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Ranking of fuzzy similar faces using relevance matrix and aggregation operators

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Abstract

In perception based imaging, Sketching With Words (SWW) is a well-established methodology in which the objects of computation are fuzzy geometric objects (f-objects). The problem of facial imaging of criminal on the basis of onlooker statement is not lack of method and measures but the modeling of onlooker(s) mind set. Because the onlooker has to give statements about different human face parts like forehead, eyes, nose, and chin etc. The concept of fuzzy similarity (f-similarity) and proper aggregation of components of face may provide more flexibility to onlooker(s). In proposed work onlooker(s) statement is recorded. Thereafter it is compared with existing statements. The f-similarity with different faces in database is estimated by using 'as many as possible' linguistic quantifier. Three types of constraints over size of parts of face 'small', 'medium', and 'large' are considered. Possibilistic constraints with linguistic hedges and negation operator like 'very long', 'not long', 'not very long' etc. are used. Moreover we have generated ranking of alike faces in decreasing order by using the concepts of f-similarity and relevance matrix.

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Keywords: Sketching With Words; fuzzy similarity; fuzzy granule; possibilistic constraints; relevance matrix

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1. Introduction

Due to exponential growth in crime, development of a fully automated facial image identification system is need of the hour. Sometimes the facial images of the miscreants are neither caught by the CCTV from the crime site nor in the database of International police. In this case face sketch of criminal on the basis of onlooker(s) statement may be very critical clue for catching the criminal. To draw the face sketch of the criminal on the basis of perceptions based verbal description of the onlooker is still in infancy. However the promising work is being carried out in image identification with a purpose of face identification^{1,2,3,4}. The problem of facial image is not lack of method and measures but the modelling of onlooker(s) mind set. Because the onlooker has to give statements about different human face parts like forehead, eyes, nose, lips, and chin etc. For example “He has very small nose and broad chin”. However in perception based imaging domain Sketching With Words (SWW)^{5,6,7} is a well-established methodology in which the objects of computation are fuzzy geometric objects(*f*-objects).The problem of sketching of face is augmented by the involvement of uncertainty in the perception based natural language statements, which is cause of concern for problems which has to be depend solely on it for its quality of results. The onlooker(s) knows different features that collectively describe the complete face, but does not know precisely how to combine these features. Hence there may be very fair chances of trapping of some innocent, where the decision of classifying a face as completely acceptable or unacceptable. In this paper we have enhanced the existing system by using the concept of fuzzy similarity (*f*-similarity) and OWA operators which are introduced by Yager⁸. The concept of *f*-similarity and proper aggregation of components of face may provide more flexibility to onlooker(s).

In proposed work facial description given by the onlooker(s) is recorded. Thereafter it is compared with stored facial images description. The *f*-similarity with different faces in database is estimated by using ‘as many as possible’ linguistic quantifier and then a ranking of face is generated by using relevance matrix.

SWW is inspired by the concept of Fuzzy Geometry (*f*-geometry)⁹, and Computing With Word (CWW)¹⁰ technique, both given by Zadeh the father, of fuzzy logic. Zadeh reveals the concept of fuzzy validity (*f*- validity) and *f*-similarity for the first time. The *f*-validity is a measure of the degree of belongingness of any *f*-objects with respect to corresponding crisp geometric objects. In *f*-geometry any two *f*-objects are said to be *f*-similar, if both of them have same shape.

