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## The concept of active elimination of vibrations in milling process

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### Abstract

Harmful self-excited vibrations called chatter occur during cutting processes (milling and turning) as a result of the trace regeneration. Moreover, dynamics of milling process is also influenced by intermittent cutting teeth during cutting. The paper presents an analysis of two-dimensional nonlinear model of milling. The analysed model takes into account susceptibility of the tool and the workpiece. The dynamics of the milling process is described by the discontinuous differential equation with a time delay, which can cause process instability for various system parameters. Therefore, stability lobes diagram was determined numerically. In order to reduce harmful vibrations the concept of the use of active piezo-elements is presented here. In addition, the work shows numerical results of chatter control in open and closed loop.

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### 1. Introduction

In the present time, the high speed machining, especially high speed milling plays an important role in manufacturing process. During the machining, at specific combination of cutting depth and spindle speed, the chatter may arise. This effect exists when flexible cutting tool or workpiece starts to vibrate due to the regeneration of the workpiece surface. A wave appearing on the workpiece causes variations in chip thickness that in turn affects variations of the cutting force and as a consequence harmful vibrations called regenerative chatter. This phenomenon is harmful, mainly because of heavy vibrations of the tool which cause low surface quality. Also the productivity of the milling process can be limited by self-excited chatter vibration. Moreover, chatter may cause rapid tool wear.

To model this phenomenon, differential equations with shifted argument are used (DDEs – Delay Differential Equations) [18,23,26]. In milling, the time delay is closely connected with rotational speed of a tool [1,2,27]. Sometimes, delay can be introduced into the system in order to control the process [13,17,20]. Significant description of the most important phenomena occurring during milling and methods for their modeling is presented in extensive publication [19]

and papers [3-6,8,25].

To overcome difficulties related to chatter vibrations one can mention several methods for the elimination or suppression regenerative chatter [15], e.g. tools with the active elimination realized by piezoelectric elements [14], a change of phase between the internal and external modulation in trace regeneration [9,12,22], and the change of dynamic properties of the tool [16,21,24]. In work [11] applied fuzzy logic controller which adaptively selects amplitude and frequency. The paper [17] presents extensive overview of the active methods of chatter elimination during machining. Whereas the use of dampers in the milling operations are discussed in [7,24]. The authors of the article [10] presented a method of modelling composite milling process. They proposed to use a periodic signal as an external excitation of a workpiece. The obtained results confirm the possibility of using excitation of the workpiece in order to eliminate self-excited chatter vibrations.

In this paper, the concept of active elimination of self-excited vibrations in milling through harmonic excitation of a workpiece is proposed. A similar idea is presented in the paper [10] but only for open-loop control and 1DOF system. Here, the system with closed-loop control realized by a *PD* controller is tested in case of 2DOF milling system.

2. 2 DOF model of milling

Models of milling process are non-smooth by nature because a cutting tool has several cutting teeth, which are in contact with a workpiece during some time intervals of cutting. For the rest of time, the cutting edge is not in contact with the workpiece. This causes discontinuities, which make difficulties in numerical simulations and analytical solutions, as well. Therefore, modelling process of milling is rather difficult and complicated from technical point of view. Here, a step function is used to this aim.

In the milling process, material is removed from a workpiece by a cutting tool, which rotates with speed  $n$  (in 1/min). A schematic representation of the 2DOF milling process is shown in Fig. 1.

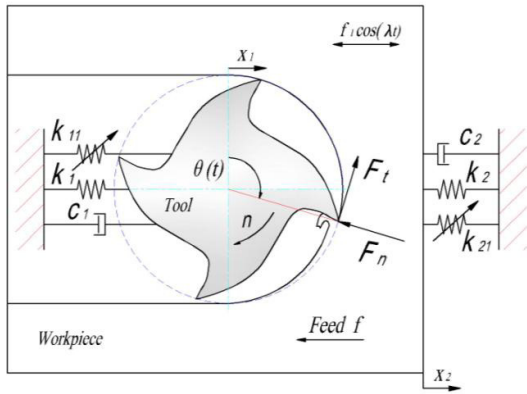


Fig. 1. 2DOF model of milling.

