



# Target mass corrections to proton spin structure functions and quark–hadron duality

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## Abstract

Target mass corrections to proton spin structure functions in deep inelastic scattering region are analysed. Moreover, Bloom–Gilman quark–hadron dualities of proton spin structure functions  $g_1$  and  $g_2$ , in inelastic resonance region, are studied. The onsets of the dualities are discussed. © 2006 Elsevier B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

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## 1. Introduction

It is known that the study of quark–hadron duality is essential to understand the physics behind the connection between perturbative QCD (pQCD) and non-perturbative QCD [1], and thus, it shows fundamental issues in strong interaction. In 2001, the new evidence of valence-like quark–hadron duality in nucleon unpolarised structure function  $F_2$  was reported by Jefferson Lab. [2]. Those new data can revisit quark–hadron duality and show that the duality works even in a rather low momentum transfer region of  $Q^2 \sim 1 \text{ GeV}^2$ . We know that the Bloom–Gilman quark–hadron duality [3] tells that prominent resonances do not disappear relative to background even at a large  $Q^2$ . Moreover, the duality also means that the average of the oscillate resonance peaks in the resonance region is the same as that of the scaling structure function at a large  $Q^2$  value. The origin of the Bloom–Gilman quark–hadron duality has been given by Rujula et al. [4] with a QCD explanation. It was also extensively studied in many works [5] through a consideration of the asymptotic perturbative QCD behaviours of the resonance electromagnetic transition amplitudes at a large momentum transfer. Recently, many interesting studies

of the quark–hadron duality were published [6–12]. Particularly, Close and Isgur [1] discussed the evolution of the nucleon structure function from coherent resonance region to incoherent inelastic scattering one. Very late review and measurement of the quark–hadron duality are referred to Ref. [13].

So far, one still has no any definitely experimental evidence about the occurrence of the Bloom–Gilman quark–hadron dualities for the nucleon spin structure functions, like  $g_1$  and  $g_2$ . It is naturally to expect that the onset of the quark–hadron duality for  $g_1$  of the proton target is at a larger  $Q^2$  point than the  $Q^2$  value of the occurrence of the duality for proton unpolarized structure function  $F_2$  [5]. This is because that very strong  $Q^2$ -dependence of  $g_1$  at low  $Q^2$  is needed by the well-known Gerasimov–Drell–Hearn (GDH) sum rule [14]. Experimental study of the quark–hadron duality in the nucleon spin structure function  $g_1$  was performed by HERMES group and Jlab recently [15]. Limited available data indicate that the onset of the duality for  $g_1$  is likely at a larger  $Q^2$  than  $1.7 \text{ GeV}^2$ . Theoretical analysis also reaches the similar conclusion that the occurrence of the duality of  $g_1^p$  is likely at  $Q^2 \sim 2 \text{ GeV}^2$  [10].

To study the Bloom–Gilman quark–hadron dualities of the proton spin structure functions  $g_1$  and  $g_2$ , one should know the structure functions both in the resonance region with small  $Q^2$  (centre-of-mass energy  $W < 2.5 \text{ GeV}$ ), and in deep inelas-

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tic scattering (DIS) region. Moreover, the role played by target mass corrections to the structure functions in DIS region should be also carefully analysed. On the one hand, we'll simply regard the parametrization forms of the proton spin structure functions  $g_1$  and  $g_2$  by Simula et al. [16] is a good choice to simulate the spin structure function  $g_1^p$  in the resonance region. Those forms contain nucleon elastic effect and the contributions of the resonances and non-resonance background with 14 parameters involved in, and fixed [16] by fitting to the available data in the resonance region. A simple Breit–Wigner shape is used to describe the  $W$  dependence of the contribution of an isolated resonance. All four-star resonances, having a total transverse photo-amplitude  $\sqrt{|A_{1/2}|^2 + |A_{3/2}|^2}$  larger than  $0.05 \text{ GeV}^{-1/2}$ , are considered. On the other hand, we will employ next-to-leading order pQCD calculations for the spin structure functions in the DIS region. There are several known calculations in the literature [17–19], like those by Glück, Reya, Stratmann and Vogelsang (GRSV) [17], and by Leader, Sidorov and Stamenov (LSS) [18]. In this Letter, we shall employ the results of GRSV to analyse the target mass corrections to the spin structure functions in the DIS region. Comparing the averages of the proton spin structure functions both in the resonance and in the DIS regions, we can phenomenologically study the Bloom–Gilman quark–hadron dualities of the proton spin structure functions.