Significant contribution in SWW is given by Imran and Beg^{11,12,13}. Different *f*-objects have been defined by¹⁴. Different fuzzy images are retrieved on the basis of perceptions^{15,16}. Since, most of the human perceptions are fuzzy¹⁷, for example when onlooker granulates face into granule label then the size of face part is a fuzzy value. Hence, the concept of fuzzy granule has been applied for face recognition. We have also applied the concept of possibility distribution¹⁸, which is useful in representing natural language statement into Generalized Constraints Language (GCL). The comprehensive range of constraints in GCL makes this language much more communicative language than the other languages of predicate logic⁹. Vishwakarma normalized the illumination of face by using fuzzy filter¹⁸ whereas Wang et al have determined the shadow and compensate the shadow of faces for recognition of different type of faces¹⁹. Wang et al have proposed fuzzy extension matrix based approach for fuzzy rule generation²⁰. In²¹, authors optimized the particle swarm for determining fuzzy measures from data. Prasad et al proposed Supervised Learning in Quest decision tree for prediction of precipitation atmospheric nimbus clouds²². The prediction of Prasad et al with an accuracy of 77.78% may be helpful in financial planning of a populous countries like India. Ibrahim et al applied neural network techniques for vehicle license plate recognition system²³. Further the authors have adapted an algorithm with proof into the solution for parking management system in the same work.

This paper is organized as follows, section 2 has the details of CWW. In section 3, we have defined proposed system. In section 4, we have explained experimental work and results. The final section consists of conclusion and future work.

2. Computing With Words

In proposed work the concept of CWW viz. Fuzzy Granule, Possibilistic Constraints, and GCL are used. These concepts are applied for the recognition of face on the basis of perceptions. Various fuzzy objects are used for describing different parts of face. This system is set out to draw the face sketch of criminal on the basis of perceptions based verbal description of the onlooker.

2.1 Representation of natural language statement into GCL

In GCL the natural language statements are expressed as a general constraint in the form of $X \text{ isr } R$ Where R is a constrained fuzzy relation, X is the constraining variable relation; ‘**isr**’ is a variable copula in which ‘ r ’ is a variable whose value describes the method in which R constrains X . In this work we are using only possibilistic constraint which will be discussed in detail later on. The detail of various constraint can be found in¹¹. In GCL, ‘ X is R ’ is the representation of different natural language statement in the form of

$$P \rightarrow X \text{ is } R$$

‘Eyes are small’

As a simple illustration, consider the proposition

$$P = \text{Nose is not fairly small}$$

In this case, the constrained variable is the Nose, which may be expressed as

$$X = \text{Size}(\text{Nose}) = \text{Part}[\text{Name} = \text{Nose}]$$

The constraining relation ‘ R ’, is given by $R = (\text{Size}^2)$ which implies that the linguistic hedges ‘fairly’ interpreted as a squaring operation, and the negation as the operation of complementation²⁴. Equivalently, R may be expressed as

$$R = \text{Short}[\text{Nose}; 1 - \mu^2]$$

2.2 Possibilistic constraint

When the value of copula r is blank in $X \text{ isr } R$. $X \text{ is } R$ abbreviated to X “ezar”. And R constrains X as possibility distribution of X . More precisely

$$\text{If } X \text{ takes values in a universe of discourse } U = \{u\},$$

$$\text{Then } \text{Poss}\{X = u\} = \pi_x(u) = \mu_R(u),$$

where μ_R is the membership function of R , and Π_X is the possibility distribution of X , that is, the fuzzy set of its possible values¹⁰.

$$X \text{ isr } R \text{ is given by}$$

$$\pi_x(u) = \mu_R(u),$$

In the following example we have explore the concept of possibility distribution in a fuzzy set. We have a set of possible values that may be taken by X are (0,1,3,4). The possibility distribution is

$$\pi_x = 1/0 + 1/1 + 0.8/4 + 0.7/3$$

$$\text{Poss}\{X = 3\} = 0.7$$

For instance ‘0.7/3’ means the possibility of that the value of X is 3 is 0.7.

2.3 Possibility distribution with hedges

In proposed work Linguistic hedges, are defined as unary operators on fuzzy sets. The linguistic hedge ‘very’ is defined as Concentration (CON) operation and ‘fairly’ is defined as Dilation (DIL) operation²⁴.

