpha(x)$. We claim that $\alpha(x) = \langle a \rangle$. Obviously, $\langle a \rangle \subseteq \alpha(x)$. Now show that $\alpha(x) \subseteq \langle a \rangle$. To do so suppose that $b \in \alpha(x)$. If a and b are comparable, then $b \leq a$ and so $b \in \langle a \rangle$. If a and b are not comparable, then there exists a maximal address m, such that $b \leq m$. It implies that (β', A) where $\beta'(x) = \alpha(x) - \{m\}$ and $\beta'(y) = \beta(y)$, for all elements $y \neq x$ of A is another maximal soft subnexus of (α, A), that is a contradiction by the uniqueness of maximal soft subnexus of (α, A). Therefore all addresses of $\alpha(x)$ is comparable with a. Hence by Theorem 2.16, (α, A) is cyclic. □

Definition 4.22. Let (α, A) be a soft set over a nexus N. Then

$$\sup\{|\alpha(x)|\,|x \in A\}.$$

It is called the integer number induced by (α, A) and denoted by $|(\alpha, A)|$.

Theorem 4.23. *Let U be the set of all soft nexuses over a nexus N. Then we have the following.*

(i) *Mapping $f : (U, \subseteq) \to (\aleph, \leq)$, defined by $f((\alpha, A)) = |(\alpha, A)|$ is a homomorphism.*
(ii) $|(\alpha, A) \times (\beta, B)| = |(\alpha, A)| \times |(\beta, B)|$.
(iii) $|(\alpha, A)\tilde{\wedge}(\beta, B)| \leq |(\alpha, A)|$.
(iv) $|(\alpha, A)\tilde{\vee}(\beta, B)| \geq |(\alpha, A)|$.

Proof. (i) If $(\alpha, A), (\beta, B) \in U$ and $(\beta, B) \subseteq (\alpha, A)$, then $B \subseteq A$ and for all $x \in B$, $\beta(x) \subseteq \alpha(x)$. Thus $|\beta(x)| \leq |\alpha(x)|$. So $\sup\{|\beta(x)| x \in B\} \leq \sup\{\alpha(x)| x \in A\}$ and hence $|(\beta, B)| \leq |(\alpha, A)|$. Thus f is a homomorphism.

(ii)

$$\begin{aligned} |(\alpha, A) \times (\beta, B)| &= \sup\{|\alpha(x)\beta(y)|(x, y) \in A \times B\} \\ &= \sup\{|\alpha(x)|; x \in A\} \times \sup\{|\beta(y)| y \in B\} \\ &= |(\alpha, A)| \times |(\beta, B)|. \end{aligned}$$

By similarity, we can see (iii) and (iv) hold. □

Theorem 4.24. *Let $n \in \aleph$. Then there exist finite set A, nexus N and mapping $\alpha : A \to P(N)$ such that (α, A) is a cyclic soft nexus over a nexus N and $|(\alpha, A)| = n$.*

Proof. If $N = \langle \underbrace{(1, 1, \ldots, 1)}_{n\text{-order}} \rangle$, $A = \{a_1, a_2, \ldots, a_n\}$ and $\alpha : A \to P(N)$, given by

$$\alpha(a_1) = \langle 1 \rangle, \alpha(a_2) = \langle (1, 1) \rangle, \ldots, \alpha(a_n) = \langle \underbrace{(1, 1, \ldots, 1)}_{n\text{-order}} \rangle.$$

Then we can see, (α, A) is a cyclic soft nexus over N and $|(\alpha, A)| = n$. □

5. Conclusion

The relation between (cyclic) soft nexus, prime and maximal subnexuses are studied; moreover we have shown that under suitable conditions the above notions are closed with respect to the main operators in soft sets theory. Finally a way for making a cyclic soft nexus respect to any natural number is presented.

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