



The LMA solution from bimaximal lepton mixing at the GUT scale by renormalization group running

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Abstract

We show that in see-saw models with bimaximal lepton mixing at the GUT scale and with zero CP phases, the solar mixing angle θ_{12} generically evolves towards sizably smaller values due to renormalization group effects, whereas the evolution of θ_{13} and θ_{23} is comparatively small. The currently favored LMA solution of the solar neutrino problem can thus be obtained in a natural way from bimaximal mixing at the GUT scale. We present numerical examples for the evolution of the leptonic mixing angles in the Standard Model and the MSSM, in which the current best-fit values of the LMA mixing angles are produced. These include a case where the mass eigenstates corresponding to the solar mass squared difference have opposite CP parity.

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1. Introduction

Recent experimental evidence strongly favors the LMA solution of the solar neutrino problem with a large but non-maximal value of the solar mixing angle θ_{12} [1–4]. An overview of the current allowed regions for the mixing angles and the mass squared differences is given in Table 1.

A big problem for model builders is to explain the deviation of θ_{12} from maximal mixing, while keeping θ_{23} maximal and θ_{13} small at the same time. The renormalization group (RG) evolution is a possible candidate for accomplishing this. Therefore, it is interesting to investigate the evolution of the mixing angles from the GUT scale to the electroweak (EW) or SUSY-breaking scale. A number of studies with three neutrinos considered the possibility of increasing a small mixing angle via RG evolution [7–10]. Others focused on the case of nearly degenerate neutrinos [11–16], on the existence of fixed points [17], or on the effect of non-zero Majorana phases on the stability of the RG evolution [18].

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Table 1

Experimental data for the neutrino mixing angles and mass squared differences. For the solar angle θ_{12} and the solar mass squared difference, the LMA solution has been assumed. The results stem from the analysis of the recent SNO data [3], the Super-Kamiokande atmospheric data [5] and the CHOOZ experiment [6]

	Best-fit value	Range (for $\theta \in [0^\circ, 45^\circ]$)	C.L.
θ_{12} [°]	32.9	26.1–43.3	99% (3σ)
θ_{23} [°]	45.0	33.2–45.0	99% (3σ)
θ_{13} [°]	–	0.0–9.2	90% (2σ)
Δm_{sol}^2 [eV ²]	5×10^{-5}	2.3×10^{-5} – 3.7×10^{-4}	99% (3σ)
$ \Delta m_{\text{atm}}^2 $ [eV ²]	2.5×10^{-3}	1.2×10^{-3} – 5×10^{-3}	99% (3σ)

We consider the see-saw scenario, i.e., the Standard Model (SM) or MSSM extended by 3 heavy neutrinos that are singlets under the SM gauge groups and have large explicit (Majorana) masses with a non-degenerate spectrum. Due to this non-degeneracy, one has to use several effective theories, with the singlets partly integrated out, when studying the evolution of the effective mass matrix of the light neutrinos [19,20]. Below the lowest mass threshold, the neutrino mass matrix is given by the effective dimension 5 neutrino mass operator in the SM or MSSM. The relevant RGE's were derived in [20–25].

In this Letter, we assume bimaximal mixing at the GUT scale with vanishing CP phases and positive mass eigenvalues. We calculate the RG running numerically in order to obtain the mixing angles at low energy and to compare them with the experimentally favored values. We include the regions above and between the see-saw scales in our study, which have not been considered in most of the previous works. We find that the solar mixing angle changes considerably, while the evolution of the other angles is comparatively small, so that values compatible with the LMA solution can be obtained. We present analytic approximations that help to understand this behavior and show that it is rather generic.

2. Bimaximal mixing at the GUT scale

At the GUT scale, we assume bimaximal mixing in the lepton sector. We restrict ourselves to the case of positive mass eigenvalues and real parameters, so that there is no CP violation. In the basis where the charged lepton Yukawa matrix is diagonal, up to phase conventions the general parametrization of the effective Majorana mass matrix of the light neutrinos is then

$$m_{\nu}^{\text{bimax}} = V \left(\frac{\pi}{4}, 0, \frac{\pi}{4} \right) \cdot \text{diag}(m_1, m_2, m_3) \cdot V^T \left(\frac{\pi}{4}, 0, \frac{\pi}{4} \right) = \begin{pmatrix} a-b & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix}, \quad (1)$$

where

$$V(\theta_{12}, \theta_{13}, \theta_{23}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}c_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix} \quad (2)$$