Nonlinear differential equations describing the dynamics of the system are presented in the form

$$\begin{aligned}
 m_1 \ddot{x}_1(t) + c_1 \dot{x}_1(t) + k_1 x_1(t) + k_{11} x_1^3(t) &= \sum_{p=1}^z F_p(t) \\
 m_2 \ddot{x}_2(t) + c_2 \dot{x}_2(t) + k_2 x_2(t) + k_{21} x_2^3(t) &= - \sum_{p=1}^z F_p(t)
 \end{aligned}
 \tag{1}$$

where:

- $m_1$  - substitute mass of the tool,
- $c_1$  - damping of the tool,
- $k_1, k_{11}$  - linear and nonlinear stiffness of the tool,
- $m_2$  - substitute mass of the workpiece,
- $c_2$  - damping of the workpiece,
- $k_2, k_{21}$  - linear and nonlinear stiffness of the workpiece.

Finally, after transformations, the equations of motion take the form

$$\begin{aligned}
 \ddot{x}_1(t) + 2\zeta_1 \omega_{n1} \dot{x}_1(t) + \omega_{n1}^2 x_1(t) + \gamma_1 x_1^3(t) &= \frac{1}{m_1} \sum_{p=1}^z F_p(t) \\
 \ddot{x}_2(t) + 2\zeta_2 \omega_{n2} \dot{x}_2(t) + \omega_{n2}^2 x_2(t) + \gamma_2 x_2^3(t) &= - \frac{1}{m_2} \sum_{p=1}^z F_p(t)
 \end{aligned}
 \tag{2}$$

where:

- $\zeta_1$  - damping coefficient of the tool,
- $\omega_{n1}$  - natural frequency of the tool,
- $\gamma_1$  - nonlinear stiffness coefficient of the tool,
- $\zeta_2$  - damping coefficient of the workpiece,
- $\omega_{n2}$  - natural frequency of the workpiece,
- $\gamma_2$  - nonlinear stiffness coefficient of the workpiece.

The resultant cutting force caused by the  $p$ -th tooth in the  $x$  direction is given by the approximate equation

$$F_p(t) = g_p(t) [-F_{ip}(t)\cos\theta_p(t) - F_{np}(t)\sin\theta_p(t)] \tag{3}$$

The cutting force  $F_p$  acting on  $p$ -th tooth ( $p=1,2,\dots,z$ ) depends on an angular tool position  $\theta_p$  of the  $p$ -th cutting tooth and consists of a tangential  $F_{ip}$  and a normal  $F_{np}$  force component (see Fig.1).  $z$  means the number of tool teeth, and  $g_p$  defines when  $p$ -th tooth is active (cuts a material). The tangential and radial cutting force acting on the tool are proportional to the axial depth of cut  $a_p$  and chip width  $w_p$  according to the equations

$$F_{ip}(t) = K_t a_p w_p(t)^\kappa, \quad F_{np}(t) = K_n a_p w_p(t)^\kappa \tag{4}$$

$K_t$  and  $K_n$  are specific cutting forces which depend on the cutting material properties. Typical relationship between  $K_t$  and  $K_n$  for classical materials is  $K_n=0.36 K_t$ . The coefficient  $\kappa$  also depends on the material, and is usually estimated from 0.75 to 1. The chip width  $w_p(t)$  is a function of the feed  $f$ , the present tool and workpiece vibrations  $x(t)$  and vibrations of the previous tooth  $x(t-\tau)$ .

$$w_p(t) = \left[ \begin{aligned} & f + (x_1(t) - x_2(t)) \\ & - (x_1(t-\tau) - x_2(t-\tau)) - f_1 \cos(\lambda t) \end{aligned} \right] \sin\theta_p(t) \tag{6}$$

Where,  $\tau=60/zn$  is the tooth passing period,  $n$  means rotational speed of the tool. Additionally, in order to control vibrations during milling process, harmonic motion of the workpiece in feed direction represented by  $f_1 \cos(\lambda t)$  is added. Where,  $f_1$  and  $\lambda$  mean the amplitude and frequency of the external excitation signal. In practice, the external excitation can be implemented using e.g. piezoelectric actuators.