It should be stressed that there were several works related to the target mass corrections for  $g_1$  in the literature [20,21]. The target mass corrections to the Bjorken sum rule were discussed in Ref. [22]. Recently, the target mass corrections to all the nucleon spin structure functions, like  $g_1$  and  $g_2$ , have been studied carefully in Refs. [23–26].

This Letter is organized as follows. In Section 2, the target mass corrections to the proton spin structure functions are studied. The corrections to the truncated moments of  $g_{1,2}$  and to the quark–hadron dualities of the proton spin structure functions will be discussed in Section 3. The final section is devoted to conclusions.

## 2. Target mass corrections

One may follow the method proposed by Georgi and Politzer [27], in the case of unpolarised structure function, to get the target mass corrections (TMCs) to the spin structure functions. The recent calculations show that [23,24]

$$g_1^{\text{TMCs}}(x, Q^2) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dn x^{-n} \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{n(n+j)!}{j!(n-1)!(n+2j)^2} \times M_1^{n+2j}(Q^2; M=0), \quad (1)$$

where  $M_1^n(Q^2; M=0)$  is the Cornwall–Norton (CN) moment of  $g_1$

$$M_1^n(Q^2; M=0) = \int_0^1 dx x^{n-1} g_1(x, Q^2; M=0) \quad (2)$$

calculated in the perturbative QCD where all the mass terms  $O(M^n/Q^n)$  are neglect, namely, the nucleon mass  $M$  vanishes. The explicit twist-2 expression of  $g_1$  with the TMCs is

$$g_1^{\text{TMCs}}(x, Q^2) = \frac{x g_1(\xi, Q^2; M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{4M^2x^2}{Q^2} \frac{x+\xi}{\xi(1+4M^2x^2/Q^2)^2} \int_{\xi}^1 \frac{d\xi'}{\xi'} g_1(\xi', Q^2; M=0) - \frac{4M^2x^2}{Q^2} \frac{(2-4M^2x^2/Q^2)}{2(1+4M^2x^2/Q^2)^{5/2}} \times \int_{\xi}^1 \frac{d\xi'}{\xi'} \int_{\xi'}^1 \frac{d\xi''}{\xi''} g_1(\xi'', Q^2, M=0), \quad (3)$$

where the Nachtmann variable [20] is

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \quad (4)$$

with  $x = \frac{Q^2}{Q^2 + W^2 - M^2}$ . Similarly, the spin structure function  $g_2$  with twist-2 contribution and with the TMCs is

$$g_2^{\text{TMCs}}(x, Q^2) = -\frac{x g_1(\xi, Q^2; M=0)}{\xi(1+4M^2x^2/Q^2)^{3/2}} + \frac{x(1-4M^2x\xi/Q^2)}{\xi(1+4M^2x^2/Q^2)^2} \int_{\xi}^1 \frac{d\xi'}{\xi'} g_1(\xi', Q^2; M=0) + \frac{3}{2} \frac{4M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}} \times \int_{\xi}^1 \frac{d\xi'}{\xi'} \int_{\xi'}^1 \frac{d\xi''}{\xi''} g_1(\xi'', Q^2, M=0). \quad (5)$$

The above  $g_2^{\text{TMCs}}$  satisfies the well-known Wandzura–Wilczek (WW) relation [28]

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y). \quad (6)$$

Namely, the WW relation is not affected by the target mass corrections.

