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Application of Grey-Markov Model in Forecasting Fire Accidents

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Abstract

Fire accidents are influenced by many complex factors, and it has the characteristic of both randomness and fluctuation, so a new forecasting model (Grey-Markov model) was established in order to forecast fire accidents effectively in this paper, which has the merits of both GM (1, 1) forecast model and Markov chain forecast model, it can reduce random fluctuation of accident data affecting forecasting precision and develop the application scope of Grey forecast. Finally, an example was analyzed, the results show that Grey-Markov model proposed in this paper has a higher forecast precision and excellent applicability.

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Keywords: GM(1,1); Grey-Markov; forecast; fire accidents

1. Introduction

The forecast of fire accidents is an important component of fire management decision-making process. However, fire accidents are influenced by many complex factors, such as environment, climate, fire investment, public’s fire safety consciousness and so on, the statistic data of fire accidents always take on the characteristic of both randomness and fluctuation, so it is quite important to select an appropriate forecasting method, this paper proposed a Grey-Markov forecasting model, which consists of GM (1, 1) model and Markov chain model, Grey system theory was proposed by Professor Deng in 1982\cite{1}, GM (1, 1) forecasting model can be used effectively to deal with some data sequences with little data, but it can not deal with some data sequences that fluctuate highly\cite{2}. However Markov chain forecasting model makes it possible to solve the problem mentioned above, so Grey-Markov model was established based on the advantage of both methods, which adopts GM (1, 1) model to study development regulation of data sequence and uses Markov model to study vibrating regularities of data sequences\cite{3,4}. Finally, Grey-Markov model was applied to forecast fire accidents in one city, and a higher forecast precision was obtained.

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Keywords: GM(1,1); Grey-Markov; forecast; fire accidents
2. Methodology

2.1. GM(1,1) forecasting model

The method of GM (1, 1) [1, 2] is as follows:

Step 1: Assume original data sequence to be:
\[ x^{(0)}(i) = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \]  
(1)

Step 2: \( x^{(1)} \) is obtained by 1-AGO (one time accumulated generating operation):
\[ x^{(1)}(k) = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\} \]  
(2)

Where \( x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad k = 1,2,3\ldots n \)

Step 3: The grey differential equation of GM (1, 1) and its whitening equation are obtained respectively:
\[ x^{(0)}(k) + ax^{(1)}(k) = b, \quad k = 2,3\ldots n \]  
(3)
\[ \frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b \]  
(4)

Where, \( a \) denotes the developing coefficient, \( b \) denotes grey input.

Step4: Let \( \hat{u} \) be the parameters vector, \( \hat{u} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y \), \( B \) denotes the accumulated matrix and \( Y \) is the constant vector, so \( a \) and \( b \) can be obtained by using least square method. Where
\[
B = \begin{pmatrix}
-\hat{z}^{(1)}(2) & 1 \\
-\hat{z}^{(1)}(3) & 1 \\
\vdots & \vdots \\
-\hat{z}^{(1)}(n) & 1 \\
\end{pmatrix}
\]
\[ Y_N = \begin{pmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n) \\
\end{pmatrix}
\]
\[ z^{(1)}(k) = \frac{x^{(1)}(k)+x^{(1)}(k-1)}{2} \quad (k = 2,3,4,\ldots n) \]  
(5)

Step5: The solution of Eq. (4) can be obtained as follows:
\[
\hat{x}^{(1)}(k+1) = (x^{(0)}(1)-\frac{b}{a})e^{-ak} + \frac{b}{a} \]  
(6)

Step6: Applying the inverse accumulated generating operation (IAGO), and then we have
\[
\hat{x}^{(0)}(k+1) = (1-e^a)(x^{(0)}(1)-\frac{b}{a})e^{-ak} \]  
(7)

2.2. Markov forecasting model

Markov chain [3, 4] is a particular type of stochastic process, and it has been used extensively to establish stochastic models in a variety of disciplines, such as physics, biology, sociology, finance etc. Moreover, Markov chain is a forecasting method which can be used to predict the future data by the occurred events.

We can get the simulation sequence by Eq. (7) as follows:
\[
\hat{x}^{(0)} = \{\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\hat{x}^{(0)}(3),\ldots,\hat{x}^{(0)}(n)\} \]  
(8)

Then \( \hat{x}^{(0)} \) is a Markov chain, we can divide it into \( n \) states according to the relative error, its any state can be denoted as:
\[
\otimes_j = [\otimes_{j-}, \otimes_{j+}] \quad \otimes_{j-} = \hat{x}^{(0)}(j) + a_j \quad \otimes_{j+} = \hat{x}^{(0)}(j) + b_j \]  
(9)

Assume \( n_j \) is the data number of original sequence, the transition probability from \( \otimes_i \) to \( \otimes_j \) can be established:
Where $P_{ij}(k)$ is the transition probability of state $\otimes_i$ transferred from state $\otimes_j$ for $k$ steps, $k$ is the number of transition steps each time, $n_i$ is the number of data in state $\otimes_i$, $n_j$ is the number of original data of state $\otimes_j$ transferred from state $\otimes_i$ for $k$ steps, its transition probability matrix can be expressed as follows:

$$P(k) = \begin{pmatrix}
P^{(k)}_{11} & P^{(k)}_{12} & \cdots & P^{(k)}_{1n} \\
P^{(k)}_{21} & P^{(k)}_{22} & \cdots & P^{(k)}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
P^{(k)}_{n1} & P^{(k)}_{n2} & \cdots & P^{(k)}_{nn}
\end{pmatrix}$$ (11)