with $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ is the (orthogonal) CKM matrix in standard parametrization, and

$$a = \frac{1}{4} (m_1 + m_2 + 2m_3), \quad (3a)$$

$$b = \frac{1}{4} (-m_1 - m_2 + 2m_3), \quad (3b)$$

$$c = \frac{m_2 - m_1}{2\sqrt{2}}. \quad (3c)$$

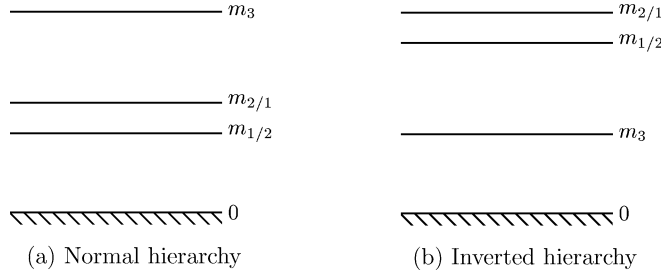


Fig. 1. Possible mass hierarchies for the light neutrinos. We use the convention that m_1 and m_2 are chosen in such a way that $0 \leq \theta_{12} \leq 45^\circ$. The LMA solution then requires $m_2 > m_1$.

Inverting Eqs. (3) yields the mass eigenvalues

$$m_1 = a - b - \sqrt{2}c, \quad (4a)$$

$$m_2 = a - b + \sqrt{2}c, \quad (4b)$$

$$m_3 = a + b. \quad (4c)$$

From Eq. (3a) we see that $a > 0$. Eqs. (4) imply that the solar mass squared difference $\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2$ is related to c , while the atmospheric one, $\Delta m_{\text{atm}}^2 = m_3^2 - m_1^2$, is controlled by b . Thus, $a > |b| > |c|$. For $b > 0$ we obtain a normal mass hierarchy, while for $b < 0$ the mass hierarchy is inverted, as illustrated in Fig. 1. For positive c , $m_1 < m_2$, otherwise $m_1 > m_2$. Hence, Δm_{sol}^2 is positive only if c is. If $a \gg |b|, |c|$, the spectrum is called degenerate. We use the convention that the mass label 2 is attached in such a way that $0 \leq \theta_{12} \leq 45^\circ$. This can always be accomplished by the replacement $c \leftrightarrow -c$.

In our see-saw scenario, the effective mass matrix of the light neutrinos is

$$m_{\nu}^{\text{bimax}} = \frac{v_{\text{EW}}^2}{2} Y_{\nu}^T M^{-1} Y_{\nu} \quad (5)$$

at the high-energy scale, with $\langle \phi \rangle = \frac{v_{\text{EW}}}{\sqrt{2}} \approx 174$ GeV. Obviously, the singlet Yukawa and mass matrices Y_{ν} and M cannot be determined uniquely from this relation, i.e., there is a set of $\{Y_{\nu}, M\}$ configurations that yield bimaximal mixing. After choosing an initial condition for Y_{ν} , M (and thus the see-saw scales) is fixed by the see-saw formula (5).

3. Solving the RGE's

To study the RG running of the leptonic mixing angles and neutrino masses, all parameters of the theory have to be evolved from the GUT scale to the EW or SUSY-breaking scale, respectively. Since the heavy singlets have to be integrated out at their mass thresholds, which are non-degenerate in general, a series of effective theories has to be used. The derivation of the RGE's and the method for dealing with these effective theories are given in [20]. Starting at the GUT scale, the strategy is to successively solve the systems of coupled differential equations of the form

$$\mu \frac{d}{d\mu} X_i^{(n)} = \beta_{X_i}^{(n)} \left(\{X_j^{(n)}\} \right) \quad (6)$$

for all the parameters $X_i^{(n)}, X_j^{(n)} \in \{\kappa^{(n)}, Y_{\nu}^{(n)}, M^{(n)}, \dots\}$ of the theory in the energy ranges corresponding to the effective theories denoted by (n) . At each see-saw scale, tree-level matching is performed. Due to the complicated structure

of the set of differential equations, the exact solution can only be obtained numerically. However, to understand certain features of the RG evolution, an analytic approximation at the GUT scale will be derived in Section 5.