The step function  $g_p(t)$  is defined in order to check whether the tool is in cut or not:

$$g_p(t) = \begin{cases} 1, & \varphi_{entry} \leq \varphi_j \leq \varphi_{exit} \\ 0, & elsewhere \end{cases} \tag{7}$$

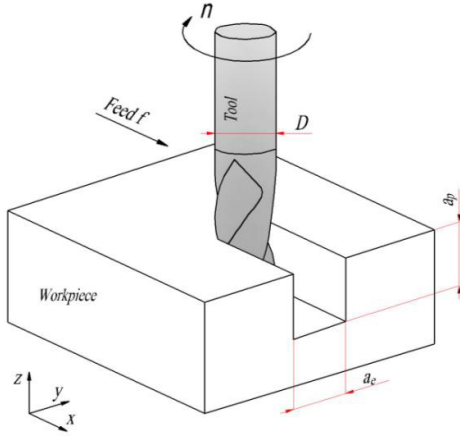


Fig. 2. View of milling process.

In milling process the entry  $\varphi_{entry}$  and exit  $\varphi_{exit}$  angles are defined as follows

$$\varphi_{entry} = \arcsin\left(\frac{D-2a_e}{D}\right), \quad \varphi_{exit} = \pi/2. \quad (8)$$

Where, D denotes the tool diameter. In analysing here case of full immersion milling (Figure 2), the entry and the exit angles take the value

$$\varphi_{entry} = 0, \quad \varphi_{exit} = \pi/2 \quad (9)$$

### 3. Numerical simulation

Based on differential equations of motion (2) the numerical simulations were performed in the environment Matlab-Simulink using the Runge-Kutta method of fourth order with variable step of integration.

Table 1. Parameters of non-linear 2DOF milling model.

Parameter	Value	Parameter	Value
$m_1$	0.1824 [kg]	$K_n$	160 [MPa]
$\omega_{n1}$	865.43 [rad/s]	$K_t$	450 [MPa]
$\zeta_1$	0.0406 [-]	$f$	0.01 [mm/tooth]
$\gamma_1$	2e12 [N/m <sup>3</sup> ]	$a_e$	10 [mm]
$m_2$	3.02 [kg]	$D$	10 [mm]
$\omega_{n2}$	318.93 [rad/s]	$z$	4 [-]
$\zeta_2$	0.0396 [-]	$\kappa$	0.75 [-]
$\gamma_2$	2e12 [N/m <sup>3</sup> ]		

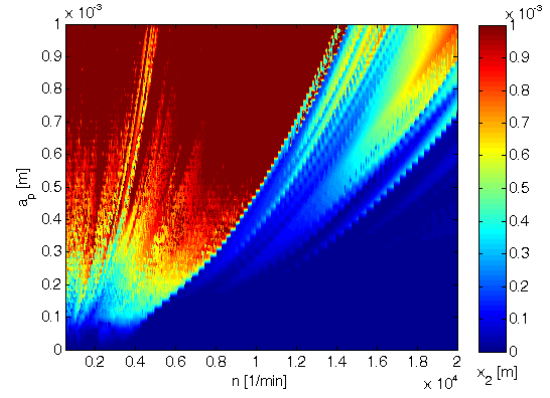
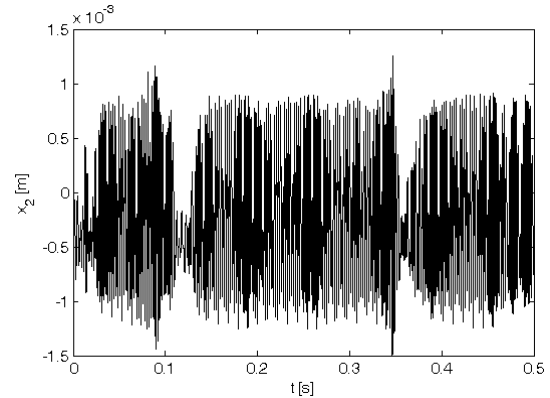


Fig. 3. Stability lobes diagram for 2DOF milling model.