The transition probability matrix $P(k)$ reflects the transition rules of the states in a system, which is the foundation of the Grey-Markov forecasting model, the future trend of the system can be predicted by studying the transition probability matrix $P(k)$. Then select the closest times from the prediction time, the transfer steps are defined as 1 steps, 2 steps and n steps respectively in terms of the distance to the predict time, in the transition probability matrix, the corresponding row vectors of the initial states are the probability that every state appears, then calculate the sum of every probability, and relative error will lie in the greatest sum of the transition probability, after confirming future state transition, the relative residual error zone $[\otimes_{j-}, \otimes_{j+}]$ is obtained, the median in $[\otimes_{j-}, \otimes_{j+}]$ is selected as the relative error, so forecasting value of original data sequence is obtained according to the above explanation.

$$\hat{y}(j) = \frac{\otimes_{j-} + \otimes_{j+}}{2} = \hat{x}^{(0)}(j) + \frac{a_j + b_j}{2}$$ (12)

### 3. Application

The original data sequence of fire accidents from 2005 to 2009 in one city are listed in Table 1, and then we are applying Grey-Markov forecasting model to forecast the fire accidents, according to the methodology proposed in section 2, the forecasting steps are as follows.

#### 3.1. Building GM(1,1) forecasting model

Based on the original data of fire accidents from 2005 to 2009, the value $(a=0.0582, b=5779.9)$ can be obtained after calculating, then the grey forecasting model is established according to Eq.(7) as follows:

$$\hat{x}^{(0)}(k+1) = (1-e^a)\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} = 5469.5e^{-0.0582k} \quad (k=1,2,\ldots,n)$$ (13)

The forecast value is calculated out by Eq. (13), see Table 1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Real value</th>
<th>Forecast value</th>
<th>Relative error (%)</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>8000</td>
<td>8000</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2006</td>
<td>5261</td>
<td>5160</td>
<td>1.92</td>
<td>3</td>
</tr>
<tr>
<td>2007</td>
<td>4657</td>
<td>4869</td>
<td>-4.03</td>
<td>1</td>
</tr>
<tr>
<td>2008</td>
<td>4728</td>
<td>4593</td>
<td>3.36</td>
<td>4</td>
</tr>
<tr>
<td>2009</td>
<td>4325</td>
<td>4336</td>
<td>-0.25</td>
<td>2</td>
</tr>
</tbody>
</table>
3.2. Dividing states by Markov forecasting model

According to relative error (see Table 1) from 2005 to 2009, four states are divided as follows, and the circumstances that relative error lies in the state are listed in Table 1.

\[ \begin{align*}
\mathbb{X}_1 &= (-5, -2], & \mathbb{X}_2 &= (-2, 0], & \mathbb{X}_3 &= (0, 2], & \mathbb{X}_4 &= (2, 4] 
\end{align*} \]

3.3. Calculating transition probability matrix

Transition probability matrix can be calculated according to the method introduced in this paper:

\[
\begin{align*}
P(1) &= \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 
\end{pmatrix}, & P(2) &= \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 
\end{pmatrix} \\
P(3) &= \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 
\end{pmatrix}, & P(4) &= \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 
\end{pmatrix}
\end{align*}
\]

Due to four states are divided, so latest four years data near to prediction time are selected to make state prediction table (Table 2), the transition steps are defined as 1,2,3,4.

Table 2. State Prediction

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Transition</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2008</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2007</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2006</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

3.4. Calculating forecasting values

The sum of all transition probability from any state to state 4 is maximal (1), and then the relative error of 2010 is in the state 3: (0, 2], the forecast value of GM (1, 1) is calculated by Eq. (13), the forecast value of 2010 obtained by GM (1, 1) is 4089, so the forecast value obtained by Grey-Markov is 4130, that is \( 4089 \times \left( 1 + \frac{0 + 2}{2} \% \right) = 4130 \), the forecast values from 2006 to 2009 are also calculated by Grey-Markov model(see Table 3).

Table 3. Forecasting Results

<table>
<thead>
<tr>
<th>Year</th>
<th>Real value</th>
<th>GM(1,1) Forecast value</th>
<th>Relative error (%)</th>
<th>State Forecast value</th>
<th>Grey-Markov Forecast value</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>5261</td>
<td>5160</td>
<td>1.92</td>
<td>3</td>
<td>5212</td>
<td>0.91</td>
</tr>
<tr>
<td>2007</td>
<td>4657</td>
<td>4869</td>
<td>-4.03</td>
<td>1</td>
<td>4699</td>
<td>-0.90</td>
</tr>
<tr>
<td>2008</td>
<td>4728</td>
<td>4593</td>
<td>3.36</td>
<td>4</td>
<td>4731</td>
<td>0</td>
</tr>
<tr>
<td>2009</td>
<td>4325</td>
<td>4336</td>
<td>-0.25</td>
<td>2</td>
<td>4293</td>
<td>0.74</td>
</tr>
</tbody>
</table>
From Table 3, it is easy to see that the relative error of Grey-Markov is lower than that of GM (1, 1), the mean absolute value of relative error of GM (1, 1) is 2.39%, and the mean absolute value of relative error of Grey-Markov is 0.64%, it is obvious that the methodology proposed in this paper is much better.

The forecast values obtained by GM (1, 1), Grey-Markov and the real value are shown in Fig.1, it shows that Grey-Markov forecasting model is better for forecasting fire accidents, the precision of Grey-Markov model is much better than GM (1, 1) model.

4. Conclusions

Fire accidents are influenced by many factors, and its data fluctuate very much, so it is suitable to forecast by Grey-Markov model, Grey-Markov forecasting model has the advantages of both GM (1, 1) model and Markov chain model, which achieves better effect in the practical application and can get a higher precision, and the forecasting precision is related with the state divided, which has no given standard, it depends on the number of history data, if there are many data, the number of states should be divided more and forecasting precision would be increased correspondingly, so the application of the model needs a further study and improvement.

References