4. Examples for the running of the mixing angles

Figs. 2 and 3 show typical numerical examples for the running of the mixing angles from the GUT scale to the EW or SUSY-breaking scale. They contain an important effect that appears for most choices of the initial parameters: The solar angle θ_{12} changes drastically, while the changes in θ_{13} and θ_{23} are comparatively small. This agrees remarkably well with the experimentally favored scenario.

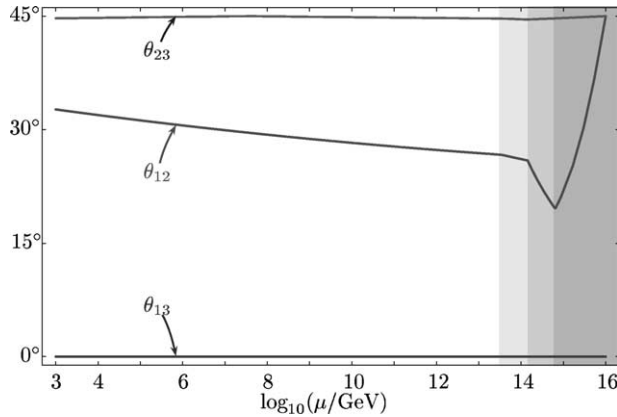


Fig. 2. RG evolution of the mixing angles from the GUT scale to the SUSY-breaking scale (taken to be ≈ 1 TeV) in the MSSM extended by heavy singlets for a normal mass hierarchy and $Y_\nu = X \text{diag}(1, \varepsilon, \varepsilon^2)$ with $\tan \beta = 5$, $\varepsilon = 0.525$, $a = 0.0675$ eV and $X = 1$. In this example, the lightest neutrino has a mass of 0.025 eV. The kinks in the plots correspond to the mass thresholds at the see-saw scales. The grey-shaded regions mark the various effective theories.

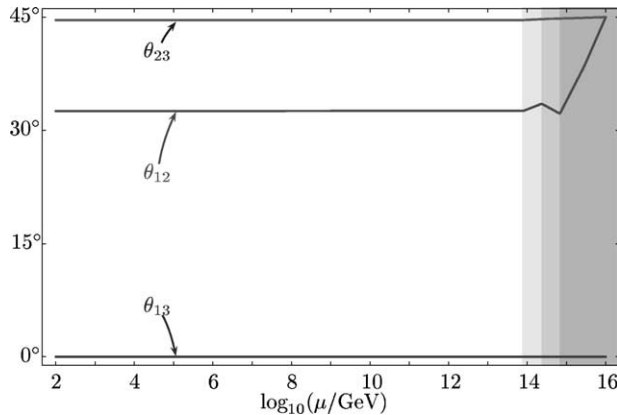


Fig. 3. Example for the RG evolution of the mixing angles in the SM extended by heavy singlets from the GUT scale to the EW scale for a normal mass hierarchy and $Y_\nu = X \text{diag}(1, \varepsilon, \varepsilon^2)$ with $\varepsilon = 0.65$, $a = 0.0655$ eV and $X = 1$. In this example, the lightest neutrino has a mass of 0.024 eV.

5. Analytic approximation for the running of the mixing angles at the GUT scale

In order to understand the effect found numerically in the previous section, we now derive an analytic approximation for the RG evolution of the mixing angles at the GUT scale. It is only affected by the part of the RGE that is not proportional to the unit matrix, which is given by

$$16\pi^2 \mu \frac{d}{d\mu} m_\nu = C_e [Y_e^\dagger Y_e]^T m_\nu + C_e m_\nu [Y_e^\dagger Y_e] + C_\nu [Y_\nu^\dagger Y_\nu]^T m_\nu + C_\nu m_\nu [Y_\nu^\dagger Y_\nu] \\ + \text{terms with trivial flavour structure} \quad (7)$$

with $C_e = -3/2$, $C_\nu = 1/2$ in the SM and $C_e = C_\nu = 1$ in the MSSM. Analogously to Eq. (1), m_ν is parametrized by

$$m_\nu(t) = V(\theta_{12}(t), \theta_{13}(t), \theta_{23}(t)) m_{\text{diag}}(t) V^T(\theta_{12}(t), \theta_{13}(t), \theta_{23}(t)), \quad (8)$$

where μ is the renormalization scale, $t := \ln \frac{\mu}{\mu_0}$, and $m_{\text{diag}} := \text{diag}(m_1, m_2, m_3)$. In general, the real Y_ν can be written as

$$Y_\nu = V(\xi_{12}, \xi_{13}, \xi_{32}) \text{diag}(y_1, y_2, y_3) V^T(\phi_{12}, \phi_{13}, \phi_{32}). \quad (9)$$