Here, it should be noted that the Nachtmann moments of the nucleon spin structure functions, shown in the literature [21,22], are

$$M_1^n(Q^2; N) = \int_0^1 dx \frac{\xi^{n+1}}{x^2} \left\{ g_1(x, Q^2) \left[ \frac{x}{\xi} - \frac{n^2}{(n+2)^2} \frac{M^2x^2}{Q^2} \frac{\xi}{x} \right] - g_2(x, Q^2) \frac{M^2x^2}{Q^2} \frac{4n}{n+2} \right\}, \quad (n=1, 3, 5, \dots) \quad (7)$$

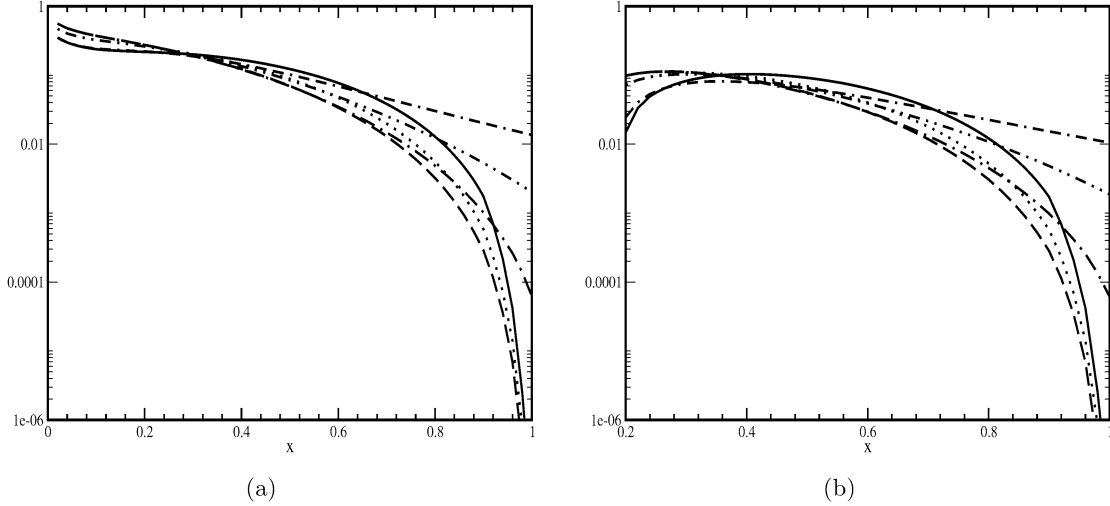


Fig. 1. Comparisons (a) of  $g_1(x, Q^2)$  with  $g_1^{\text{TMCs}}(x, Q^2)$  and (b) of  $-g_2(x, Q^2)$  with  $-g_2^{\text{TMCs}}(x, Q^2)$ . The solid ( $Q^2 = 1 \text{ GeV}^2$ ), dotted ( $Q^2 = 3 \text{ GeV}^2$ ) and dashed curves ( $Q^2 = 10 \text{ GeV}^2$ ) are the results without the target mass corrections; whereas two-dashed-dotted ( $Q^2 = 1 \text{ GeV}^2$ ), two-dotted-dashed ( $Q^2 = 3 \text{ GeV}^2$ ) and dotted-dashed ( $Q^2 = 10 \text{ GeV}^2$ ) curves represent the results with TMCs, respectively. The scaling structure function  $g_1$  is from GRSV [17].

and

$$M_2^n(Q^2; N) = \int_0^1 dx \frac{\xi^{n+1}}{x^2} \left\{ \frac{x}{\xi} g_1(x, Q^2) + \left[ \frac{n}{n-1} \frac{x^2}{\xi^2} - \frac{n}{n+1} \frac{M^2 x^2}{Q^2} \right] g_2(x, Q^2) \right\},$$

$(n = 3, 5, \dots)$  (8)

The difference between the CN and Nachtmann moments comes from the trace terms appearing in the matrix elements of the operators of definite spin, which are disregarded in the CN moments, but kept in the Nachtmann moments [26].  $M_{1,2}^n(Q^2; N)$  in Eqs. (7) and (8) are constructed to protect the moments of the structure functions from the TMCs (with kinematical origin) and they contain only dynamical higher twists, which are the ones related to the correlations among parton [16].

