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Green–Schwarz, Nambu–Goto actions, and Cayley's hyperdeterminant

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Abstract

It has been recently shown that Nambu–Goto action can be re-expressed in terms of Cayley's hyperdeterminant with the manifest $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ symmetry. In the present Letter, we show that the same feature is shared by Green–Schwarz σ-model for $N=2$ superstring whose target space–time is $D=2+2$. When its zweibein field is eliminated from the action, it contains the Nambu–Goto action which is nothing but the square root of Cayley's hyperdeterminant of the pull-back in superspace $\sqrt{\mathcal{D}\mathrm{et}(\Pi_{i\alpha\dot{\alpha}})}$ manifestly invariant under $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$. The target space–time $D=2+2$ can accommodate self-dual supersymmetric Yang–Mills theory. Our action has also fermionic κ-symmetry, satisfying the criterion for its light-cone equivalence to Neveu–Schwarz–Ramond formulation for $N=2$ superstring.

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1. Introduction

Cayley's hyperdeterminant [1], initially an object of mathematical curiosity, has found its way in many applications to physics [2]. For instance, it has been used in the discussions of quantum information theory [3,4], and the entropy of the STU black hole [5,6] in four-dimensional string theory [7].

More recently, it has been shown [8] that Nambu–Goto (NG) action [9,10] with the $D=2+2$ target space–time possesses the manifest global $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \equiv [SL(2,\mathbb{R})]^3$ symmetry. In particular, the square root of the determinant of an inner product of pull-backs can be rewritten exactly as a Cayley's hyperdeterminant [1] realizing the manifest $[SL(2,\mathbb{R})]^3$ symmetry.

It is to be noted that the space–time dimensions $D=2+2$ pointed out in [8] are nothing but the consistent target space–time of $N=2$[1] NSR superstring [13–19]. However, the NSR formulation [16,17] has a drawback for rewriting it purely in terms of a determinant, due to the presence of fermionic superpartners on the 2D world-sheet. On the other hand, it is well known that a GS formulation [12] without explicit world-sheet supersymmetry is classically equivalent to a NSR formulation [11] on the light-cone, when the former has fermionic κ-symmetry [15,20]. From this viewpoint, a GS σ-model formulation in [14] of $N=2$ superstring [16–18] seems more advantageous, despite the temporary sacrifice of world-sheet supersymmetry. However, even the GS formulation [14] itself has an obstruction, because obviously the kinetic term in the

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[1] The $N=2$ here implies the number of world-sheet supersymmetries in the Neveu–Schwarz–Ramond (NSR) formulation [11]. Its corresponding Green–Schwarz (GS) formulation [12–14] might be also called '$N=2$' GS superstring in the present Letter. Needless to say, the number of world-sheet supersymmetries should *not* be confused with that of space–time supersymmetries, such as $N=1$ for type I superstring, or $N=2$ for type IIA or IIB superstring [15].

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GS action is not of the NG-type equivalent to a Cayley's hyperdeterminant.

In this Letter, we overcome this obstruction, by eliminating the zweibein (or 2D metric) *via* its field equation which is *not* algebraic. Despite the *non*-algebraic field equation, such an elimination is possible, just as a NG action [9,10] is obtained from a Polyakov action [21]. Similar formulations are known to be possible for Type I, heterotic, or Type II superstring theories, but here we need to deal with $N = 2$ superstring [16] with the target space–time $D = 2 + 2$ instead of 10D. We show that the same global $[SL(2,\mathbb{R})]^3$ symmetry [8] is inherent also in $N = 2$ GS action in [14] with $N = (1,1)$ supersymmetry in $D = 2 + 2$ as the special case of [13], when the zweibein field is eliminated from the original action, re-expressed in terms of NG-type determinant form.

As is widely recognized, the quantum-level equivalence of NG action [9,10] to Polyakov action [21] has not been well established even nowadays [22]. As such, we do not claim the quantum equivalence of our formulation to the conventional $N = 2$ NSR superstring [16,17] or even to $N = 2$ GS string [13] itself. In this Letter, we point out only the existence of fermionic κ-symmetry and the manifest global $[SL(2,\mathbb{R})]^3$ symmetry with Cayley's hyperdeterminant as classical-level symmetries, after the elimination of 2D metric from the classical GS action [14] of $N = 2$ superstring [16,17].

As in $N = 2$ NSR superstring [16,17], the target $D = (2,2;2,2)$[2] superspace [19] of $N = 2$ GS superstring [14] can accommodate self-dual supersymmetric Yang–Mills (SDSYM) multiplet [18,19] with $N = (1,1)$ space–time supersymmetry [13,14,19], which is supersymmetric generalization of purely bosonic YM theory in $D = 2 + 2$ [23]. The importance of the latter is due to the conjecture [24] that all the bosonic integrable or soluble models in dimensions $D \leqslant 3$ are generated by self-dual Yang–Mills (SDYM) theory [23]. Then it is natural to 'supersymmetrize' this conjecture [24], such that all the supersymmetric integrable models in $D \leqslant 3$ are generated by SDSYM in $D = 2 + 2$ [18,19], and thereby the importance of $N = 2$ GS σ-model in [14] is also reemphasized.

