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I-balls

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Abstract

We find that there exists a soliton-like solution "I-ball" in theories of a real scalar field if the scalar potential satisfies appropriate conditions. Although the I-ball does not have any topological or global $U(1)$ charges, its stability is ensured by the adiabatic invariance for the oscillating field.

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1. Introduction

Scalar fields play important roles in theories of the early universe. It is believed that our universe experienced quasi-exponential expansion phase (= inflation) in its very early stage, which solves the flatness and horizon problems of the standard cosmology and explains the origin of the density fluctuations of the universe such as observed by COBE [1] and other experiments [2–4]. The inflationary universe scenario is realized by the vacuum energy of some scalar field (inflaton). After inflation, the inflaton starts to oscillate and decays into other particles which reheat the universe through thermalization processes.

Similar dynamics is found in the Affleck–Dine mechanism for baryogenesis [5] which is a promising candidate for explaining the matter–antimatter asymmetry of the universe. The mechanism makes use of a scalar field (AD field) corresponding to a flat direction in the scalar potential of the minimal supersymmetric standard model. During inflation the AD field has a large field value and oscillates when the effective mass becomes smaller than the Hubble parameter after inflation. When the AD field starts oscillation, the baryon number is generated through the baryon number violating term in the potential.

Recently, it was found that the oscillating AD field deforms into lumps of the scalar condensate called Q balls [6–9]. The Q ball is a non-topological soliton and its stability comes from the global charge (= baryon number) conservation. The existence of the Q ball is crucial because it may significantly change the scenario of the Affleck–Dine baryogenesis [10]. The fragmentation into scalar lumps may also take place for the inflaton field. In fact, Enqvist et al. [11]

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pointed out that the oscillating inflaton field can fragment into Q balls.

Since the Q ball is stable owing to the charge conservation, the scalar field responsible for the Q ball must be complex. Then, a question arises whether or not a real scalar field deforms into lumps similar to the Q balls. At first glance, no stable lumps are formed because any conservation quantities like a global charge do not exist for the system of a real scalar field. However, the previous studies on the dynamics of scalar fields showed that some soliton-like objects are formed. For example, "oscillons" are formed for phase the double well potentials [14] and the axion field fragments into "axitons" [15]. In both cases the numerical simulations showed the existence of some scalar lumps inside which the scalar fields are rapidly oscillating. However, the reason why such quasi-stable scalar lumps are formed was not clear at all. Moreover, recently, McDonald [12] pointed out that in a hybrid inflation model the inflaton field can fragment into scalar lumps even if the scalar field has any conserved charges (see also Ref. [13]).

Thus, it has been seen that real scalar fields fragment into quasi-stable lumps in numerical simulations for various situations. In this Letter, therefore, we face the important problem concerning the real scalar dynamics, that is, what makes the scalar lump quasi-stable? Because the conserved baryon number plays a crucial role for stability of the Q ball, we need similar conservation quantity to stabilize the real scalar lump. In classical mechanics it is well known that the adiabatic invariant exists for oscillating phenomena [16]. As will be seen later, we find that the adiabatic invariant can be extended to the field theories. Then, the existence of the stable lump can be explained by the adiabatic charge I (see Section 2 for definition) for the oscillating scalar field. We call this scalar lump "I-ball", since the adiabatic charge I plays the same role as the global $U(1)$ charges in the case of Q balls. We obtain the condition on the form of the scalar potential for the I-ball formation and derive the equation which determines the field configuration of the I-ball. In particular, it is found that the adibaticity requires the scalar potential to be dominated by a quadratic term. We also perform numerical simulations to confirm the existence of the I-balls for two types of simple potentials.

2. Adiabatic charge

In this section we derive the conservation of an adiabatic charge, which guarantees the stability of the I-ball as discussed later. First we shortly review an adiabatic invariant in a mechanical system, according to Ref. [16]. Let us suppose that a system is executing a finite motion in one dimension and characterized by some parameter $\lambda(t)$ which specifies the properties of the external field. We assume that λ varies slowly enough (i.e., "adiabatically"):

$$\left|\frac{\dot{\lambda}}{\lambda}\right| \ll T^{-1}, \tag{1}$$

where an overdot represents a derivative with respect to time, and T is the period of the motion. If λ is constant, the system executes a strictly periodic motion with a constant energy E. For slowly varying λ, the energy E varies slowly, while there is a quantity which remains constant, called an adiabatic invariant. This is written as