Piccione and Ridolfi [23] have compared the moments of Eqs. (7) and (8) with those of Eqs. (3) and (5). They argued that Eqs. (7) and (8) are not directly applicable in a full analysis of the polarized DIS data because the target mass corrected reduced matrix elements of the relevant operators in operator production expansion (OPE), like  $a_n$  and  $d_n$ , were expressed in terms of the polarized structure functions, if taking the TMCs into account; these expressions reduce to the moments of the structure functions in the massless limit, but do not have a simple parton model interpretation in the case of  $M \neq 0$ . Thus, they claimed that Eqs. (3) and (5) have the advantage that the moments of the polarized structure functions are expressed as the functions of the reduced operator matrix elements and the effects of the TMCs on the nucleon spin structure functions, which are of pure kinematical origin, can be explicitly seen.

To see the effects of the TMCs, we, in Fig. 1, explicitly show the comparisons of  $g_1(x, Q^2)$  with  $g_1^{\text{TMCs}}(x, Q^2)$ , and of  $-g_2(x, Q^2)$  with  $-g_2^{\text{TMCs}}(x, Q^2)$ . From Figs. 1(a) and (b), we see that the TMCs play an remarkably role. They enlarge the

values of the spin structure functions, particularly in the large  $x$  region. The figures reasonably show that the smaller the momentum transfer  $Q^2$  is, the larger the effects of the TMCs are. Moreover, we see that in the limit  $x \rightarrow 1$ ,  $g_{1,2}^{\text{TMCs}}$  do not vanish, although  $g_{1,2}(x, Q^2; M = 0) \rightarrow 1$ . This phenomenon can be easily understood since  $\xi(x = 1) < 1$  in the limit, for example,  $\xi(x = 1, Q^2) \sim 0.64, 0.87$  and  $0.92$  for  $Q^2 = 1, 5$  and  $10 \text{ GeV}^2$ , respectively. In Fig. 1(b), we show the results starting from  $x \geq 0.2$ . It should be mentioned that  $-g_2$  and  $-g_2^{\text{TMCs}}$  are negative in a very small  $x$ -region, so that the Burkhardt–Cottingham (BC) [29] sum rule is satisfied.

### 3. Truncated moments and the quark–hadron dualities of the proton spin structure functions

We know that the Bloom–Gilman quark–hadron duality means that the smooth scaling curve seen at a high  $Q^2$  region is an average over the resonance bumps seen at low  $Q^2$  region. To check this quark–hadron duality, we, first of all, calculate the truncated Cornwall–Norton moments. These truncated moments are proposed by Rujula et al. [4] to justify the Bloom–Gilman duality. One of them is defined in the resonance production regions as

$$\bar{M}_{1,2}^n(Q^2) = \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}(\xi, Q^2),$$

(9)

and other one is

$$\bar{A}_{1,2}^n(Q^2) = \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}^{\text{TMCs}}(\xi, Q^2),$$

(10)

where  $\xi_\pi$  (or  $\xi^*$ ) in the integrated interval stands for  $\xi$  with the minimum of the center-of-mass energy  $W = M + m_\pi$  (or the maximum of  $W_{\text{max}} = 2.5 \text{ GeV}$ ). Here, the moments are truncated ones and they are very sensitive to the Bloom–Gilman