In the next two sections, we present our total action of $N = 2$ GS σ-model [14] whose target superspace is $D = (2,2;2,2)$ [19], and show the existence of fermionic κ-symmetry [20] as well as $[SL(2,\mathbb{R})]^3$ symmetry, due to the Cayley's hyperdeterminant for the kinetic terms in the NG form. We next confirm that our action is derivable from the $N = 2$ GS σ-model [14] which is light-cone equivalent to $N = 2$ NSR superstring [16,17], by eliminating a zweibein or a 2D metric.

[2] We use in this Letter the symbol $D = (2,2;2,2)$ for the target superspace, meaning $2 + 2$ bosonic coordinates, plus 2 chiral and 2 anti-chiral fermionic coordinates [14,19]. In terms of supersymmetries in the *target* $D = 2+2$ space–time, this superspace corresponds to $N = (1,1)$ [14,19], which should not be confused with $N = 2$ on the world-sheet. In other words, $D = (2,2;2,2)$ is superspace for $N = (1,1)$ supersymmetry realized on $D = 2 + 2$ space–time. Maximally, we can think of $N = (4,4)$ supersymmetry for SDSYM [18], but we focus only on $N = (1,1)$ supersymemtry in this Letter.

2. Total action with $[SL(2,\mathbb{R})]^3$ symmetry

We first give our total action with manifest global $[SL(2,\mathbb{R})]^3$ symmetry, then show its fermionic κ-symmetry [20]. Our action has classical equivalence to the GS σ-model formulation [14] of $N = 2$ superstring [16,17] with the right $D = (2,2;2,2)$ target superspace that accommodates self-dual supersymmetric YM multiplet [14,17–19]. In this section, we first give our total action of our formulation, leaving its derivation or justifications for later sections.

Our total action $I \equiv \int d^2\sigma\, \mathcal{L}$ has the fairly simple Lagrangian

$$\mathcal{L} = +\sqrt{-\det(\Gamma_{ij})} + \epsilon^{ij}\Pi_i{}^A\Pi_j{}^B B_{BA} \tag{2.1a}$$

$$= +\sqrt{+\mathcal{D}\text{et}(\Pi_{i\alpha\dot\alpha})}\big(1 + 2\Pi_-{}^A\Pi_+{}^B B_{BA}\big)$$
$$\equiv \mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZNW}}, \tag{2.1b}$$

where respectively the two terms $\mathcal{L}_{\text{NG}}$ and $\mathcal{L}_{\text{WZNW}}$ are called 'NG-term' and 'WZNW-term'. The indices $i, j, \ldots = 0, 1$ are for the curved coordinates on the 2D world-sheet, while $+, -$ are for the light-cone coordinates for the local Lorentz frames, respectively defined by the projectors

$$P_{(i)}{}^{(j)} \equiv \frac{1}{2}\big(\delta_{(i)}{}^{(j)} + \epsilon_{(i)}{}^{(j)}\big),$$
$$Q_{(i)}{}^{(j)} \equiv \frac{1}{2}\big(\delta_{(i)}{}^{(j)} - \epsilon_{(i)}{}^{(j)}\big), \tag{2.2}$$

where $(i), (j), \ldots = (0), (1), \ldots$ are used for local Lorentz coordinates, and $(\eta_{(i)(j)}) = \text{diag}(+,-)$. Note that $\delta_+{}^+ = \delta_-{}^- = +1$, $\epsilon_+{}^+ = -\epsilon_-{}^- = +1$, $\eta_{++} = \eta_{--} = 0$, $\eta_{+-} = \eta_{-+} = 1$. Whereas $\Pi_i{}^A$ is the superspace pull-back, Γ_{ij} is a product of such pull-backs:

$$\Pi_i{}^A \equiv \big(\partial_i Z^M\big)E_M{}^A, \tag{2.3a}$$
$$\Gamma_{ij} \equiv \eta_{\underline{a}\underline{b}}\Pi_i{}^{\underline{a}}\Pi_j{}^{\underline{b}} = \Pi_i{}^{\underline{a}}\Pi_{j\underline{a}}, \tag{2.3b}$$

for the target superspace coordinates Z^M. The $(\eta_{\underline{a}\underline{b}}) = \text{diag}(+,+,-,-)$ is the $D = 2 + 2$ space–time metric. We use the indices $\underline{a}, \underline{b}, \ldots = 0,1,2,3$ (or $\underline{m}, \underline{n}, \ldots = 0,1,2,3$) for the bosonic local Lorentz (or curved) coordinates. The $E_M{}^A$ is the flat background vielbein [25] for $D = (2,2;2,2)$ target superspace [14,19]. Its explicit form is

$$\big(E_M{}^A\big) = \begin{pmatrix} \delta_{\underline{m}}{}^{\underline{a}} & 0 \\ -\frac{i}{2}(\sigma^{\underline{a}}\theta)_{\underline{\mu}} & \delta_{\underline{\mu}}{}^{\underline{\alpha}} \end{pmatrix},$$
$$\big(E_A{}^M\big) = \begin{pmatrix} \delta_{\underline{a}}{}^{\underline{m}} & 0 \\ +\frac{i}{2}(\sigma^{\underline{m}}\theta)_{\underline{\alpha}} & \delta_{\underline{\alpha}}{}^{\underline{\mu}} \end{pmatrix}. \tag{2.4}$$