The values of parameters used in the simulation are shown in Table 1. The parameters are taken from author's research. As a result of the numerical research a graph of stability region in milling process is obtained (Figure 3). The colors indicate the value of vibrations amplitude of the workpiece. The unstable lobes presented on stability lobes diagram (SLD) in Figure 3 are a bit different from classical ones known in literature. This is because our model has the cubic nonlinearity expressed by the coefficient  $\gamma_1$  and  $\gamma_2$ . Knowledge of the stability curves has practical importance because it is possible to avoid chatter vibrations if we choose the proper depth of cut  $a_p$  and the rotational speed  $n$ . This method to avoid chatter has a disadvantage because the proper values of  $a_p$  and  $n$  are chosen only once at the beginning of the cutting process. Thus, this is a passive method.

Fig. 4. Time history for  $a_p=0.6$  mm,  $n=7000$  1/min.

Another, active method to suppress chatter can be realized by a controller which can regulate external excitation. The concept of chatter control by means of external excitation is applied here for milling process. Milling is discontinuous operation by nature. A set of control parameters of workpiece excitation, that is the amplitude  $f_j$  and the frequency  $\lambda$  is tested here in order to find such of them which can decrease chatter vibrations amplitude. For further analysis the point

described by parameters:  $a_p=0.6\text{ mm}$ ,  $n=7000\text{ 1/min}$  is selected. This point exists in the area of unstable milling. The amplitude of vibration of the workpiece for these parameters is about one millimeter. In this case the time history of workpiece displacement is shown in Figure 4.

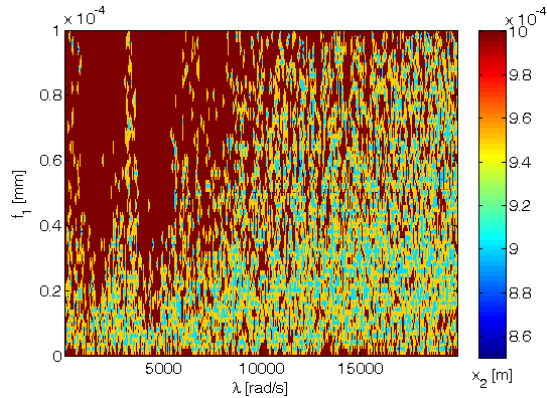


Fig. 5. Influence of an external excitation parameters on the amplitude of vibration for  $a_p=0.6\text{ mm}$ ,  $n=7000\text{ 1/min}$ .

In order to eliminate self-excited vibration in milling process we have used the external displacement  $f_1\cos(\lambda t)$  that introduces the additional workpiece vibrations in feed direction. Properly selected parameters ( $f_1$ ,  $\lambda$ ), of these vibrations acting on the workpiece can reduce the amplitude of the negative vibrations. The problem of selecting these parameters is still open question. Here the numerical study is proposed to evaluate the applicability of the proposed solution where an open and closed loop control is applied. Figure 5 shows the effect of an external displacement parameters ( $f_1$  and  $\lambda$ ) on the amplitude of vibration using the open-loop control for the depth of cut  $a_p=0.6\text{ mm}$  and the speed  $n=7000\text{ 1/min}$ . The colors indicate the value of vibrations amplitude of the workpiece. There are small areas, where a slight decrease of vibration is visible.

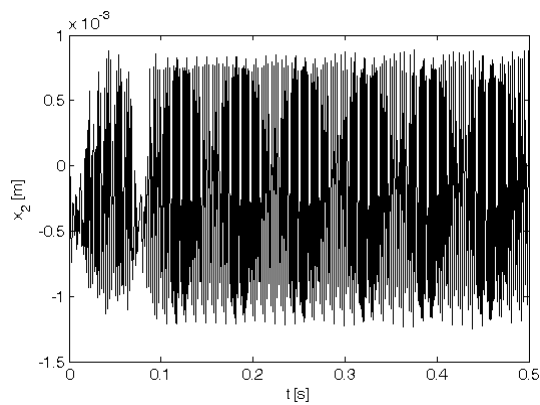


Fig. 6. Time history for  $a_p=0.6\text{ mm}$ ,  $n=7000\text{ 1/min}$  with external excitation  $f_1=0.008\text{ mm}$  and  $\lambda=3093\text{ rad/s}$ .

Adequate selection of the vibrations amplitude and frequency of the external force can only reduce the vibrations level but not eliminate them. For example the time history of workpiece vibrations for  $f_1=0.008\text{ mm}$  and  $\lambda=3093\text{ rad/s}$ . is shown in Figure 6. From a practical point of view to maintain required parameters of external force during machining can be impossible.