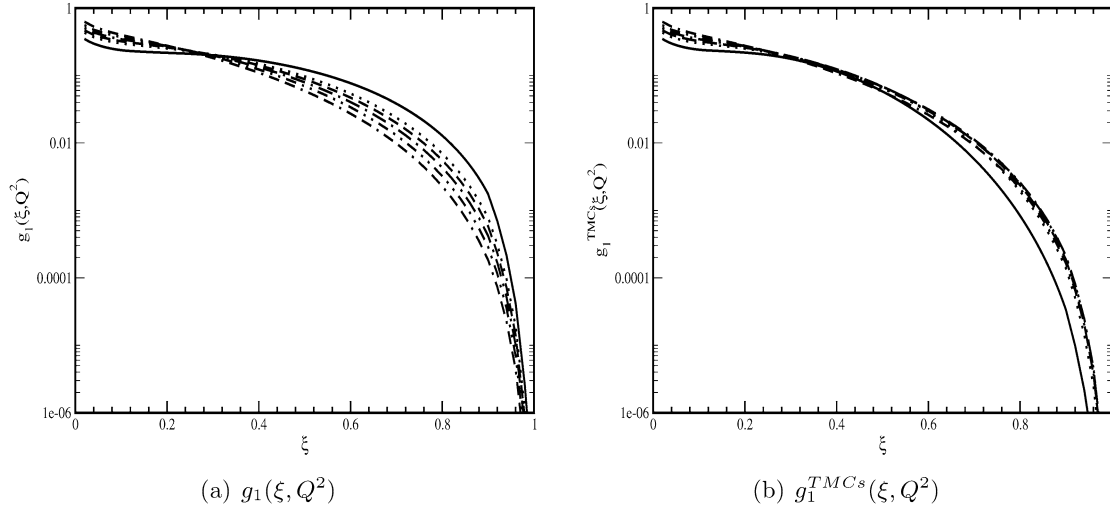


Fig. 2. Proton spin structure function  $g_1(\xi, Q^2)$  (a) without and (b) with the target mass corrections. The solid, dotted, dashed, dotted-dashed double-dotted-dashed and dotted-double-dashed curves stand for the cases of  $Q^2 = 1, 2, 3, 5, 10$  and  $30 \text{ GeV}^2$ , respectively. The scaling structure function  $g_1$  is from GRSV [17].

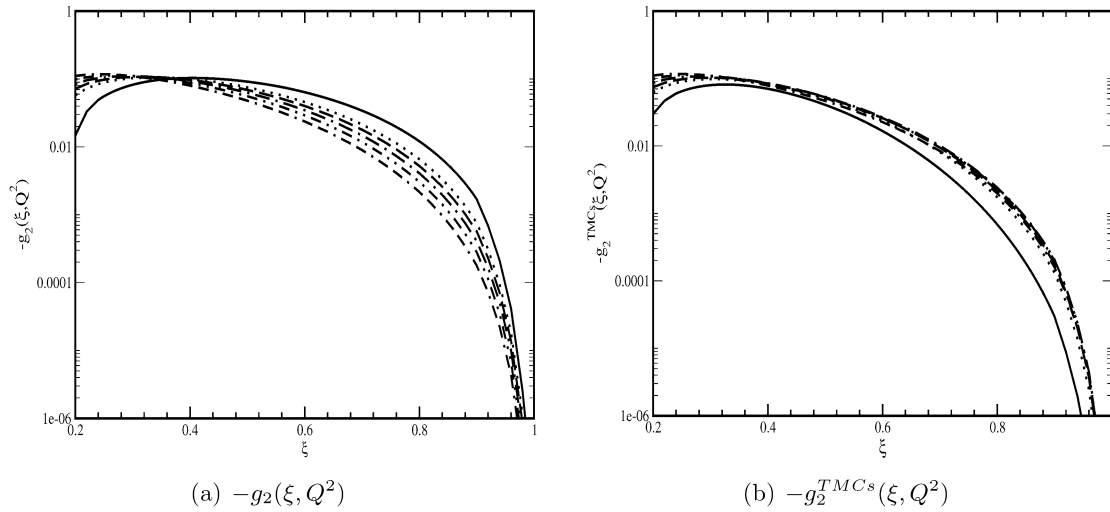


Fig. 3. Proton spin structure function  $-g_2(\xi, Q^2)$  (a) without and (b) with the target mass corrections. Notations as in Fig. 2.

quark–hadron duality with respect to un-truncated ones because most of the contributions of the resonance bumps are involved in. Eq. (9) can be calculated by taking into account the parametrizations given by Ref. [16]. It should be mentioned that the integrated variables in Eqs. (9) and (10) are the Nachtmann variable  $\xi$  which is the correct one in studying QCD scaling violations in the nucleon because the integral with respect to  $\xi$  partially takes the TMCs into account [30].

In order to see the effects of the TMCs on the truncated moments of the nucleon spin structure functions, we, in Fig. 2, respectively plot  $g_1(\xi, Q^2)$  and  $g_1^{TMCs}(\xi, Q^2)$ . Here, we replace the argument  $x$  in  $g_1(x, Q^2)$  directly by the Nachtmann variable  $\xi$ , and employ the result of the next-to-leading order pQCD prediction of GRSV [17] for the scaling spin structure function  $g_1(x, Q^2; M = 0)$ . In the figures several different values of  $Q^2$  are selected. Moreover, in Fig. 3, we respectively show the corresponding results for the proton spin structure function  $-g_2$  starting from  $\xi \geq 0.2$ . It should be reiterated that the some of the results with small  $\xi$  are negative so that the BC sum rule is satisfied.