However, the effective mixing matrix m_ν is invariant under the transformations $Y_\nu \rightarrow V^T Y_\nu$ and $M \rightarrow V^T M V$, which correspond to a change of basis for the heavy sterile neutrinos. Thus, $V(\xi_{12}, \xi_{13}, \xi_{32})$ in Eq. (9) can be absorbed into M , leading to the simpler parametrization

$$Y_\nu(y_1, y_2, y_3, \phi_{12}, \phi_{13}, \phi_{32}) = \text{diag}(y_1, y_2, y_3) V^T(\phi_{12}, \phi_{13}, \phi_{32}). \quad (10)$$

Furthermore, we use the approximation that the effect of the charged lepton Yukawa matrices Y_e can be neglected compared to that of the neutrino Yukawa matrix. Note that in the MSSM a large $\tan\beta$ can yield a relatively large Y_e , which can also have sizable effects that are neglected in this approximation.

We now differentiate Eq. (8) w.r.t. t and insert the RGE (7). For the evolution of the mixing angles at the GUT scale with bimaximal mixing as initial condition, we thus obtain both in the SM and in the MSSM the ratios

$$\left. \frac{\dot{\theta}_{12}}{\dot{\theta}_{13}} \right|_{M_{\text{GUT}}} = \frac{2\sqrt{2}(m_1 + m_2)(m_3 - m_1)(m_3 - m_2)F_1}{(m_2 - m_1)[8(m_3^2 - m_1 m_2)F_2 + 4\sqrt{2}(m_2 - m_1)m_3 F_3]} \\ \approx \begin{cases} \pm \frac{1}{2\sqrt{2}} \frac{m_2 + m_1}{m_2 - m_1} \frac{F_1}{F_2} & \text{for hierarchical neutrino masses}^1, \\ \frac{1}{2\sqrt{2}} \frac{\Delta m_{\text{atm}}^2}{\Delta m_{\text{sol}}^2} \frac{F_1}{F_2} & \text{for degenerate neutrino masses,} \end{cases} \quad (11a)$$

$$\left. \frac{\dot{\theta}_{12}}{\dot{\theta}_{23}} \right|_{M_{\text{GUT}}} = \frac{2\sqrt{2}(m_1 + m_2)(m_3 - m_1)(m_3 - m_2)F_1}{(m_2 - m_1)[8(m_2 - m_1)m_3 F_2 + 4\sqrt{2}(m_3^2 - m_1 m_2)F_3]} \\ \approx \begin{cases} \pm \frac{1}{2} \frac{m_2 + m_1}{m_2 - m_1} \frac{F_1}{F_3} & \text{for hierarchical neutrino masses}^1, \\ \frac{1}{2} \frac{\Delta m_{\text{atm}}^2}{\Delta m_{\text{sol}}^2} \frac{F_1}{F_3} & \text{for degenerate neutrino masses} \end{cases} \quad (11b)$$

with

$$F_1 = (y_1^2 - y_2^2) \{ \cos(2\phi_{12}) [(\cos(2\phi_{13}) - 3) \sin(2\phi_{23}) - 6 \cos^2(\phi_{13})] - 4 \cos(2\phi_{23}) \sin(2\phi_{12}) \sin(\phi_{13}) \}$$

¹ Note that this approximation is also valid for a relatively weak hierarchy, where m_3 is a few times larger or smaller than m_1, m_2 .

$$+ (y_1^2 + y_2^2 - 2y_3^2) [\cos(2\phi_{13})(\sin(2\phi_{23}) - 3) + (1 + \sin(2\phi_{23}))], \quad (12a)$$

$$F_2 = 2(y_1^2 - y_2^2) \cos(\phi_{13}) \sin(2\phi_{12}) (\sin(\phi_{23}) - \cos(\phi_{23})) - (y_1^2 + y_2^2 - 2y_3^2 + (y_1^2 - y_2^2) \cos(2\phi_{12})) \\ \times \sin(2\phi_{13}) (\cos(\phi_{23}) + \sin(\phi_{23})), \quad (12b)$$

$$F_3 = (y_1^2 - y_2^2) [\cos(2\phi_{12}) (\cos(2\phi_{13}) - 3) \cos(2\phi_{23}) + 4 \sin(2\phi_{12}) \sin(\phi_{13}) \sin(2\phi_{23})] \\ + 2(y_1^2 + y_2^2 - 2y_3^2) \cos^2(\phi_{13}) \cos(2\phi_{23}). \quad (12c)$$