We use the underlined Greek indices: $\underline{\alpha} \equiv (\alpha, \dot\alpha)$, $\underline{\beta} \equiv (\beta, \dot\beta), \ldots$ for the pair of fermionic indices, where $\alpha, \beta, \ldots = 1, 2$ are for chiral coordinates, and $\dot\alpha, \dot\beta, \ldots = \dot 1, \dot 2$ are for anti-chiral coordinates [19]. The indices $\underline{\mu}, \underline{\nu}, \ldots = 1,2,3,4$ are for curved fermionic coordinates. Similarly to the superspace for the Minkowski space–time with the signature $(+,-,-,-)$ [25], a bosonic index is equivalent to a pair of fermionic indices, e.g., $\Pi_i{}^{\underline{a}} \equiv \Pi_i{}^{\alpha\dot\alpha}$. In (2.4), we use the expressions like $(\sigma^{\underline{a}}\theta)_{\underline{\alpha}} \equiv -(\sigma^{\underline{a}})_{\underline{\alpha}\underline{\beta}}\theta^{\underline{\beta}}$ for the σ-matrices in $D = 2 + 2$ [19,26]. Relevantly, the only non-vanishing supertorsion components are

[14,19]

$$T_{\underline{\alpha}\underline{\beta}}{}^{\underline{c}} = i\left(\sigma^{\underline{c}}\right)_{\underline{\alpha}\underline{\beta}} = \begin{cases} +i(\sigma_{\underline{c}})_{\alpha\dot{\beta}}, \\ +i(\sigma_{\underline{c}})_{\dot{\alpha}\beta} = +i(\sigma_{\underline{c}})_{\beta\dot{\alpha}}. \end{cases} \tag{2.5}$$

The antisymmetric tensor superfield B_{AB} has the superfield strength

$$G_{ABC} \equiv \frac{1}{2}\nabla_{[A}B_{BC)} - \frac{1}{2}T_{[AB|}{}^{D}B_{D|C)}. \tag{2.6}$$

Our anti-symmetrization rule is such as $M_{[AB)} \equiv M_{AB} - (-1)^{AB}M_{BA}$ *without* the factor 1/2. The flat-background values of G_{ABC} is [14,19]

$$G_{\underline{\alpha}\underline{\beta}\underline{c}} = +\frac{i}{2}(\sigma_{\underline{c}})_{\underline{\alpha}\underline{\beta}} = \begin{cases} +\frac{i}{2}(\sigma_{\underline{c}})_{\alpha\dot{\beta}}, \\ +\frac{i}{2}(\sigma_{\underline{c}})_{\dot{\alpha}\beta} = +\frac{i}{2}(\sigma_{\underline{c}})_{\beta\dot{\alpha}}. \end{cases} \tag{2.7}$$

In our formulation, the Lagrangian (2.1a) needs the 'square root' of the matrix Γ_{ij}, analogous to the zweibein $e_i{}^{(j)}$ as the 'square root' of the 2D metric g_{ij}, defined by

$$\gamma_i{}^{(k)}\gamma_{j(k)} = \Gamma_{ij}, \qquad \gamma_{(k)}{}^{i}\gamma^{(k)j} = \Gamma^{ij}, \tag{2.8a}$$

$$\gamma_i{}^{(k)}\gamma_{(k)}{}^{j} = \delta_i{}^{j}, \qquad \gamma_{(i)}{}^{k}\gamma_k{}^{(j)} = \delta_{(i)}{}^{(j)}. \tag{2.8b}$$

Relevantly, we have $\gamma = \sqrt{-\Gamma}$ for $\Gamma \equiv \det(\Gamma_{ij})$ and $\gamma \equiv \det(\gamma_i{}^{(j)})$. We define $\Pi_\pm{}^A \equiv \gamma_\pm{}^i\Pi_i{}^A$ for the $\pm$ local light-cone coordinates. For our formulation with (2.1), we always use the γ's to convert the curved indices $i, j, \ldots = 0, 1$ into local Lorentz indices $(i), (j), \ldots = (0), (1)$.

From (2.8), it is clear that we can always define the 'square root' of Γ_{ij} of (2.3b) just as we can always define the zweibein $e_i{}^{(j)}$ out of a 2D metric g_{ij}. In fact, (2.8) determines $\gamma_i{}^{(j)}$ up to 2D local Lorentz transformations $O(1,1)$, because (2.8) is covariant under arbitrary $O(1,1)$. However, (2.8) has much more significance, because if the curved indices ij of Γ_{ij} are converted into 'local' ones, then it amounts to

$$\begin{aligned}\Gamma_{(i)(j)} &= \gamma_{(i)}{}^{k}\gamma_{(j)}{}^{l}\Gamma_{kl} = \gamma_{(i)}{}^{k}\gamma_{(j)}{}^{l}\left(\gamma_k{}^{(m)}\gamma_{l(m)}\right) \\ &= \left(\gamma_{(i)}{}^{k}\gamma_k{}^{(m)}\right)\left(\gamma_{(j)}{}^{l}\gamma_{l(m)}\right) = \delta_{(i)}{}^{(m)}\eta_{(j)(m)} \\ &= \eta_{(i)(j)} \Longrightarrow \Gamma_{(i)(j)} = \eta_{(i)(j)}.\end{aligned} \tag{2.9}$$

In terms of light-cone coordinates, this implies formally the Virasoro conditions [27]

$$\Gamma_{++} \equiv \Pi_+{}^{\underline{a}}\Pi_{+\underline{a}} = 0, \qquad \Gamma_{--} \equiv \Pi_-{}^{\underline{a}}\Pi_{-\underline{a}} = 0, \tag{2.10}$$

because $\eta_{++} = \eta_{--} = 0$. The only caveat here is that our $\gamma_i{}^{(j)}$ is not exactly the zweibein $e_i{}^{(j)}$, but it differs only by certain factor, as we will see in (4.6).