Figs. 2 and 3 show that the shapes of  $g_{1,2}^{TMCs}(\xi, Q^2)$  at large  $\xi$  are more insensitive to  $Q^2$  than those of  $g_{1,2}$  in the case of  $Q^2 \geq 2 \text{ GeV}^2$ . Thus, leading to an approximately dual relation

$$g_{1,2}^{TMCs}(\xi, Q^2) \sim g_{1,2}(\xi, Q_{\text{high}}^2) \quad (11)$$

which is valid for  $\xi \geq 0.4$  and  $Q^2 \geq 2 \text{ GeV}^2$ . This result means that the  $\xi$ -shapes of the target mass corrected scaling structure functions  $g_{1,2}^{TMCs}$  with  $Q^2 \geq 2 \text{ GeV}^2$  are quite similar to the  $\xi$ -shapes of the scaling spin structure functions  $g_1(\xi, Q^2; M = 0)$  seen at a high  $Q_{\text{high}}^2$  (say  $Q_{\text{high}}^2 = 30 \text{ GeV}^2$ , for example). It also turns out that the TMCs almost compensate the effects due to the pQCD evolution. The above conclusion is similar to the one of Ref. [12] in the case of the unpolarized structure function  $F_2^p$ . In addition, the  $x$ - (or  $\xi$ -) dependences of the target mass corrected structure functions  $g_{1,2}^{TMCs}$  are different from those without the TMCs, particularly, in the large  $x$  (or  $\xi$ ) region. We know that  $\xi < x$ , and the  $\xi$ -range extends up to  $\xi_{\text{max}} = 1$ , corresponding to unphysical, but finite values of

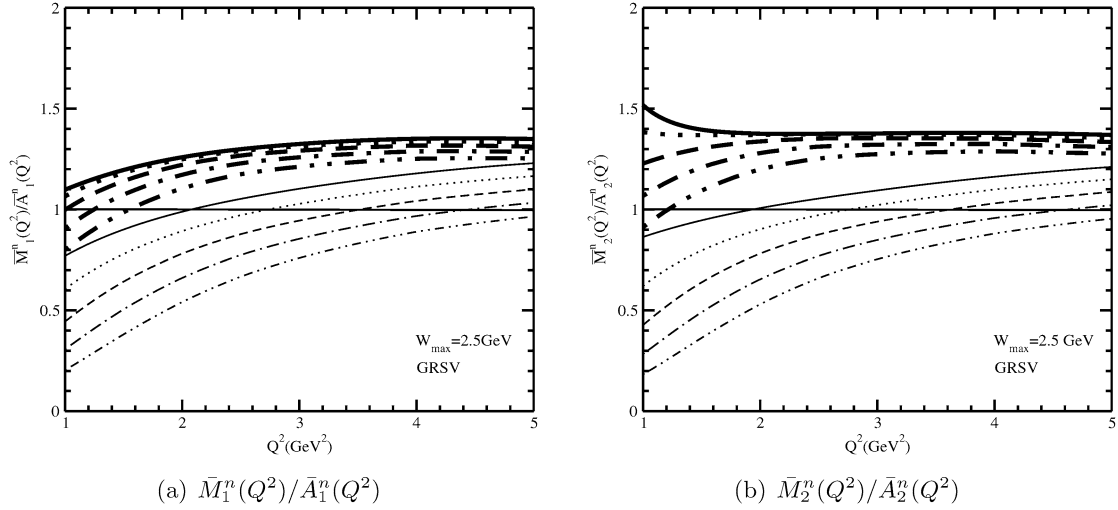


Fig. 4. (a) The ratios  $\bar{M}_1^n(Q^2)/\bar{A}_1^n(Q^2)$  (thick lines) and  $\bar{M}_1^n(Q^2)/\bar{A}_1^n(Q^2; S)$  (thin lines), and (b) the ratios  $\bar{M}_2^n(Q^2)/\bar{A}_2^n(Q^2)$  (thick lines) and  $\bar{M}_2^n(Q^2)/\bar{A}_2^n(Q^2; S)$  (thin lines) with the GRSV scaling structure function. The solid, dotted, dashed, dotted-dashed and double-dotted-dashed curves correspond to  $n = 1, 2, 3, 4$  and  $5$ , respectively.