2.3.1.1 Concentration:

$$\text{very}(X) = \text{CON}(X) = \mu_{\text{CON}(X)}(u) = \mu_x(u)^2$$

$$X \text{ is very small} \rightarrow \Pi_x = (\text{small})^2$$

$$\Pi_x(u) = \mu_{\text{small}}(u)^2$$

2.3.1.2 Dilation

$$\text{fairly}(X) = \text{DIL}(X) = \mu_{\text{DIL}(X)}(u) = \mu_x(u)^{1/2}$$

$$X \text{ is fairly small} \rightarrow \Pi_x = (\text{small})^{1/2}$$

$$\Pi_x(u) = \mu_{\text{small}}(u)^{1/2}$$

2.3.2 Negation possibility distribution

$$X \text{ is not small} \rightarrow \Pi_x = (\text{small})'$$

$$\Pi_x = 1 - \mu_{\text{small}}(u)$$

$$X \text{ is not very small} \rightarrow \Pi_x = ((\text{small})')^2$$

$$\Pi_x(u) = 1 - \mu_{\text{small}}(u)^2$$

The ‘very’ hedges make stronger the positive meaning of true, while fairly weakens its positive meaning²⁴. So we

have used a parametric representation of linguistic hedges of fuzzy logic. In²⁴ authors have established basic linguistic truth expressions concomitant with corresponding parameters as follows.

2.3.3 Parametric representation of ‘very linguistic hedges’

$$\begin{aligned}
 X \text{ is large} &\rightarrow \Pi_x(u) = \mu_{large}(u) \\
 X \text{ is very large} &\rightarrow \Pi_x = (large)^{1/2} \\
 \Pi_x(u) &= \mu_{large}(u)^{1/2}
 \end{aligned}$$

2.3.4 Parametric representation of ‘fairly linguistic hedges’ of fuzzy logic

$$\begin{aligned}
 X \text{ is small} &\Pi_x(u) = \mu_{small}(u) \\
 X \text{ is fairly small} &\rightarrow \Pi_x = (small)^2 \\
 \Pi_x(u) &= \mu_{small}(u)^2
 \end{aligned}$$

3. Proposed system

In proposed system a set of description of criminal’s face is stored in database along with membership values of different parts of face. Whenever any input statement is submitted by onlooker(s), a set of possible facial images are retrieved. Moreover, the f-similarity of stored images is estimated with respect to input statements by using ‘as many as possible’ fuzzy quantifier. However the concept of relevance matrix is used for ranking these images in decreasing order.

3.1 Formalization of different parts of face

In this work we are taking the length and breadth of face as 8 and 6 inches respectively. We are reserving eye area 3 x 5 inches as shown in Fig.1.(a). The area for nose length is assumed as 5 inches. Rest of the area is reserved for lips, chin and forehead.

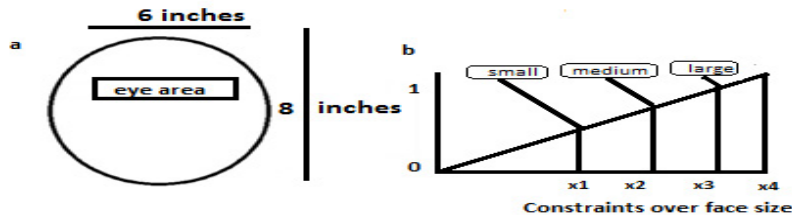


Fig.1. (a) framework for components of face; (b) face size as a fuzzy granule with different constraints.

The size and type of shape may vary as per requirements. The membership functions of parts of face given by (1).

$$\mu(parts) = \left\{ \begin{array}{ll} x/b & \text{if } 0 < x < b \\ 0 & \text{otherwise} \end{array} \right\} \tag{1}$$

where ‘b’ is a real number. The value of b=8 inches, because it is the largest number and has been used for normalizing rest of the component of face. As shown in Fig.1(a) the length of face is 8 inches, which is maximum value. Where the variables x₁, x₂, x₃, and x₄ are taken from Fig.1(b), and variable x₄ is the largest value. All the parts of face are normalized by x₄ i.e. 8 inches. The values of x₁, x₂, and x₃ are used for small, medium, and large constraints respectively. The shape of foreheads, eyes, nose, lips and chins are considered as fuzzy semicircle (f-semicircle),fuzzy (f-circle), fuzzy triangle (f-triangle),fuzzy circle (f-circle) and fuzzy semicircle (f-semicircle) respectively, which are shown in Fig. 2(a),(b),and Fig 3 (a),(b). The size of different parts of face is shown in ‘Table

1' with 'small', 'medium', and 'large' constraints.