The result (2.10) is not against the original results in NG formulation [9,10]. At first glance, since the NG action has no metric, it seems that Virasoro condition [27] will not follow, unless a 2D metric is introduced as in Polyakov formulation [21]. However, it has been explicitly shown that the Virasoro conditions follow as first-order constraints, when canonical quantization is performed [10]. Naturally, this quantum-level result is already reflected at the classical level, i.e., the Virasoro condition (2.10) follows, when the ij indices on $\Gamma_{ij} \equiv \Pi_i{}^{\underline{a}}\Pi_{j\underline{a}}$ are converted into 'local Lorentz indices' by using the γ's in (2.8).

Most importantly, $\mathcal{D}\text{et}(\Pi_{i\alpha\dot{\alpha}})$ in (2.1b) is a Cayley's hyperdeterminant [1,8], related to the ordinary determinant in (2.1a) by

$$\begin{aligned}\mathcal{D}\text{et}(\Pi_{i\alpha\dot{\alpha}}) &= -\frac{1}{2}\epsilon^{ij}\epsilon^{kl}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\gamma}\dot{\delta}}\Pi_{i\alpha\dot{\alpha}}\Pi_{j\beta\dot{\beta}}\Pi_{k\gamma\dot{\gamma}}\Pi_{l\delta\dot{\delta}} \\ &= -\det(\Gamma_{ij}),\end{aligned} \tag{2.11a}$$

$$\Gamma_{ij} \equiv \Pi_i{}^{\underline{a}}\Pi_{j\underline{a}} = \Pi_i{}^{\alpha\dot{\alpha}}\Pi_{j\alpha\dot{\alpha}} = \epsilon^{\alpha\beta}\epsilon^{\dot{\gamma}\dot{\delta}}\Pi_{i\alpha\dot{\gamma}}\Pi_{j\beta\dot{\delta}}. \tag{2.11b}$$

The global $[SL(2,\mathbb{R})]^3$ symmetry of our action I is more transparent in terms of Cayley's hyperdeterminant, because of its manifest invariance under $[SL(2,\mathbb{R})]^3$. For other parts of our Lagrangian, consider the infinitesimal transformation for the first factor group[3] of $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ with the infinitesimal real constant traceless 2 by 2 matrix parameter p as

$$\begin{aligned}&\delta_p\Pi_i{}^A = p_i{}^j\pi_j{}^A, \\ &\delta_p\gamma_{(i)}{}^j = -p_k{}^j\gamma_{(i)}{}^k \quad \left(p_i{}^i = 0\right).\end{aligned} \tag{2.12}$$

The latter is implied by the definition of $\Gamma_{ij} \equiv \Pi_i{}^{\underline{a}}\Pi_{j\underline{a}}$ and $\gamma_{(i)}{}^j$ in (2.8). Eventually, we have $\delta_p\Pi_{(i)}{}^A = 0$, while $\mathcal{L}_{\text{WZNW}}$ is also invariant, thanks to $\delta_p\Pi_{(i)}{}^A = 0$. This concludes $\delta_p\mathcal{L} = 0$.

The second and third factor groups in $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ act on the fermionic coordinates α and $\dot{\alpha}$ in $D = (2,2;2,2)$, which need an additional care. We first need the alternative expression of $\mathcal{L}_{\text{WZNW}}$ by the use of Vainberg construction [28,29]:

$$I_{\text{WZNW}} = i\int d^3\hat{\sigma}\,\hat{\epsilon}^{\hat{i}\hat{j}\hat{k}}\hat{\Pi}_{\hat{i}\alpha\dot{\alpha}}\hat{\Pi}_{\hat{j}}{}^{\alpha}\hat{\Pi}_{\hat{k}}{}^{\dot{\alpha}}. \tag{2.13}$$

We need this alternative expression, because superfield strength G_{ABC} is less ambiguous than its potential superfield B_{AB} avoiding the subtlety with the indices α and $\dot{\alpha}$. In the Vainberg construction [28,29], we are considering the extended 3D 'world-sheet' with the coordinates $(\hat{\sigma}^{\hat{i}}) \equiv (\sigma^i, y)$ $(\hat{i} = 0,1,2)$, where $\hat{\sigma}^2 \equiv y$ is a new coordinate with the range $0 \leqslant y \leqslant 1$. Relevantly, $\hat{\epsilon}^{\hat{i}\hat{j}\hat{k}}$ is totally antisymmetric constant, and $\hat{\epsilon}^{2\hat{i}\hat{j}} = \epsilon^{ij}$. All the *hatted* indices and quantities refer to the new 3D. Any *hatted* superfield as a function of $\hat{\sigma}^i$ should satisfy the conditions [28], e.g.,

$$\hat{Z}^M(\sigma, y=1) = Z^M(\sigma), \qquad \hat{Z}^M(\sigma, y=0) = 0. \tag{2.14}$$