$x > 1$  in the case of  $Q > M$ . Clearly, the larger the  $Q^2$  is, the smaller the difference between  $g_{1,2}$  and  $g_{1,2}^{\text{TMCs}}$  becomes.

To study the Bloom–Gilman quark hadron dualities of the nucleon spin structure functions explicitly, we respectively plot the ratios of  $\bar{M}_{1,2}^n(Q^2)/\bar{A}_{1,2}^n(Q^2)$  with different weight factor  $\xi^{n-1}$  (like the  $n$ th CN moment) in Fig. 4. Moreover, to check the effects of the TMCs, we also display the ratios of  $\bar{M}_{1,2}^n(Q^2)/\bar{A}_{1,2}^n(Q^2; S)$  (by the thin curves) with

$$\bar{A}_{1,2}^n(Q^2; S) = \int_{x^*}^{x_\pi} dx x^{n-1} g_{1,2}(x, Q^2; M = 0). \quad (12)$$

Clearly, when the target mass corrections vanish, we have  $\xi \rightarrow x$ , and  $g_{1,2}^{\text{TMCs}}(\xi, Q^2) \rightarrow g_{1,2}(x, Q^2)$ . Thus,  $\bar{A}_{1,2}^n(Q^2)$  turns to be  $\bar{A}_{1,2}^n(Q^2; S)$ . However, when  $M \neq 0$ , the effects of the TMCs can be explicitly seen from the difference between the two truncated moments of Eqs. (10) and (12).

From Fig. 4 one finds that the  $Q^2$ -dependences of the ratio  $\bar{M}_2^n(Q^2)/\bar{A}_2^n(Q^2)$  are much more smooth than those of  $\bar{M}_2^n(Q^2)/\bar{A}_2^n(Q^2; S)$  in the large  $Q^2$ -region with  $Q^2 \geq 2 \text{ GeV}^2$ . The obvious differences between the two ratios shown by Figs. 4(a) and (b) indicate the significant role played by the target mass corrections. Moreover, the feature that the shapes of the ratios  $\bar{M}_2^n(Q^2)/\bar{A}_2^n(Q^2)$ , being almost  $n$ -independent and  $Q^2$ -independent in the region of  $Q^2 \geq 2 \text{ GeV}^2$ , show that the occurrences of the quark–hadron dualities for  $g_{1,2}^p$  are expected to be at around  $\sim 2 \text{ GeV}^2$ . Thus, we conclude that the quark–hadron dualities of the nucleon spin structure functions can easily preserve if the target mass corrections are included.

#### 4. Conclusions

To sum up, we have studied the Bloom–Gilman quark–hadron dualities of the proton spin structure functions  $g_1$  and  $g_2$  with the truncated moments simultaneously and phenomenologically. The twist-2 contributions with the target mass correc-

tions are explicitly included. Our results show that the TMCs play a remarkable role on the nucleon spin structure functions, particular in the low  $Q^2$  and large  $x$  (or  $\xi$ ) region. The  $\xi$ -dependences of  $g_{1,2}^{\text{TMCs}}(\xi, Q^2)$  show that when  $Q^2 \geq 2 \text{ GeV}^2$  we have  $g_{1,2}^{\text{TMCs}}(\xi, Q^2) \sim g_{1,2}(\xi, Q_{\text{high}}^2)$  and consequentially

$$\bar{A}_{1,2}^n(Q^2) \sim \int_{\xi^*}^{\xi_\pi} d\xi \xi^{n-1} g_{1,2}(\xi, Q_{\text{high}}^2). \quad (13)$$