This result can also be obtained from the formulae derived in [26]. The constants F_1 , F_2 and F_3 clearly depend on the choice of $Y_\nu(M_{\text{GUT}})$. However, unless the parameters $\{y_1, y_2, y_3, \phi_{12}, \phi_{13}, \phi_{32}\}$ are fine-tuned, we expect the ratios F_1/F_2 and F_1/F_3 to be of the order one. Consequently, the RG change of θ_{12} is larger than that of the other angles if the mass-dependent factors in Eqs. (11a) and (11b) are large. This is always the case for degenerate neutrino masses, since $\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{sol}}^2$. As $(m_1 - m_2)$ is related to the small solar mass squared difference, it is also true for non-degenerate mass schemes, unless m_1 is very small, in which case the ratio approaches 1. This corresponds to a normal mass hierarchy and a strongly hierarchical mass scheme. Finally, it can be shown that the running of θ_{12} is always enhanced compared to that of θ_{13} and θ_{23} for inverted schemes. Hence, we conclude that this is a generic effect.

6. Parameter space regions compatible with the LMA solution

6.1. Parameters at the GUT scale

The considerable change of the solar mixing angle found in the previous sections raises the question whether the parameter region of the LMA solution might be reached by RG evolution, if one starts with bimaximal mixing at high energy. We will investigate this possibility by further numerical calculations in the following. To reduce the parameter space for the numerical analysis, we choose a specific neutrino Yukawa coupling Y_ν at the GUT scale. We assume that it is diagonal and of the form

$$Y_\nu = X \text{diag}(1, \varepsilon, \varepsilon^2). \quad (13)$$

Y_ν and M are now determined by the parameters $\{\varepsilon, X, a, b, c\}$. Moreover, we fix the GUT scale values of b and c by the requirement that the solar and atmospheric mass squared differences obtained at the EW scale after the RG evolution be compatible with the allowed experimental regions. Thus, we are left with the free parameters X , ε and a . The parameter ε controls the hierarchy of the entries in Y_ν and thus the degeneracy of the see-saw scales, while a determines the mass of the lightest neutrino. The dependence of physical quantities on ε and a is shown in Fig. 4. The effect of changing the scale X of the neutrino Yukawa coupling will be discussed in Section 6.3. As mentioned above, we work in the basis where the Yukawa matrix of the charged leptons is diagonal.

6.2. Allowed parameter space regions

The parameter space regions in which the RG evolution produces low-energy values compatible with the LMA solution are shown in Fig. 5 for the SM and the MSSM ($\tan \beta = 5$) with a normal mass hierarchy. We find that for the form of Y_ν under consideration, hierarchical and degenerate neutrino mass schemes as well as degenerate and non-degenerate see-saw scales are possible. For inverted neutrino mass spectra, allowed parameter space regions exist as well.

We would like to stress that the shape of the allowed parameter space regions strongly depends on the choice of the initial value of Y_ν at the GUT scale. One also has to ensure that the sign of Δm_{sol}^2 is positive, as the LMA solution requires this if the convention is used that the solar mixing angle is smaller than 45° . With bimaximal mixing at the GUT scale, the sign of Δm_{sol}^2 is not defined by the initial conditions. Using the analytic approximation

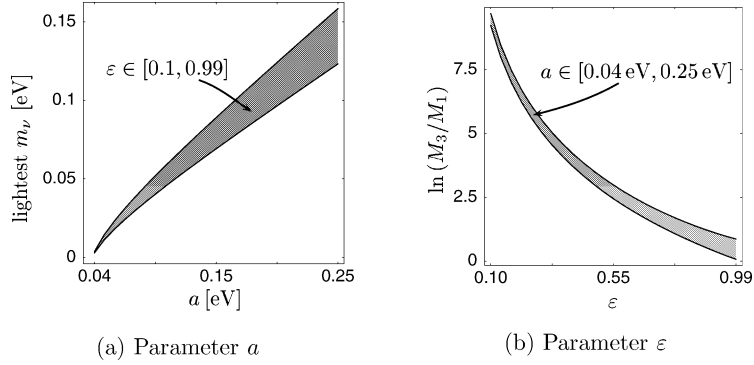


Fig. 4. Plot (a) shows the mass of the lightest neutrino (at low energy) as a function of a for the SM and the MSSM with normal mass hierarchy, $X = 1$ and $\varepsilon \in [0.1, 0.99]$ (grey region). Plot (b) shows the degeneracy of the see-saw scales, parametrized by $\ln(M_3/M_1)$ (at the GUT scale), as a function of ε for the same cases with $a \in [0.04 \text{ eV}, 0.25 \text{ eV}]$ (grey region).