Consider next the isomorphism $SL(2,\mathbb{R}) \approx Sp(1)$ [30] for the last two groups in $SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \approx SL(2,\mathbb{R}) \times Sp(1) \times Sp(1)$. These two $Sp(1)$ groups are acting respectively on the spinorial indices α and $\dot{\alpha}$. The contraction matrices $\epsilon_{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$ are the metrics of these two $Sp(1)$ groups, used for raising/lowering these spinorial indices. Now the infinitesimal transformation parameters of $Sp(1) \times Sp(1)$ can be 2 by 2 real constant symmetric matrices $q_{\alpha\beta}$ and $r_{\dot{\alpha}\dot{\beta}}$ acting as

$$\delta_q\hat{\Pi}_{\hat{i}\alpha} = -q^\alpha{}_\beta\hat{\Pi}_{\hat{i}}{}^\beta, \qquad \delta_q\hat{\Pi}_{\hat{i}\alpha\dot{\alpha}} = q_\alpha{}^\gamma\hat{\Pi}_{\hat{i}\gamma\dot{\alpha}}, \tag{2.15a}$$

[3] In a sense, this invariance is trivial, because $SL(2,\mathbb{R}) \subset GL(2,\mathbb{R})$, where the latter is the 2D general covariance group.

$$\delta_r \hat{\Pi}_{\hat{i}}{}^{\dot{\alpha}} = -r^{\dot{\alpha}}{}_{\dot{\beta}} \hat{\Pi}_{\hat{i}}{}^{\dot{\beta}}, \qquad \delta_r \hat{\Pi}_{\hat{i}\alpha\dot{\alpha}} = r_{\dot{\alpha}}{}^{\dot{\gamma}} \hat{\Pi}_{\hat{i}\alpha\dot{\gamma}}, \tag{2.15b}$$

where $q^\alpha{}_\beta \equiv \epsilon^{\alpha\gamma} q_{\gamma\beta}$, $r^{\dot{\alpha}}{}_{\dot{\beta}} \equiv \epsilon^{\dot{\alpha}\dot{\gamma}} r_{\dot{\gamma}\dot{\beta}}$, etc. Then it is easy to confirm for $\mathcal{L}_{\text{WZNW}}$ that

$$\delta_q\big(\hat{\Pi}_{\hat{i}\alpha\dot{\alpha}} \hat{\Pi}_{\hat{j}}{}^{\alpha} \hat{\Pi}_{\hat{k}}{}^{\dot{\alpha}}\big) = 0, \qquad \delta_r\big(\hat{\Pi}_{\hat{i}\alpha\dot{\alpha}} \hat{\Pi}_{\hat{j}}{}^{\alpha} \hat{\Pi}_{\hat{k}}{}^{\dot{\alpha}}\big) = 0, \tag{2.16}$$

because of $q_\alpha{}^\gamma = +q^\gamma{}_\alpha$ and $r_{\dot{\alpha}}{}^{\dot{\gamma}} = +r^{\dot{\gamma}}{}_{\dot{\alpha}}$. We thus have the total invariances $\delta_q \mathcal{L} = 0$ and $\delta_r \mathcal{L} = 0$. Since $\delta_p \mathcal{L} = 0$ has been confirmed after (2.12), this concludes the $[SL(2,\mathbb{R})]^3$-invariance proof of our action (2.1).

It was pointed out in Ref. [8] that 'hidden' discrete symmetry also exists in NG-action under the interchange of the three indices for $[SL(2,\mathbb{R})]^3$. In our system, however, this hidden triality seems absent. This can be seen in (2.1b), where the Cayley's hyperdeterminant or $\mathcal{L}_{\text{NG}}$ indeed possesses the discrete symmetry for the three indices $i\alpha\dot{\alpha}$, while it is lost in $\mathcal{L}_{\text{WZNW}}$. This is because the mixture of $\Pi_{i\alpha\dot{\alpha}}$ and $\Pi_i{}^\alpha$ or $\Pi_i{}^{\dot{\alpha}}$ *via* the non-zero components of B_{AB} breaks the exchange symmetry among $i\alpha\dot{\alpha}$, *unlike* Cayley's hyperdeterminant.

3. Fermionic invariance of our action

We now discuss our fermionic κ-invariance. Our action (2.1) is invariant under

$$\big(\delta_\kappa Z^M\big) E_M{}^{\underline{\alpha}} = +i(\sigma_{\underline{b}})_{\underline{\alpha}}{}^{\underline{\beta}} \kappa_{-\underline{\beta}} \Pi_+{}^{\underline{b}} \equiv +i(\not{\Pi}_+ \kappa_-)^{\underline{\alpha}}, \tag{3.1a}$$

$$(\delta_\kappa Z^M) E_M{}^{\underline{a}} = 0, \tag{3.1b}$$

$$\begin{aligned}\delta_\kappa \Gamma_{ij} &= +\big[\kappa_-{}^{\underline{\alpha}} (\sigma_{\underline{a}}\sigma_{\underline{c}})_{\underline{\alpha}}{}^{\underline{\beta}} \Pi_{(j|\underline{\beta}}\big] \Pi_+{}^{\underline{a}} \Pi_{|i)}{}^{\underline{c}} \\ &\equiv +(\bar{\kappa}_- \not{\Pi}_+ \not{\Pi}_{(i} \Pi_{j)}).\end{aligned} \tag{3.1c}$$

The $\kappa_-{}^{\underline{\alpha}}$ is the parameter for our fermionic symmetry transformation, just as in the conventional Green–Schwarz superstring [12,20]. Since Z^M is the only fundamental field in our formulation, (3.1c) is the necessary condition of (3.1a) and (3.1b).