$$I_{1\mathrm{dim}} \equiv \frac{1}{2\pi}\oint p\,dq, \tag{2}$$

where q and p are the coordinate and momentum, and the integral is taken over the variation of the coordinate during one period. Note that the periodicity plays an essential role in the proof of the existence for the adiabatic invariant. Before we go on to the case of the field theory, it will be useful to discuss a multi-dimensional system. Let us consider a system with any number of degrees of freedom $\{q_i, p_i\}$, executing a finite motion in all the coordinates. It is assumed the variables are separable so that the action can be written as the sum of functions each depending on only one coordinate. As shown in Ref. [16], the motion of the system is in general not strictly periodic, but *conditionally periodic* since the system passes arbitrarily close to a given state in the corse of a sufficient time. In fact, it is periodic only if the frequencies of all degrees of freedom are commensurable for arbitrary values of $\{q_i\}$. In this case, there exists only one adiabatic invariant in the system. On the other hand, if the variables are not separable, there is no adiabatic invariant in general. However, if the Hamiltonian of the system differs only by small terms from one which allows separation of the variables, the properties of the motion are close to the periodic motion.

Thus, in order to extend the adiabatic invariant to the case of the field theory, we have to impose the following two assumptions. First, the gradient energy is always sub-dominant everywhere, since the action of a scalar field is separable except the gradient term. Second, the scalar potential is quadratic, which would ensure the strictly periodic motion if it were not for the gradient term. In the following arguments, we adopt these two assumptions.

Let us consider the system of a real scalar field ϕ, whose motion is finite and characterized by a parameter $\lambda(\mathbf{x}, t)$. We suppose that λ varies adiabatically, and that its dependence on the position $\mathbf{x}$ is weak enough as well. In the limit of constant λ, the motion of the scalar field is homogeneous and periodic with the period T. The Lagrangian is given as,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi, \lambda), \tag{3}$$

$$V(\phi, \lambda)\big|_{t<0} = \frac{1}{2}m^2\phi^2, \tag{4}$$

where we assumed that the potential is quadratic with the mass equal to m before the external field $\lambda(\mathbf{x}, t)$ is turned on at $t = 0$. If λ were constant, the energy–momentum conservation law would be given as

$$\partial_\nu T^{\mu\nu} = 0, \tag{5}$$

$$T^{\mu\nu} \equiv \partial^\mu\phi\,\partial^\nu\phi - \eta^{\mu\nu}\mathcal{L}. \tag{6}$$

We turn attention to its time component,

$$\partial_\mu j^\mu = 0, \tag{7}$$

$$j^\mu \equiv T^{\mu 0} = T^{0\mu} = \dot{\phi}\,\partial^\mu\phi - \eta^{\mu 0}\mathcal{L}, \tag{8}$$

where an overdot represents ∂_t, and $\eta_{\mu\nu} = \mathrm{diag}(+, -, -, -)$. With slowly varying λ for $t > 0$, the motion of the scalar field changes adiabatically. Then the energy–momentum current j^μ is no longer conserved:

$$\partial_\mu j^\mu = (\partial_\mu\lambda)\frac{\partial j^\mu}{\partial\lambda}. \tag{9}$$

If we average Eq. (9) over the period T, we have

$$\begin{aligned}\overline{\partial_\mu j^\mu} &= (\partial_\mu\lambda)\frac{1}{T}\int_t^{t+T} dt\,\frac{\partial j^\mu}{\partial\lambda} \\ &= (\partial_\mu\lambda)\left(\oint\frac{d\phi}{\dot{\phi}}\right)^{-1}\oint\frac{\partial j^\mu}{\partial\lambda}\frac{d\phi}{\dot{\phi}},\end{aligned} \tag{10}$$

where the overline represents the average over the period of the motion:

$$\overline{Z} \equiv \frac{1}{T}\int_t^{t+T} dt\,Z. \tag{11}$$

In deriving Eq. (10), we used the two assumptions, one of which states that λ varies adiabatically:

$$\left|\frac{\dot{\lambda}}{\lambda}\right| \ll T^{-1}. \tag{12}$$

Therefore, λ can be regarded as a constant, when $\oint d\phi$ is integrated over the total motion of ϕ during one period at fixed $\mathbf{x}$. The other assumption is that λ depends on the position weakly enough:

$$\left|\frac{\nabla\lambda}{\lambda}\right| \ll T^{-1}, \tag{13}$$

where T can be interpreted as a typical spatial scale of the oscillating system. This condition is necessary because otherwise the gradient term becomes too large to be negligible. Note that the large gradient term also changes the value of $\overline{\phi^2}$ significantly during one period, leading to the violation of the adiabaticity. For a small but non-zero $\nabla\lambda$, the motion is not strictly periodic, and the deviation from the orbit obtained as if λ were constant can be estimated to be of the order of $\delta\phi \sim \phi T\nabla\lambda$ after one period. We neglect such small corrections, and would like to focus attention on the leading terms in the following argument. Thus the motion of ϕ is approximated to be both periodic over T and homogeneous in the volume $V = [\mathbf{x} - T/2, \mathbf{x} + T/2]$. Hence there are four conserved quantities $J^\mu \equiv \int_V T^{\mu 0}\,d^3x$ along the path of the motion, and one can regard $\partial^\mu\phi$ as a function of (ϕ, J^μ, λ).

By differentiating the equation: $j^\nu(\phi, \partial_\mu\phi, \lambda) = J^\nu$ with λ, we have

$$\begin{aligned}&\frac{\partial j^\nu}{\partial\partial_\mu\phi}\frac{\partial\partial_\mu\phi}{\partial\lambda} + \frac{\partial j^\nu}{\partial\lambda} = 0 \\ &\iff \frac{\partial j^\mu}{\partial\lambda} = -\dot{\phi}\frac{\partial\partial^\mu\phi}{\partial\lambda}.\end{aligned} \tag{14}$$

In deriving Eq. (14), we have used the relation

$$\frac{\partial j^\nu}{\partial(\partial^\mu\phi)} \simeq \dot{\phi}\delta^\nu{}_\mu, \tag{15}$$

where the non-diagonal components are higher order in $\dot{\lambda}$ and $\nabla\lambda$, and hence we neglect them. With the use of Eq. (14), Eq. (10) can be rewritten as

$$\begin{aligned}
\overline{\partial_\mu j^\mu} &= -\partial_\mu\lambda\left(\oint \frac{d\phi}{\dot{\phi}}\right)^{-1}\oint \frac{\partial(\partial^\mu\phi)}{\partial\lambda}\,d\phi \\
&\Longleftrightarrow \oint\left(\partial_\mu\lambda\frac{\partial(\partial^\mu\phi)}{\partial\lambda} + \overline{\partial_\mu j^\mu}\frac{1}{\dot{\phi}}\right)d\phi = 0 \\
&\Longleftrightarrow \oint\left(\partial_\mu\lambda\frac{\partial(\partial^\mu\phi)}{\partial\lambda} + \overline{\partial_\mu j^\nu}\frac{\partial(\partial^\mu\phi)}{\partial j^\nu}\right)d\phi = 0.
\end{aligned} \tag{16}$$

Hence we have

$$\overline{\partial_\mu\left(\oint d\phi\,\partial^\mu\phi\right)} = 0, \tag{17}$$

where the time component ($\mu = 0$) is the leading term, and has a definite physical meaning. Thus, we are led to define the adiabatic charge I as

$$I \equiv \frac{1}{2m}\int d^3x\,\overline{\dot{\phi}^2}, \tag{18}$$

where the factor $1/m$ in the definition here is introduced to make I dimensionless. Apparently I is conserved,

$$\frac{dI}{dt} = 0. \tag{19}$$

3. Conditions for existence of I-balls

With the use of the adiabatic charge derived in the previous section, let us consider the condition that I-balls are formed. We would like to focus attention on λ in the first place. It specifies the properties of the external field, hence the energy of the system is no longer conserved for varying λ (see Eq. (9)). Alternatively, it could be argued that the role of λ is played by the self-interaction of the scalar field, which must be such that the adiabatic conditions, (12) and (13) are satisfied. Then the total energy of the system including the interaction would be conserved, although the energy of the free part varies due to the self-coupling. In other words, there are two invariants, the energy and adiabatic charge for the whole system including 'the external field'.

