#### Physics Letters B 714 (2012) 55-61

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



# On the universality of CP violation in $\Delta F = 1$ processes

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#### ARTICLE INFO

Article history: Received 12 March 2012 Received in revised form 20 May 2012 Accepted 21 June 2012 Available online 25 June 2012 Editor: B. Grinstein

#### ABSTRACT

We show that new physics which breaks the left-handed  $SU(3)_Q$  quark flavor symmetry induces contributions to CP violation in  $\Delta F = 1$  couplings which are approximately *universal*, in that they are not affected by flavor rotations between the up and the down mass bases. (Only the short distance contributions are universal, while observables are also affected by hadronic matrix elements.) Therefore, such flavor violation cannot be aligned, and is constrained by the strongest bound from either the up or the down sectors. We use this result to show that the bound from  $\epsilon'/\epsilon$  prohibits an  $SU(3)_Q$  breaking explanation of the recent LHCb evidence for CP violation in *D* meson decays. Another consequence of this universality is that supersymmetric alignment models with a moderate mediation scale are consistent with the data, and are harder to probe via CP violating observables. With current constraints, therefore, squarks need not be degenerate. However, future improvements in the measurement of CP violation in  $D-\overline{D}$  mixing will start to probe alignment models.

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#### 1. Introduction

Measurements of flavor-changing neutral-current (FCNC) processes in the quark sector put strong constraints on New Physics (NP) at the TeV scale and provide a crucial guide for model building. Generically, NP models can avoid existing bounds by aligning the flavor structure with one of the quark Yukawa matrices. However, new flavor breaking sources involving only the  $SU(2)_L$  doublet quarks  $Q_i$  (i.e., breaking only the  $SU(3)_Q$  quark flavor symmetry) cannot be simultaneously diagonalized in both the up and the down quark mass bases, and new contributions to FCNCs are necessarily generated. To constrain such models of flavor alignment, processes involving both up and down type quark transitions need to be measured. Consequently, one would naïvely conclude that robust constraints on the corresponding microscopic flavor structures come from the *weaker* of the bounds in the up and the down sectors.

Below we argue, however, that in a large class of models, contrary to flavor violation in  $\Delta F = 2$  processes [1], in the case of  $\Delta F = 1$  CP violation, it is the *strongest* of the up and down sec-

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0370-2693 © 2012 Elsevier B.V. Open access under CC BY license. http://dx.doi.org/10.1016/j.physletb.2012.06.050 tor constraints which applies. We show that in these scenarios, to a good approximation, the sources of  $\Delta F = 1$  CP violation are universal, namely they do not transform under flavor rotations between the up and the down mass bases. This is particularly important for the NP interpretation of the recent LHCb evidence for CP violation in *D* decays. Employing the  $\epsilon'/\epsilon$  constraint on new CP violating  $\Delta s = 1$  operators, we exclude sizable contributions of  $SU(3)_Q$  breaking NP operators to the direct CP asymmetries in singly-Cabibbo-suppressed *D* decays, in particular to  $\Delta a_{CP}$  measured by the LHCb experiment [2].

Furthermore, applying our argument to rare semileptonic *K* and *B* decays, we show how the present and future measurements of these processes constrain the sources of CP violation in rare semileptonic *D* decays and FCNC top decays. In particular, the observation of non-SM CP asymmetries in these processes would, barring cancellations, signal the presence of new sources of  $SU(3)_{U,D}$  flavor symmetry breaking.

Finally, an additional implication of our result is that in viable flavor alignment models the universal flavor and CP violating phases are naturally small. Applying this insight to supersymmetric (SUSY) alignment models leads to the conclusion that the first two generation squarks can have mass splittings as large as 30% at the TeV scale, consistent with mass anarchy at a supersymmetry breaking mediation scale as low as 10 TeV.