We can confirm $\delta_\kappa I = 0$ easily, once we know the intermediate results:

$$\delta_\kappa \mathcal{L}_{\text{NG}} = +\sqrt{-\Gamma}\big(\bar{\kappa}_- \not{\Pi}_+ \not{\Pi}_{(i)} \Pi^{(i)}\big), \tag{3.2a}$$

$$\delta_\kappa \mathcal{L}_{\text{WZNW}} = -\epsilon^{ij} (\bar{\kappa}_- \not{\Pi}_+ \not{\Pi}_i \Pi_j). \tag{3.2b}$$

By using the relationships, such as $\sqrt{-\Gamma}\epsilon^{(k)(l)} = +\epsilon^{ij}\gamma_i{}^{(k)}\gamma_j{}^{(l)}$, with the most crucial equation (2.10), we can easily confirm that the sum (3.2a) + (3.2b) vanishes:

$$\begin{aligned}\delta_\kappa \mathcal{L} &= \delta_\kappa (\mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZNW}}) \\ &= +2\sqrt{-\Gamma}(\bar{\kappa}_- \Pi_-)\Pi_+{}^{\underline{a}} \Pi_{+\underline{a}} = 0.\end{aligned} \tag{3.3}$$

Thus the fermionic κ-invariance $\delta_\kappa I = 0$ works also in our formulation, despite the absence of the 2D metric or zweibein. The existence of fermionic κ-symmetry also guarantees the light-cone equivalence of our system to the conventional $N = 2$ GS superstring [14].

4. Derivation of Lagrangian and fermionic symmetry

In this section, we start with the conventional GS σ-model action [14] for $N = 2$ superstring [16,17], and derive our Lagrangian (2.1) with the fermionic transformation rule (3.1). This procedure provides an additional justification for our formulation.

The $N = 2$ GS action $I_{\text{GS}} \equiv \int d^2\sigma\, \mathcal{L}_{\text{GS}}$ [14] which is light-cone equivalent to $N = 2$ NSR superstring [16,17] has the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{GS}} &= +\frac{1}{2}\sqrt{-g}\, g^{ij} \Pi_i{}^{\underline{a}} \Pi_{j\underline{a}} + \epsilon^{ij} \Pi_i{}^A \Pi_j{}^B B_{BA} \\ &= +e\Pi_+{}^{\underline{a}} \Pi_{-\underline{a}} + 2e\Pi_-{}^A \Pi_+{}^B B_{BA},\end{aligned} \tag{4.1}$$

where $g \equiv \det(g_{ij})$ is for the 2D metric g_{ij}, while $e \equiv \det(e_i{}^{(j)}) = \sqrt{-g}$ is for the zweibein $e_i{}^{(j)}$. The action I_{GS} is invariant under the fermionic transformation rule [15,20][4]

$$\delta_\lambda E^{\underline{\alpha}} = +i(\sigma_{\underline{a}})^{\underline{\alpha\beta}} \lambda^i{}_{\underline{\beta}} \Pi_i{}^{\underline{a}} = +i\big(\not{\Pi}_i \lambda^i\big)^{\underline{\alpha}}, \tag{4.2a}$$

$$\delta_\lambda E^{\underline{a}} = 0, \tag{4.2b}$$

$$\delta_\lambda e_-{}^i = -\big(\lambda_-{}^{\underline{\alpha}} \Pi_{-\underline{\alpha}}\big) e_+{}^i \equiv -(\bar{\lambda}_- \Pi_-) e_+{}^i, \tag{4.2c}$$

$$\delta_\lambda e_+{}^i = 0, \tag{4.2d}$$

where λ has only the negative component: $\lambda_{(i)}{}^{\underline{\alpha}} \equiv Q_{(i)}{}^{(j)} \lambda_{(j)}{}^{\underline{\alpha}}$. Only in this section, the local Lorentz indices are related to curved ones through the zweibein as in $\Pi_{(i)}{}^A \equiv e_{(i)}{}^j \Pi_j{}^A$, *instead of* $\gamma_i{}^{(j)}$ in the last section. In the routine confirmation of $\delta_\lambda \mathcal{L}_{\text{GS}} = 0$, we see its parallel structures to $\delta_\kappa \mathcal{L} = 0$.

We next derive our Lagrangians $\mathcal{L}_{\text{NG}}$ and $\mathcal{L}_{\text{WZNW}}$ from $\mathcal{L}_{\text{GS}}$ in (4.1). To this end, we first get the 2D metric field equation from I_{GS}[5]

$$g_{ij} \doteq +2\big(g^{kl} \Pi_k{}^{\underline{b}} \Pi_{l\underline{b}}\big)^{-1} \big(\Pi_i{}^{\underline{a}} \Pi_{j\underline{a}}\big) \equiv 2\Omega^{-1} \Gamma_{ij} \equiv h_{ij}, \tag{4.3a}$$

$$\Omega \equiv g^{ij} \Pi_i{}^{\underline{a}} \Pi_{j\underline{a}} = g^{ij} \Gamma_{ij}. \tag{4.3b}$$