We assume that the scalar potential $V(\phi)$ is given as

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_1(\phi), \tag{20}$$

where the self-interaction $V_1(\phi)$ is small enough to respect the adiabatic conditions. That is to say, the adiabatic conditions are assumed to be satisfied by the dynamics of the system. First, we separate ϕ into the rapidly oscillating part ($\widetilde{P}$) and slowly varying part (Φ) as

$$\phi(\mathbf{x}, t) = \Phi(\mathbf{x}, t)\widetilde{P}(\mathbf{x}, t). \tag{21}$$

For the general potentials, $\widetilde{P}$ might strongly depend on $\mathbf{x}$, which leads to the large gradient energy $(\nabla\phi)^2$ and violates the adiabatic condition (13). However, we can safely adopt the ansatz that $\widetilde{P}$ is homogeneous over a sufficiently large scale in which we are interested, since the periodicity of the system is guaranteed if the adiabatic conditions are satisfied. Thus,

$$\widetilde{P}(\mathbf{x}, t) = P(t), \tag{22}$$

where $P(t)$ oscillates between -1 and 1 with the period T. We also define O(1) constants c_n ($n = 1, 2, 3, \ldots$) for the later use.

$$\overline{\phi^{2n}} = c_n\Phi^{2n}, \tag{23}$$

$$\overline{\dot{\phi}^2} = c_1 m^2\Phi^2, \tag{24}$$

where we used the fact that the scalar potential is dominated by the quadratic term.

We take advantage of the method of Lagrange multipliers to look for the minimum of the energy E at fixed I, and minimize

$$\begin{aligned}
E_\omega &= \overline{E} + \tilde{\omega}\left(I - \frac{1}{2m}\int d^3x\,\overline{\dot{\phi}^2}\right) \\
&= \int d^3x\left(\left(1 - \frac{\tilde{\omega}}{m}\right)\frac{1}{2}\overline{\dot{\phi}^2} + \frac{1}{2}\overline{|\nabla\phi|^2} + \overline{V(\phi)}\right) \\
&\quad + \tilde{\omega}I \\
&= \int d^3x\,c_1\left(\frac{1}{2}|\nabla\Phi|^2 - \frac{\omega m}{2}\Phi^2 + V(\Phi)\right) \\
&\quad + (\omega + m)I,
\end{aligned} \tag{25}$$

where $\tilde{\omega} \equiv \omega + m$, and $V(\Phi) \equiv \overline{V(\phi)}/c_1$. Assuming the bounce solution is spherically symmetric,

$$\begin{aligned}
E_\omega &= \int dr\,4\pi r^2 c_1\left(\frac{1}{2}\left(\frac{d\Phi}{dr}\right)^2 + V(\Phi) - \frac{\omega m}{2}\Phi^2\right) \\
&\quad + (\omega + m)\,I.
\end{aligned} \tag{26}$$

In order to minimize E_ω with respect to Φ, we have to seek for the bounce solution, which is equivalent to solving the equation,

$$\frac{d^2\Phi}{dr^2}+\frac{2}{r}\frac{d\Phi}{dr}+\frac{dU}{d\Phi}=0, \tag{27}$$

where

$$U(\Phi)\equiv\frac{\omega m}{2}\Phi^2-V(\Phi). \tag{28}$$

The bounce solution should satisfy the following boundary conditions (see Fig. 1):

$$\left.\frac{d\Phi}{dr}\right|_{r=0}=0, \tag{29}$$

$$\Phi(\infty)=0. \tag{30}$$

Hence, the bounce solution (I-ball solution) exists if the following inequalities are satisfied.

$$\min\left[\frac{2V(\Phi)}{\Phi^2}\right]<\omega m<m^2. \tag{31}$$

Now we investigate the constraint on the interaction V_1. For the field configuration to satisfy the adiabatic condition (13), we require $|(d\Phi/dr)/\Phi|\ll m$. Since the spatial scale of the I-ball solution is determined by the mass scale of $U(\Phi)$, this requirement is equivalent to

$$\left|\frac{d^2U(\Phi)}{d\Phi^2}\right|\sim\left|\frac{d^2V_1(\Phi)}{d\Phi^2}\right|\ll m^2, \tag{32}$$

where we used the inequality (31). If the interaction satisfies this constraint, the periodicity of the system is maintained, and the I-ball configuration which minimizes the total energy of the system is also gently-sloping enough.