#### 2. Universal contributions to CP violation with two generations

It is well known that the gauge sector of the Standard Model (SM) respects a large global flavor symmetry. In the quark sector, the corresponding flavor group,  $\mathcal{G}_F = SU(3)_Q \times SU(3)_U \times SU(3)_D$ , is broken by the up and the down Yukawa matrices  $Y_{u,d}$ , formally transforming as  $(3, \overline{3}, 1)$  and  $(3, 1, \overline{3})$  under  $\mathcal{G}_F$ , respectively. From these, one can construct two independent sources of  $SU(3)_Q$  breaking,

$$\mathcal{A}_{u} \equiv \left(Y_{u}Y_{u}^{\dagger}\right)_{tt}, \qquad \mathcal{A}_{d} \equiv \left(Y_{d}Y_{d}^{\dagger}\right)_{tt}, \tag{1}$$

which both transform as (8, 1, 1) under  $\mathcal{G}_F$ , where t denotes the traceless part. Our argument applies to all NP flavor structures, X, which can be written in the form

$$\mathcal{O}_L = \left[ (X_L)^{ij} \bar{Q}_i \gamma^{\mu} Q_j \right] L_{\mu}. \tag{2}$$

Here  $Q_i$  stands for the left-handed quark doublets, *i* and *j* are generation indices and  $X_L$  is a traceless Hermitian flavor matrix. The  $L_{\mu}$  denotes a flavor-singlet current, such as

$$L_{\mu} = \sum_{q} \bar{q} \gamma_{\mu} q, \quad \sum_{\ell} \bar{\ell} \gamma_{\mu} \ell, \quad H^{\dagger} D_{\mu} H, \quad \dots,$$
(3)

that is, a sum over quarks or leptons, a Higgs current, etc. Note that the way that color and spinor indices are contracted in Eq. (2) is irrelevant for our argument below.

It is easy to show that in the two generation limit, the unique Jarlskog invariant relevant for  $\Delta F = 1$  CP violation sourced by  $\mathcal{O}_L$  ( $X_L$ ) is proportional to  $X_L^{CPV} \equiv \text{Tr}(X_L \cdot J)$  [3], where

$$J \equiv i[\mathcal{A}_u, \mathcal{A}_d]. \tag{4}$$

The expression for  $X_L^{CPV}$  is manifestly reparametrization invariant and thus basis independent. A nontrivial feature of such  $SU(2)_Q$ breaking is that  $A_u$ ,  $A_d$ , J form a complete basis in the threedimensional space of traceless Hermitian  $2 \times 2$  matrices, and that J is orthogonal to the other two directions, i.e.,  $\text{Tr}(A_{u,d} \cdot J) = 0$ . It follows that the imaginary (CP violating) part of  $X_L$ , proportional to  $X_L^{CPV}$ , is also orthogonal to the plane of  $A_u$ ,  $A_d$ . It is thus invariant under flavor rotations in this plane and in particular under the two-dimensional real CKM rotation between the up and the down quark mass bases. Consequently, the amount of CP violation generated by  $X_L$  in both up and down quark transitions is the same, irrespective of the direction of the projection of  $X_L$  in the  $(A_u, A_d)$ plane.

Explicitly, the up and down quark components of  $\mathcal{O}_L$  in their relevant mass bases are

$$\left[ \left( X_L^u \right)_{ij} \bar{u}_L^i \gamma^\mu u_L^j \right] L_\mu, \qquad \left[ \left( X_L^d \right)_{ij} \bar{d}_L^i \gamma^\mu d_L^j \right] L_\mu, \tag{5}$$

where  $X_L^{u,d}$  are  $X_L$  rotated to the up and down mass bases, respectively. The universality of CP violation in  $\Delta F = 1$  transitions can now be expressed explicitly as

$$\operatorname{Im}(X_L^u)_{12} = \operatorname{Im}(X_L^d)_{12} \propto \operatorname{Tr}(X_L \cdot J).$$
(6)

Another simple way to understand this universality of CP violation is to notice that in the up or down mass basis, *J* is proportional to the Pauli matrix  $\sigma_2$ , which is invariant under *SO*(2) rotations. A consequence of Eq. (6) is that CP violation in both the up and the down sectors vanishes if  $X_L$  is in the plane of  $\mathcal{A}_u$  and  $\mathcal{A}_d$  (and in particular if  $X_L$  is aligned with  $\mathcal{A}_u$  or  $\mathcal{A}_d$ ), as can also be seen from the vanishing of the Jarlskog invariant. The difference compared to  $\Delta F = 2$  flavor violation follows from the fact that in the latter case CP violation is proportional to  $\text{Im}[(X_L^{u,d})^2] = 2 \text{Im} X_L^{u,d} \text{Re} X_L^{u,d}$  [1,3], which depends also on the non-universal real part. In addition, many CP violating observables also depend on hadronic matrix elements, which modify the universal short distance contributions we focus on, but do not introduce dependence on any new invariants.