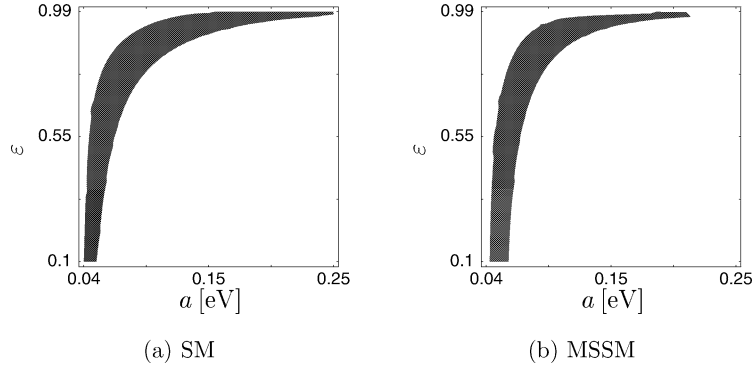


Fig. 5. Parameter space regions compatible with the LMA solution of the solar neutrino problem for the example $Y_\nu = \text{diag}(1, \varepsilon, \varepsilon^2)$. The initial condition at the GUT scale $M_{\text{GUT}} = 10^{16} \text{ GeV}$ is bimaximal mixing, and the comparison with the experimental data is performed at the EW scale or at 1 TeV for the SM and the MSSM, respectively. The white regions of the plots are excluded by the data (LMA) at 3σ . For this example, we consider the case of a normal neutrino mass hierarchy and $X = 1$ for the scale factor of the neutrino Yukawa couplings.

of Section 5, the sign just below the GUT scale can be calculated. We find $\Delta m_{\text{sol}}^2 > 0$ for $F_1 < 0$ and vice versa. However, in order to predict the sign of Δm_{sol}^2 at low energy, the numerical RG evolution has to be used. This excludes some of the possible choices for the neutrino Yukawa coupling Y_ν at the GUT scale. For example, among the possibilities with diagonal Y_ν it excludes $Y_\nu = \text{diag}(\varepsilon^2, \varepsilon, 1)$.

6.3. Dependence on the scale X of the neutrino Yukawa coupling

For small values of X , the contribution from Y_ν to the evolution of the mixing angles above the largest see-saw scale is suppressed by a factor of X^2 . Nevertheless, the evolution to the LMA solution is still possible, as can be seen from the example in Fig. 6. Here the large change of θ_{12} also seems to be generic but takes place between the see-saw scales, which shows the importance of carefully studying the RG behavior in these intermediate regions [20]. Note that in this case the analytic approximation of Section 5 cannot be applied, since it is only valid at the GUT scale.

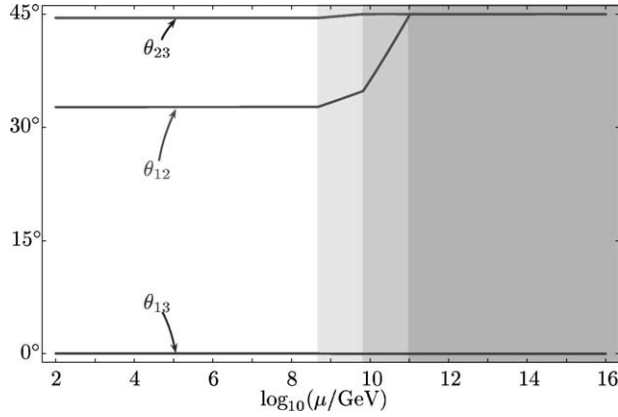


Fig. 6. RG evolution in the SM for $X = 0.01$, $\varepsilon = 0.3$, $a = 0.0535$ eV and a normal mass hierarchy. The running from bimaximal mixing to the LMA solution now takes place between the see-saw scales. In this example, the lightest neutrino has a mass of 0.017 eV.