Lastly, we comment on the cases that the scalar potential is not dominated by the quadratic term. Up to here we have shown that the adiabatic invariant can be found for a scalar field with the quadratic potential in the external field, and that the I-ball configuration minimizes the energy of the whole system including the interaction, which plays a role of the external field, for the fixed adiabatic invariant. Thus it is not evident from the preceding arguments whether the lumps are formed for other potentials. However, we performed the numerical calculations, and found that no quasi-stable lumps are formed for the several examples of such potentials. Hence this fact strongly suggests that the existence of I-balls is peculiar to the case where the quadratic term dominates the potential.

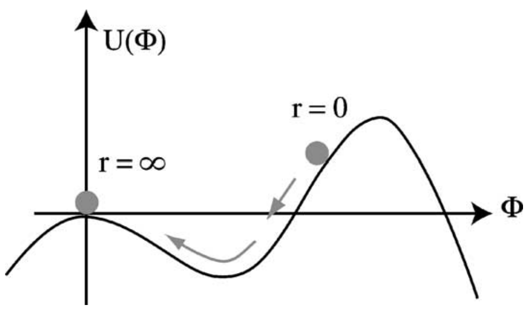

Fig. 1. Potential $U(\Phi)$.

4. Numerical simulation

Now that we have shown that the I-balls minimize the energy for fixed I, it must be then investigated whether such soliton-like objects are really formed. For this purpose we perform numerical calculation which follow the time evolution of the system. This is a quite non-trivial question, since the requirement that the system should vary adiabatically and its spatial distribution be gently-sloping enough ("adiabaticity condition") must be satisfied dynamically. As concrete examples we take both the gravity-mediation like potential and $m^2\phi^2-\phi^4$ potential.

First let us suppose the potential is given as

$$V(\phi)=\frac{1}{2}m^2\phi^2\left(1+K\log\left(\frac{\phi^2}{2M_*^2}\right)\right), \tag{33}$$

where M_* is a renormalization point to define the mass, and the K term is the one-loop correction which is assumed to be negative. The I-ball equation Eq. (27) reads

$$\frac{d^2\Phi}{dr^2}+\frac{2}{r}\frac{d\Phi}{dr}+\omega_0^2\Phi-m^2\Phi K\log\left(\frac{\Phi^2}{4M_*^2}\right)=0, \tag{34}$$

where ω_0 is defined as

$$\omega_0^2\equiv\omega m-m^2(1+K). \tag{35}$$

From numerical calculations, it is suggested that a Gaussian ansatz is a reasonable approximation to the I-ball solution for this potential. If we insert the Gaussian ansatz,

$$\Phi(r)=\Phi(0)e^{-r^2/R_I^2}, \tag{36}$$

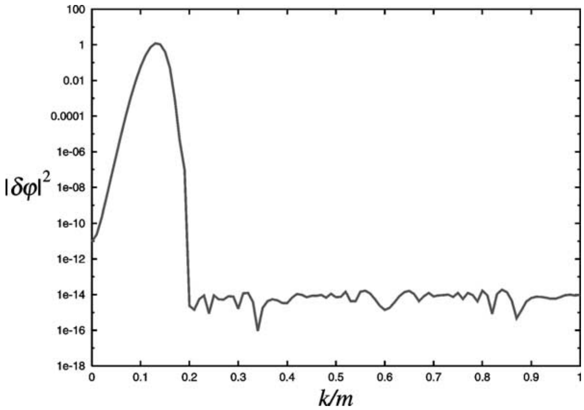

Fig. 2. Instability band for the gravity-mediation like potential with $K = -0.1$.

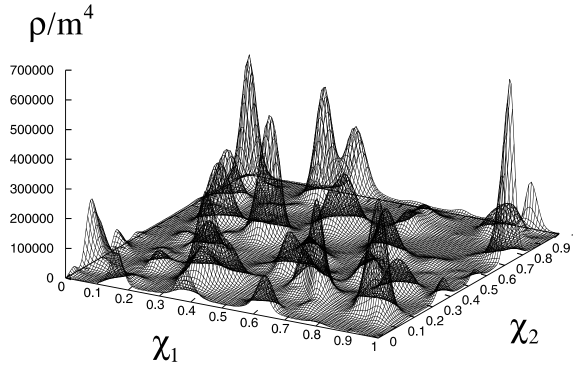

Fig. 3. Spatial distribution of the energy density ρ for the gravity-mediation like potential. Inside the I-balls, the scalar field ϕ oscillates rapidly.