As is well known in string σ-models, this field equation is *not* algebraic for g_{ij}, because the r.h.s. of (4.3) again contains g^{ij} *via* the factor Ω. Nevertheless, we can formally delete the metric from the original Lagrangian, using a procedure similar to getting NG string [9,10] from Polyakov string [21], or NG action out of type II superstring action [12], as

$$\begin{aligned}\frac{1}{2}\sqrt{-g}\, g^{ij} \Gamma_{ij} &= \frac{1}{2}\sqrt{-g}\,\Omega \doteq \frac{1}{2}\sqrt{-\det(h_{ij})}\,\Omega \\ &= \frac{1}{2}\sqrt{-\det\big(2\Omega^{-1}\Gamma_{ij}\big)}\,\Omega \\ &= \Omega^{-1}\sqrt{-\det(\Gamma_{ij})}\,\Omega = \sqrt{-\Gamma} = \mathcal{L}_{\text{NG}}.\end{aligned} \tag{4.4}$$

Thus the metric disappears completely from the resulting Lagrangian, leaving only $\sqrt{-\Gamma}$ which is nothing but $\mathcal{L}_{\text{NG}}$ in (2.1). As for $\mathcal{L}_{\text{WZNW}}$, since this term is metric-independent, this is exactly the same as the second term of (4.1).

We now derive our fermionic transformation rule (3.1) from (4.2). For this purpose, we establish the on-shell relationships between $e_i{}^{(j)}$ and our newly-defined $\gamma_i{}^{(j)}$. By taking the 'square root' of (4.3a), we get the $e_i{}^{(j)}$-field equation expressed in terms of the Π's, that we call $f_i{}^{(j)}$ which coincides with $e_i{}^{(j)}$

4 We use the parameter λ instead of κ due to a slight difference of λ from our κ (cf. Eq. (4.8)).

5 We use the symbol $\doteq$ for a field equation to be distinguished from an algebraic one.

only *on-shell*:

$$e_i{}^{(j)} \doteq f_i{}^{(j)} = f_i{}^{(j)}(\Pi_k{}^A), \tag{4.5a}$$

$$f_{i(k)} f_j{}^{(k)} = h_{ij}, \qquad f^{(k)i} f_{(k)}{}^j = h^{ij},$$
$$f_i{}^{(k)} f_{(k)}{}^j = \delta_i{}^j, \qquad f_{(i)}{}^k f_k{}^{(j)} = \delta_{(i)}{}^{(j)} \tag{4.5b}$$

Note that the f's is proportional to the γ's by a factor of $\sqrt{\Omega/2}$, as understood by the use of (4.3), (4.5) and (2.8):

$$e_i{}^{(j)} \doteq f_i{}^{(j)} = \sqrt{\frac{2}{\Omega}}\gamma_i{}^{(j)},$$
$$e_{(i)}{}^j \doteq f_{(i)}{}^j = \sqrt{\frac{\Omega}{2}}\gamma_{(i)}{}^j. \tag{4.6}$$

Recall that the factor Ω contains the 2D metric or zweibein which might be problematic in our formulation, while $\gamma_i{}^{(j)}$, $\gamma_{(i)}{}^j$ are expressed only in terms of the $\Pi_i{}^A$'s. Fortunately, we will see that Ω disappears in the end result.

Our fermionic transformation rule (3.1a) is now obtained from (4.2a), as

$$\delta_\lambda E^{\underline{\alpha}} = i(\not{\Pi}_i\lambda^i)^{\underline{\alpha}} \doteq i f^{(i)j}(\not{\Pi}_j\lambda_{(i)})^{\underline{\alpha}}$$
$$= i\sqrt{\frac{\Omega}{2}}\gamma^{(i)j}(\not{\Pi}_j\lambda_{(i)})^{\underline{\alpha}}$$
$$= i\gamma^{(i)j}\left[\not{\Pi}_j\left(\sqrt{\frac{\Omega}{2}}\lambda_{(i)}\right)\right]^{\underline{\alpha}} = i(\not{\Pi}^{(i)}\kappa_{(i)})^{\underline{\alpha}} = \delta_\kappa E^{\underline{\alpha}}, \tag{4.7}$$

where λ and κ are proportional to each other by

$$\kappa_{(i)} \equiv \sqrt{\frac{\Omega}{2}}\lambda_{(i)}. \tag{4.8}$$

Such a re-scaling is always possible, due to the arbitrariness of the parameter λ or κ.

As an additional consistency confirmation, we can show the κ-invariance of (2.10), using the convenient lemmas

$$(\delta_\kappa\gamma_+{}^i)\gamma_i{}^+ = (\delta_\kappa\gamma_-{}^i)\gamma_i{}^- = \frac{1}{2}\Omega^{-1}\delta_\kappa\Omega,$$
$$(\delta_\kappa\gamma_+{}^i)\gamma_i{}^- = 0, \qquad (\delta_\kappa\gamma_-{}^i)\gamma_i{}^+ = -(\bar{\kappa}_-\not{\Pi}_-). \tag{4.9}$$

Combining these with (3.1c), we can easily confirm that $\delta_\kappa\Gamma_{++} = 0$ and $\delta_\kappa\Gamma_{--} = 0$, as desired for consistency of the 'built-in' Virasoro condition (2.10).