It turns out that the effects of the TMCs compensate the role due to the pQCD evolution. Moreover, we find that the ratios  $\bar{M}_{1,2}^n(Q^2)/\bar{A}_{1,2}^n(Q^2)$  with the TMCs become almost  $Q^2$ -independent and the weight factor  $\xi^{n-1}$ -independent when  $Q^2$  is large (say about  $2 \text{ GeV}^2$ ). It means that the averages of the resonance bumps are similar to the truncated moments of the scaling structure functions with the TMCs (or with a high  $Q^2$ ). Thus, it shows that the onsets of the Bloom–Gilman quark–hadron dualities of  $g_1$  and  $g_2$  in the inelastic region, including nucleon resonance with  $M + m_\pi \leq W \leq W_{\text{max}} = 2.5 \text{ GeV}$ , are expected in the region of  $Q^2 \geq 2 \text{ GeV}^2$ . It should be noted that this conclusion remains the same for other pQCD predictions of the scaling structure function  $g_1$ , like LSS [18]. The remarkable role of the TMCs shows that they should be included for producing the Bloom–Gilman quark–hadron dualities of the nucleon spin structure functions. Here, we stress that we only examine the truncated moments which are limited to the nucleon resonance production region. The dualities of those truncated moments cannot have any operator production expansion based justification [12].

Our present results depend on the parametrizations of  $g_1$  and  $g_2$  in the resonance region [16]. They also depend on the pQCD predictions for  $g_1$  and twist-2 WW relation for  $g_2$ . It should be pointed out that although the ratios, in Fig. 4, indicate that they are almost  $Q^2$ -independent in the large  $Q^2$  region, they are not exact unity as the Bloom–Gilman quark–hadron duality means. Therefore, our calculation only means that the averages of the



spin structure functions in the truncated resonance region are similar to those of the scaling spin structure functions in the DIS region.

In fact, this phenomenon also appears in the study of  $F_2^p$  [2,12]. One reason is mainly due to the difficulty in accurately modelling the large  $\xi$  behaviour of the scaling spin structure functions. With increasing  $Q^2$ , the moment of Eq. (10) is determined by a smaller and smaller region near  $\xi \sim 1$ . For example, the values of the upper (or lower) limit of  $\xi_\pi$  (or  $\xi^*$ ) in the integral are about 0.563 (0.154), 0.832 (0.464) and 0.903 (0.628) for  $Q^2 = 1 \text{ GeV}^2$ ,  $5 \text{ GeV}^2$ , and  $10 \text{ GeV}^2$ , respectively. The corresponding values of  $x_\pi$  (or  $x^*$ ) are about 0.780 (0.157), 0.947 (0.482), and 0.973 (0.651), respectively. Those kinematic regions are already beyond the ones of the world data used in the pQCD predictions of the nucleon scaling spin structure functions. For example, in LSS calculation [18], the world data, covering the kinematic region of  $0.005 \leq x \leq 0.75$  and  $1 \leq Q^2 \leq 58 \text{ GeV}^2$ , are employed. To estimate the uncertainty in the large  $\xi$  region, we replace  $\xi_\pi$  in the integral of Eq. (10) by  $\xi'_\pi = \xi(x_{\max} = 0.75, Q^2)$ . Then, the values of the upper limit turn to be 0.55, 0.685, and 0.714 for the three cases of  $Q^2$ . Comparing the results of Eq. (10) with

$$\bar{B}_{1,2}^n(Q^2) = \int_{\xi^*}^{\xi'_\pi} d\xi \xi^{n-1} g_{1,2}^{\text{TMCs}}(\xi, Q^2), \quad (14)$$

we find that the contributions from the large  $\xi$  region (with  $x \geq 0.75$ ) to the integral are around 1%, 8% and 30% for the three  $Q^2$  values, respectively. One concludes that the scaling structure functions in the large  $x$  region play a more important role on the truncated integral with a large  $Q^2$  value than on the integral with a small  $Q^2$  value. Unfortunately, there is a very limited amount of the data of the nucleon spin structure functions in DIS region currently available at large  $\xi$  region. It should be mentioned that the high-precision nucleon resonance data set in the large  $x$  ( $\xi$ ) region is required to test the duality quantitatively. If the duality is quantitatively confirmed, this would allow for a precise verification of our knowledge of large- $x$  parton distribution functions.

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