into the I-ball equation Eq. (34), we obtain

$$\left(\frac{4r^2}{R_I^4} - \frac{6}{R_I^2}\right)\Phi + \left(\omega_0^2 + \frac{2Km^2}{R_I^2}r^2\right)\Phi = 0, \tag{37}$$

where we have set $M_* = \Phi(0)/2$. Thus we see that the same form is obtained in the first and second terms. This requires that

$$R_I = \frac{\sqrt{2}}{\sqrt{|K|}m}, \tag{38}$$

which roughly corresponds to the inverse of the most amplified mode $k_{\max}$, and this correspondence is checked numerically (see Fig. 2). Also the maximum growth rate of the linear perturbation is estimated to be of the order $|K|m$ in the similar way as in the case of Q balls [7,11]. Hence the adiabaticity condition is satisfied for the gravity-mediation like potential.

We perform the numerical simulation to confirm whether the I-balls are formed with the radius obtained above. For the numerical calculation, we take the variables to be dimensionless as follows.

$$\varphi = \frac{\phi}{m}, \qquad \tau = mt,$$
$$\chi_i = mx_i, \qquad \kappa_i = k_i/m, \tag{39}$$

where k_i is a wave number in the x_i direction. The initial conditions are taken as

$$\varphi(0) = 1.6 \times 10^5 + \delta_1,$$
$$\varphi'(0) = -\frac{1}{3} + \delta_2, \tag{40}$$

where the prime denotes the derivative with τ, and we set $K = -0.1$. δ's are fluctuations which originate from the quantum fluctuations, and their amplitudes are taken to be 10^{-7} times smaller than the homogeneous mode. We have confirmed that the smaller fluctuations just delay the formation of I-balls. First we check that the adiabatic condition is satisfied up to the linear growth of perturbations. Fig. 2 shows the numerical result of the instability band for the initial conditions stated above. It can be seen that the instability band roughly corresponds to the inverse of the radius R_I obtained analytically. We present the result of numerical simulation in two dimensional lattices in Fig. 3, from which one can see that the energy density ρ deforms into lumps, identified as I-balls. Also Fig. 4 represents the profile of the scalar field Φ inside the I-ball, and the analytic solution obtained above agrees quite well with the numerical result, which suggests the Gaussian ansatz is appropriate. Though there is some deviation in the outer region, it is irrelevant since the absolute value of the scalar field is much smaller than that at the center of the I-ball.[1]

Next we take the following potential.

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{a}{4}\phi^4 + \frac{b}{6m^2}\phi^6, \tag{41}$$

[1] Some reasons can be adduced: (1) The adiabatic conditions might not be satisfied well around the surface, since the homogeneous mode is relatively small compared to the fluctuations. (2) Actually the I-balls are not isolated.

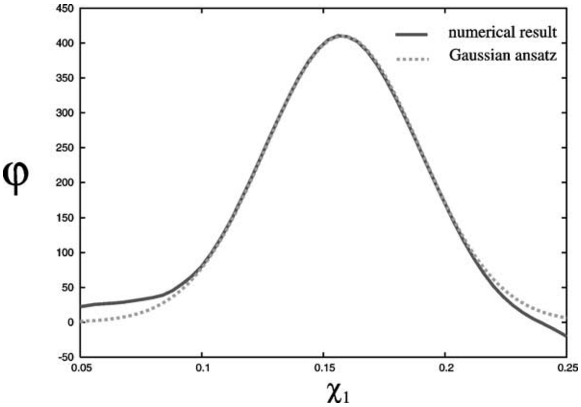

Fig. 4. Profile of the scalar field inside the typical I-ball. The analytic solution (dotted line) agrees very well with the actual profile (solid line).

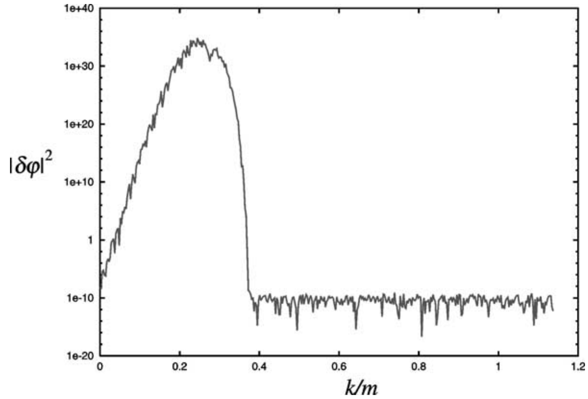

Fig. 5. Instability band for $\phi_i = m$, $\dot{\phi}_i = 0$, $a = 0.1$ and $b = 0.005$ in the potential Eq. (41).