Table 1. Size of different parts of face in inches

Parts of face	Small (x_1)	Medium (x_2)	Large (x_3)
Forehead	2	4.8	5.4
Eye	2	3.2	3.6
Nose	2	3.2	4.8
Lips/chin	1	3.2	3.6

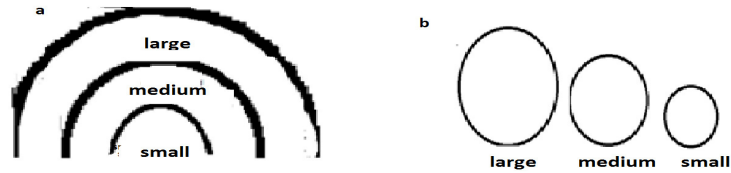


Fig.2. (a) different types of forehead with semi-circle shapes; (b) different types of eyes with circular shapes.

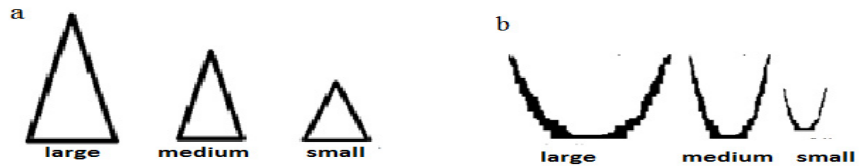


Fig.3. (a) different types of nose with triangle shapes (b) different types of chins with semi-circle shapes

First of all, we have pre-processed the input query by potter stemmer algorithm. After pre-processing the statement is converted into GCL i.e. in the form of X is R or

$$X \rightarrow R \text{ or } \text{Nose} \rightarrow \text{very small}$$

The membership values of different parts on the basis of 'Possibilistic constraints' have shown in Table 2, 3, and 4.

Table2. Membership values of different parts of face with small constraints and 'fairly' linguistic hedges

Parts of face	small	not small	fairly small	not fairly small
Forehead	0.25	0.75	0.5	0.5
Eye	0.25	0.75	0.5	0.5
Nose	0.125	0.875	0.353	0.646
Lips/chin	0.125	0.875	0.353	0.646

Table 3. Membership values of different parts of face with medium constraints and large constraints with 'very' linguistic hedges

Parts of face	medium	not medium	large	not large	very large	not very large
Forehead	0.4	0.6	0.675	0.325	0.455	0.544
Eye	0.4	0.6	0.45	0.55	0.2025	0.797
Nose	0.4	0.6	0.6	0.4	0.36	0.64
Lips/chin	0.4	0.6	0.45	0.55	0.2025	0.7975

Table4. Membership values of different parts of face after applying parametric representation of linguistic hedges 'fairly' and 'very' on small and large constraint respectively

Parts of face	small	fairly small	not fairly small	large	very large	not very large
Forehead	0.25	0.0625	0.325	0.675	0.455625	0.544375
Eye	0.25	0.0625	0.55	0.45	0.2025	0.7975
Nose	0.125	0.0625	0.4	0.6	0.36	0.64
Lips/chin	0.125	0.015625	0.55	0.45	0.2025	0.7975

3.2 Ordered Weighted Averaging

Ordered Weighted Averaging (OWA), was originally introduced by Yager⁸. OWA operators are applicable, where the decision lie some-where between OR-ness and AND-ness. Detail discussion about the behaviour of operators is in⁸.

The OWA operation involves following three steps.