The complete disappearance of Ω in our transformation rule (3.1) is desirable, because Ω itself contains the metric that is *not* given in a closed algebraic form in terms of $\Pi_i{}^A$. If there were Ω involved in our transformation rule (3.1), it would pose a problem due to the metric g_{ij} in Ω. To put it differently, our action (2.1) its fermionic symmetry (3.1) are expressed only in terms of the fundamental superfield Z^M *via* $\Pi_i{}^A$ with no involvement of g_{ij}, $e_i{}^{(j)}$ or Ω, thus indicating the total consistency of our system. This concludes the justification of our fermionic κ-transformation rule (3.1), based on the $N = 2$ GS σ-model [14] light-cone equivalent to $N = 2$ NSR superstring [16,17].

5. Concluding remarks

In this Letter, we have shown that after the elimination of the 2D metric at the classical level, the NG-action part I_{NG} of GS σ-model action [14] for $N = 2$ superstring [16,17] is entirely expressed as the square root of a Cayley's hyperdeterminant with the manifest $[SL(2,\mathbb{R})]^3$ symmetry. In particular, this is valid in the presence of target superspace background in $D = (2,2;2,2)$ [19]. From this viewpoint, $N = 2$ GS σ-model [14] seems more suitable for discussing the $[SL(2,\mathbb{R})]^3$ symmetry *via* a Cayley's hyperdeterminant. We have seen that the $[SL(2,\mathbb{R})]^3$ symmetry acts on the three indices $i, \alpha, \dot{\alpha}$ carried by the pull-back $\Pi_{i\alpha\dot{\alpha}}$ in $\mathcal{D}\text{et}(\Pi_{i\alpha\dot{\alpha}})$ in $D = (2,2;2,2)$ superspace [14,19]. The hidden discrete symmetry pointed out in [8], however, seems absent in $N = 2$ string [14,17,19] due to the WZNW-term $\mathcal{L}_{\text{WZNW}}$.

We have also shown that our action (2.1) has the classical invariance under our fermionic κ-symmetry (3.1), despite the elimination of zweibein or 2D metric. Compared with the original I_{GS} [14], our action has even simpler structure, because of the absence of the 2D metric or zweibein. Due to its fermionic κ-symmetry, we can also regard that our system is classically equivalent to NSR $N = 2$ superstring [16,17], or $N = 2$ GS superstring [13]. As an important by-product, we have confirmed that the Virasoro condition (2.10) are inherent even in the NG reformulation of $N = 2$ GS string [14] at the classical level. This is also consistent with the original result that Virasoro condition is inherent in NG string [9,10].

One of the important aspects is that our action (2.1) and the fermionic transformation rule (3.1) involve neither the 2D metric g_{ij}, the zweibein $e_i{}^{(j)}$, nor the factor Ω containing these fields. This indicates the total consistency of our formulation, purely in terms of superspace coordinates Z^M as the fundamental independent field variables.

In this Letter, we have seen that neither the 2D metric g_{ij} nor the zweibein $e_i{}^{(j)}$, but the superspace pull-back $\Pi_{i\alpha\dot{\alpha}}$ is playing a key role for the manifest symmetry $[SL(2,\mathbb{R})]^3$ acting on the three indices $i\alpha\dot{\alpha}$. In particular, the combination $\Gamma_{ij} \equiv \Pi_i{}^{\underline{a}}\Pi_{j\underline{a}}$ plays a role of 'effective metric' on the 2D world-sheet. This suggests that our field variables Z^M alone are more suitable for discussing the global $[SL(2,\mathbb{R})]^3$ symmetry of $N = 2$ superstring [14,16,17].

As a matter of fact, in $D = 2+2$ *unlike* $D = 3+1$, the components α and $\dot{\alpha}$ are *not* related to each other by complex conjugations [18,19,26]. Additional evidence is that the signature $D = 2+2$ seems crucial, because $SO(2,2) \approx SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ [30], while $SO(3,1) \approx SL(2,\mathbb{C})$ for $D = 3+1$ is not suitable for $SL(2,\mathbb{R})$. Thus it is more natural that the NG reformulation of $N = 2$ GS superstring [14] with the target superspace $D = (2,2;2,2)$ is more suitable for the global $[SL(2,\mathbb{R})]^3$ symmetry acting on the three independent indices i, α and $\dot{\alpha}$.

It seems to be a common feature in supersymmetric theories that certain non-manifest symmetry becomes more manifest only after certain fields are eliminated from an original Lagrangian. For example, in $N = 1$ local supersymmetry in 4D, it is well known that the σ-model Kähler structure shows up, only after all the auxiliary fields in chiral multiplets are eliminated [31]. This viewpoint justifies to use a NG-formulation with the 2D metric eliminated, instead of the original $N = 2$ GS formulation [13,14], in order to elucidate the global $[SL(2, \mathbb{R})]^3$ symmetry of the latter, *via* a Cayley's hyperdeterminant.

It has been well known that the superspace $D = (2, 2; 2, 2)$ is the natural background for SDYM multiplet [14,17–19]. Moreover, SDSYM theory [14,18,19] is the possible underlying theory for all the (supersymmetric) integrable systems in space–time dimensions lower than four [24]. All of these features strongly indicate the significant relationships among Cayley's hyperdeterminant [1,8], $N = 2$ superstring [16,17], or $N = 2$ GS superstring [13,14] with $D = (2, 2; 2, 2)$ target superspace [14,19], its NG reformulation as in this paper, the STU black holes [5,6], SDSYM theory in $D = 2 + 2$ [14,18,19], and supersymmetric integrable or soluble models [14,17,19,24] in dimensions $D \leqslant 3$.

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