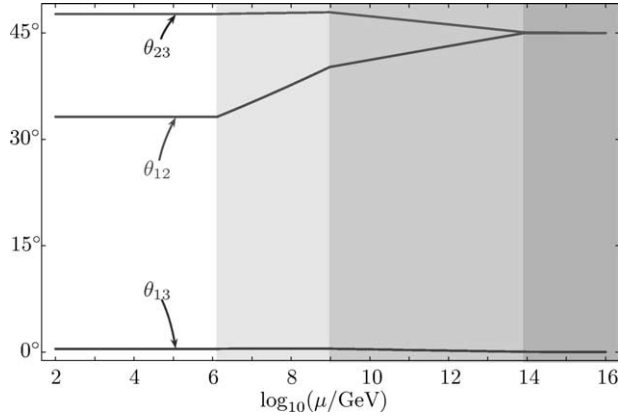


Fig. 7. RG evolution in the SM with a negative CP parity for m_2 , $X = 0.5$, $\varepsilon = 3.5 \times 10^{-3}$ and a normal mass hierarchy. The running from bimaximal mixing to the LMA solution takes place between the see-saw scales. In this example, we consider a strongly hierarchical mass spectrum. The lightest neutrino has a mass of 0.004 eV. Note that the cases $\theta_{23} > 45^\circ$, $\Delta m_{23}^2 > 0$ and $\tilde{\theta}_{23} := 90^\circ - \theta_{23} < 45^\circ$, $\Delta \tilde{m}_{23}^2 := -\Delta m_{23}^2$ are indistinguishable in neutrino oscillations.

6.4. Effect of neutrino CP parities

An example for the running of the mixing angles to the LMA solution with a negative CP parity for the state with mass m_2 is shown in Fig. 7. For this we have chosen a different diagonal structure for Y_ν ,

$$Y_\nu = X \text{diag}(\varepsilon^2, \varepsilon, 1), \quad (14)$$

at the GUT scale. Here, the evolution to the LMA solution is possible due to running between the see-saw scales. A more detailed study of the effect of CP phases will be given in a forthcoming paper [27].

The large RG effects in this case seem surprising at first sight, since previous studies, e.g. [18,26], found that opposite CP parities for m_1 and m_2 prevent a sizable change of the solar mixing angle by RG evolution. However, these works did not consider the energy region between the see-saw scales, where the largest change occurs in our example. This fact explains the apparent discrepancy.

6.5. Low scale values of θ_{13} and θ_{23}

The mixing angles θ_{13} and θ_{23} are affected by the RG evolution as well, i.e., they do not stay at their initial values $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$. However, lower bounds on their changes cannot be given unless a specific model is chosen. As one can see from the previous examples, the changes can be tiny. For instance, the evolution of Fig. 2 gives $\Delta\theta_{13} = 0.02^\circ$, which corresponds to $\sin^2(2\theta_{13}) = 5 \times 10^{-7}$, and $\Delta\theta_{23} = 0.28^\circ$. On the other hand, other choices of Y_ν at the GUT scale produce $\Delta\theta_{13}$ and $\Delta\theta_{23}$ that come close to the experimental bounds. This can make it possible to discriminate between models with different initial values $Y_\nu(M_{\text{GUT}})$.

7. Summary and conclusions

We have shown that in see-saw scenarios the experimentally favored neutrino mass parameters with the LMA solution of the solar neutrino problem can be obtained in a rather generic way from bimaximal mixing at the GUT scale by renormalization group running. We have concentrated on the case of vanishing CP phases, which implies positive mass eigenvalues. In an example where the mass eigenstates corresponding to the solar mass squared difference have opposite CP parity, we have demonstrated that an evolution towards the LMA solution is possible in this case as well. The general case of arbitrary CP phases is beyond the scope of this Letter and will be studied elsewhere [27]. The mixing angles evolved down to the electroweak scale show a strong dependence on the mass scale of the lightest neutrino, on the degeneracy of the see-saw scales, and on the form of the neutrino Yukawa coupling. A generic feature of the renormalization group evolution is that the solar mixing angle θ_{12} evolves towards sizably smaller values, whereas the change of θ_{13} and θ_{23} is comparatively small. In the SM and MSSM, we find extensive regions in parameter space which are compatible with the LMA solution for normal and inverted neutrino mass hierarchies and for large and small absolute scales of the neutrino Yukawa couplings. Thus, RG running may provide a natural explanation for the observed deviation of the LMA mixing angles from bimaximality.

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