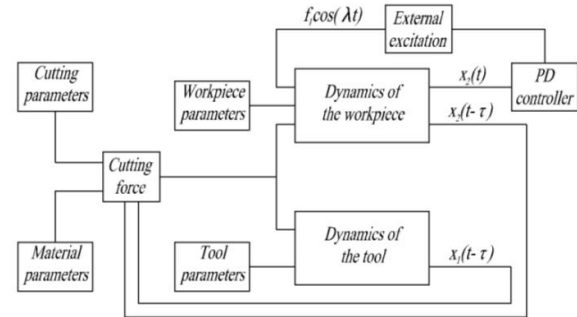


Fig. 7. Diagram of the control system.

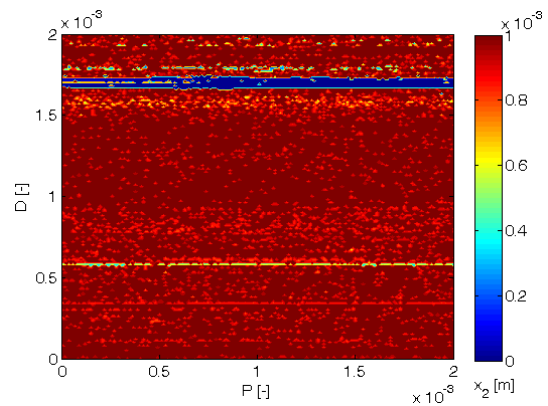


Fig. 8. Influence PD controller settings parameters on the amplitude of vibration for  $a_p=0.6\text{ mm}$ ,  $n=7000\text{ 1/min}$ .

Therefore, on-line chatter control should be better. Figure 7 shows the diagram of the control system. In order to increase the efficiency of chatter reduction system, the closed-loop control with PD (Proportional-Derivative) controller is proposed at this point. The influence of parameters  $P$  and  $D$  of the controller are shown in Figure 8. The colors indicate the value of vibrations amplitude of the workpiece for different  $P$  and  $D$  controller parameters. In this case, was observed the blue area where the level of workpiece vibration is reduced to zero.

Choosing the parameters  $P=0.0012$  and  $D=0.0017$  a significant reduction of workpiece vibration was observed as presented in time series of  $x_2$  in Figure 9. By using closed-loop control with a PD controller are achieved much better results than open-loop control.

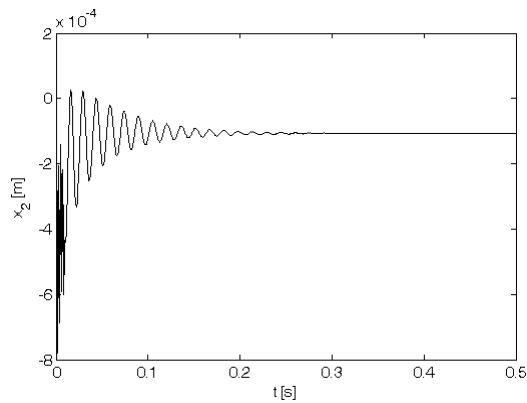


Fig. 9. Time history for  $a_p=0.6$  mm,  $n=7000$  1/min with controller  $P=0.0012$  and  $D=0.0017$ .

#### 4. Conclusions

The paper presents the application of an external excitation to control chatter in the milling process. Numerical studies of non-linear model of milling were carried out. It is proved that chatter vibrations can be reduced by additional workpiece motion. Simultaneously, the method of the external excitation control is the key aspect which should be taken into consideration. When open loop control is applied only small reduction of chatter vibrations is possible. Nevertheless, the selection of the excitation parameters is quite difficult. Moreover, in a real milling process which is variable and to some extent stochastic by nature, excitation parameters should be adjust in real time. Therefore closed-loop control with a PD controller should be much better. Unfortunately, application of closed-loop control requires an expensive real time controller. Nevertheless, in both cases, thanks to the external force the reduction in vibration level is observed. This solution can be applied industry where piezo-element could be introduced as a activator. Lublin University of Technology has a laboratory equipped with such system but the problem of the proper control algorithm has not been solved yet. Nevertheless, the successful numerical results let us believe that application of the proper controller may eliminate chatter vibrations.

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