3.2.1 Rearrangement of inputs

The input parameter $(x_1, x_2, x_3, \dots, x_n)$, rearranged in decreasing order $y_1 \geq y_2 \geq y_3 \geq \dots \geq y_m$. However, the weights w_j of the operator R are not related with any exact value of x_j , instead they are related with the ordinal position of y_j .

3.2.2 Weight determination

The mathematical representation of relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ (r - a)/(b - a) & \text{if } a \leq r \leq b \\ 1 & \text{if } r > b \end{cases} \quad (2)$$

where $a, b, r \in [0,1]$.

Yager calculates the weights w_j from the function Q describing the quantifier, with m number of criteria.

$$W_j = Q(j/m) - Q((j - 1)/m) \quad (3)$$

where $j=1,2,\dots,m$ and $Q(0) = 0$.

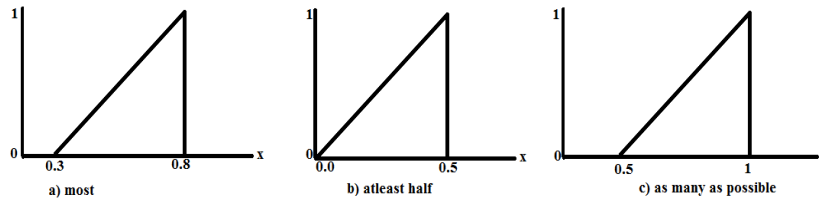


Fig.4. linguistic quantifier

The relative quantifiers pictorially represented as in Fig.4. This shows the relative quantifiers “most”, “at least half” and “as many as possible” taking the parameter ‘a’ and ‘b’ as (0.3, 0.8), (0, 0.5) and (0.5, 1) respectively.

3.2.3 Aggregation of weights with input

OWA determines the f -similarity in f -objects by aggregating the multiplication of decreasing order of input parameter and weight as shown in equation (4). Where $X=(x_1, x_2, x_3, \dots, x_m)$ are input parameters with the multi-criteria of size m . The y_i is the j^{th} largest input parameter.

$$OWA(x_1, x_2, x_3, \dots, x_m) = \sum_{j=1}^m w_j y_j \quad (4)$$

for $j = 1$ to m

4. Experimental works and results

For experimental work ‘query statement’ submitted by onlooker(s) is taken as input. These are supposed to be description of facial image. We have estimated relevancy of input image with other stored images by using relevance matrix. The ranking of different faces are shown in Fig.5 in decreasing order of f -similarity

Query Statement “He has large forehead. His eyes are large. He has a large nose. He has large lips. He has a large chin”

Description of Facial Image1 “He has large forehead. His eyes are large. He has a large nose. He has large lips. He has a large chin”

Description of Facial Image2 “He has a very large forehead. His eyes are large. He has a large nose. He has large

lips. He has a large chin"

Description of Facial Image3 "He has a large forehead. Eyes are large. He has a large nose. He has small lips. He has a small chin"

Description of Facial Image4 "He has large forehead. Eyes are small. He has a large nose. He has small lips. He has a large chin"

Description of Facial Image5 "He has very large forehead. His eyes are small. He has a large nose. He has large lips. He has a large chin"

Example1:The size of large forehead, eyes, nose, lips, and chin in inches are {5.4,3.6,4.8,3.6,3.6} in turns membership {0.675,0.45,0.6,0.45,0.45} respectively by using (1).

For estimating f-similarity of a face having the description of all the facial images the membership values are given by equation (1). Let the relevance matrix R with face description set {D₁, D₂, D₃, D₄, D₅} and set of membership values of features (μ₁,μ₂,μ₃,μ₄,μ₅) be given as

$$R = \begin{matrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \end{matrix} \begin{bmatrix} 0.675 & 0.45 & 0.6 & 0.45 & 0.45 \\ 0.6 & 0.45 & 0.45 & 0.45 & 0.45 \\ 0.675 & 0.45 & 0.6 & 0.125 & 0.125 \\ 0.675 & 0.45 & 0.6 & 0.25 & 0.125 \\ 0.6 & 0.45 & 0.456 & 0.45 & 0.25 \end{bmatrix}$$

For the feature description D="as many as possible" {D₁,D₂,D₃,D₄,D₅} we would have the weights w₁=0.0,w₂=0.0,w₃=0.2,w₄=0.4,w₅=0.4, which are generated by (4) and (5) for 'm'=5. i.e. in this work we have considered five main parts of face viz. forehead, eyes, nose, lips and chin.