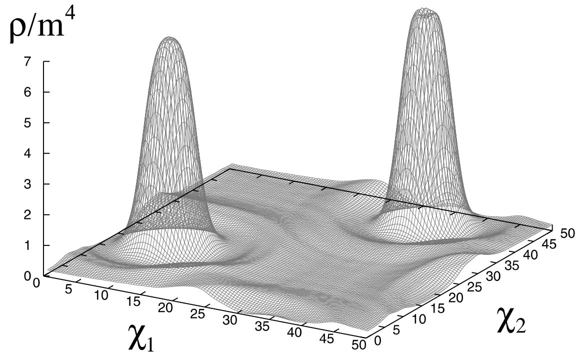

Fig. 6. Spatial distribution of the energy density ρ for the $m^2\phi^2 - \phi^4$ potential at time corresponding to $\tau = 450$, when the I-balls are newborn. Inside the I-balls, the scalar field ϕ oscillates rapidly.

where we have added the ϕ^6 term for the stability of the vacuum. Here a is positive to satisfy the condition (31). Actually, for negative a, no I-balls are produced in the numerical simulations. As in the previous case, we take the variables to be dimensionless as Eq. (39). The initial conditions are taken as

$$\varphi(0) = 1.0 + \delta_1,$$
$$\varphi'(0) = 0 + \delta_2. \tag{42}$$

We set $a = 0.1$, $b = 0.005$, and δ's amplitudes are taken to be 10^{-5} times smaller than the homogeneous mode. As long as $-\phi^4$ term is much smaller than the mass term, the growth rate α and the instability mode $\mathbf{k}$ are given by [12]

$$\alpha(\mathbf{k}) \simeq \left(\frac{3a\Phi^2}{8m^2}\right)^{1/2} |\mathbf{k}|, \quad 0 < \mathbf{k}^2 < \frac{3}{2}a\Phi^2. \tag{43}$$

Since we take $\phi = m$ as an initial condition, $\alpha(\mathbf{k}_{\max})$ and $\sqrt{\mathbf{k}^2_{\max}}$ are much smaller than m, so the adiabaticity condition is satisfied up to linear perturbation. The numerical result of the instability band for the initial conditions stated above is shown in Fig. 5. We can see that the instability band coincides exactly with that obtained analytically.

We present the result of numerical calculation in two-dimensional lattices in Fig. 6. The energy density ρ deforms into I-balls in this case, too. We have performed numerical simulations with different a, and found that all of these results generally look alike. When the I-balls are newborn, they are almost spherically symmetric, and the gradient energy is subdominant everywhere, suggesting that our formulation is valid. As the system evolves in time, their shapes deviate from the spherically symmetric one, and become irregular. Finally, they decay into random phases where the kinetic, potential and gradient energies are all same order. The lifetime of the I-balls τ becomes shorter and they are formed earlier as a increases, but its typical value is $O(10^3\ \mathrm{m}^{-1})$ for $a = 0.1$. This is consistent with the facts pointed out in Refs. [14,15], that the oscillons and axitons have very long but finite lifetime. Hence, it is certain that the I-balls for this type of potential decay in the end. The decay might be induced by the small deviation from the adiabaticity of the dynamics, which induces the decoherence of the oscillating scalar field inside I-balls. But it needs further investigations to make clear how the decay proceeds.

5. Conclusion

We have studied the system of a real scalar field and found the solution of the quasi-stable scalar lump, I-ball. The stability of the I-ball can be explained by the adiabatic invariant charge I, which does stem from the dynamics of the system, not any symmetries. For the I-ball solution to exist, the scalar potential should be dominated by the quadratic term ($m^2\phi^2$) and satisfy the condition (31) which is almost same as that for the Q ball.

Furthermore, we have performed numerical simulations and have found that the quasi-stable I-balls are really produced and their properties are in agreement with the theoretical predictions. Since scalar fields prevail in theories of the early universe, the I-balls may be formed and play important roles in various cosmological processes [17].

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