The ordered relevance matrix with respect to the description is then given as follows,

$$DR = \begin{bmatrix} 0.0 \times 0.675 & 0.0 \times 0.675 & 0.2 \times 0.675 & 0.4 \times 0.600 & 0.4 \times 0.600 \\ 0.0 \times 0.450 & 0.0 \times 0.450 & 0.2 \times 0.450 & 0.4 \times 0.450 & 0.4 \times 0.450 \\ 0.0 \times 0.600 & 0.0 \times 0.600 & 0.2 \times 0.600 & 0.4 \times 0.456 & 0.4 \times 0.450 \\ 0.0 \times 0.450 & 0.0 \times 0.450 & 0.2 \times 0.450 & 0.4 \times 0.250 & 0.4 \times 0.125 \\ 0.0 \times 0.450 & 0.0 \times 0.450 & 0.2 \times 0.250 & 0.4 \times 0.125 & 0.4 \times 0.125 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.45 \\ 0.19 \\ 0.24 \\ 0.37 \end{bmatrix}$$

Decreasing order sort on the elements of description relevance vector DR gives the results of the query as D₁≥D₂>D₅>D₄>D₃

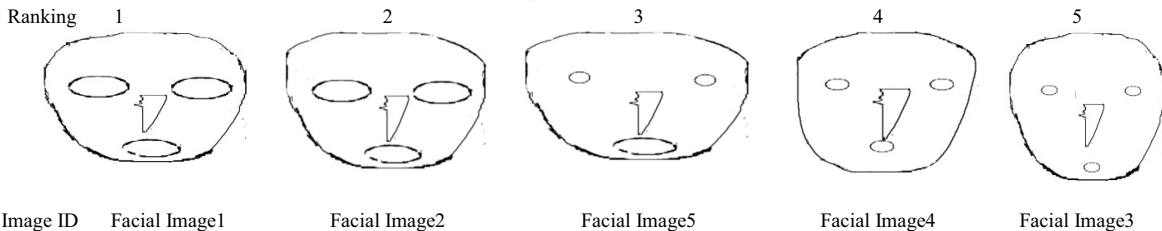


Fig.5. ranking of facial images in decreasing order of relevancy

5. Conclusion and Future Directions

The concepts of f-geometry and CWW are combined for the identification of miscreant's face. The concepts of fuzzy granule, possibilistic constraints, and Generalize Constraints Language, are the foundation of CWW. The concept of fuzzy granule has been applied for modelling onlooker(s) mind set. Various fuzzy objects are used for describing different parts of face. For that we have estimated the membership values. Afterwards these facial parts are stored along with the membership values. The shapes of foreheads, eyes, nose, lips and chins are considered as f-semicircle, f-circle, f-triangle, f-circle, and f-semicircle respectively. We have considered three types of constraints over size of parts of face 'small', 'medium', and 'large'. Possibilistic constraints with linguistic hedges 'fairly', 'very' and negation operator are used. Yager's quantifier 'as many as possible' is used for aggregating forehead,

eyes, nose, lips, and chin to make a complete face. Moreover the concept of f -similarity is used for listing similar faces in decreasing order of relevancy. Providing a number of similar faces helps onlooker(s) identifying criminal. SWW may provide a scientific basis for human face recognition system by simulating the forensic sketch expert in the discipline of computational forensics. Identification of face sketch of criminal on the basis of onlooker's statement may open door for identification of imprecise image.

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