The two-generation limit can only be considered as approximate for the strange and charm sectors. Furthermore, it is not immediately clear how it can be relevant for  $\Delta F = 1$  transitions involving the third generation quarks. We address these two issues in turn. In both cases we use the fact that the SM possesses an approximate  $SU(2)_Q$  flavor symmetry, which is broken only by  $(m_{c,s}^2 - m_{u,d}^2)/m_{t,b}^2$  and the  $\theta_{13}$  and  $\theta_{23}$  CKM mixing angles.

# 3. Universal contributions to CP violation with three generations

# 3.1. CP violation involving the first two generations within the three flavor framework

To describe  $\Delta c$ ,  $\Delta s = 1$  processes in the context of three generations, we can decompose  $X_L$  according to the  $SU(2)_Q$  symmetry. Taking advantage of the SM  $SU(2)^3$  symmetry obtained when the first two generation masses are neglected [4], one can choose a basis which isolates the large eigenvalues in the up and down Yukawa matrices. These become block diagonal in the limit where the small CKM mixing angles  $\theta_{13}$  and  $\theta_{23}$  are neglected. In this basis,  $(X_L)_{33}$  does not transform under the  $SU(2)_0$  symmetry. The upper 2 × 2 block of  $X_L$ , which we denote  $(X_L)_2$ , transforms as an adjoint of  $SU(2)_0$  corresponding to the two-generation case discussed above. In addition,  $X_L$  consists of an extra doublet of  $SU(2)_Q$ , which we denote  $x_L$ , composed of  $(X_L)_{13}$  and  $(X_L)_{23}$ . At leading order,  $x_L$  only mediates  $\Delta b$ ,  $\Delta t = 1$  processes, while  $\Delta c$ ,  $\Delta s = 1$  processes can be generated at order  $x_L^2$ . Thus, we expect its contributions to  $\Delta c$ ,  $\Delta s = 1$  processes to be subdominant and also generically independent of those of  $(X_L)_2$ . In practice, our result in Eq. (6) applies separately to contributions from  $(X_L)_2$  and  $x_1^2$ , barring cancellations. Further corrections to Eq. (6) come from the SM breaking of the  $SU(2)_Q$ , but these are suppressed by powers of  $m_c^2/m_t^2$  or  $m_s^2/m_b^2$ .

#### 3.2. CP violation in third generation transitions

The universality of CP violation also holds for flavor transitions involving third generation quarks. A useful approximation is again to neglect the masses of the first two generation quarks. The breaking of the flavor symmetry by  $Y_{u,d}$  is then characterized by SU(3)/SU(2) [4]. In this limit, the 1-2 rotation and the phase of the CKM matrix become unphysical, and we can, for instance, apply an SU(2) rotation to the first two generations to "undo" the 1-3 rotation. Therefore, the CKM matrix is effectively reduced to a real matrix with a single rotation angle  $\theta \simeq \sqrt{ heta_{13}^2 + heta_{23}^2}$  between an "active" light flavor,  $q_a$ , and the third generation [3]. Such a pattern of flavor breaking respects an approximate  $U(1)_0$  symmetry for the combination of light quarks that effectively decouples, thus ensuring that all interactions of this "sterile" flavor,  $q_s$ , are CP conserving. What remains is an effective two-generation system with a single measure of CP violation in transitions between the third generation and the active light flavor. It is given again by  $Tr(X_L \cdot J)$ , which is flavor basis independent, and thus universal (see [5] for an extended discussion). Therefore, to leading order, there is a universal relation for CP violation involving transitions between the third generation and the up and down component of the active states,

$$\operatorname{Im}(X_L^u)_{a3} = \operatorname{Im}(X_L^d)_{a3},\tag{7}$$

where the active states coincide with the second generation quarks up to  $\mathcal{O}(\lambda_C)$  [3], where  $\lambda_C \simeq 0.23$  is the sine of the Cabibbo angle.

The leading corrections to Eq. (7), in the massless twogeneration limit, can be understood by decomposing  $X_L$  to its representations under the SU(2) third generation-active flavor group,  $SU(2)_{3a}$ . Besides the adjoint contribution of  $(X_L)_{a3}$ , the entries  $(X_L)_{sa}$  and  $(X_L)_{s3}$  form an  $SU(2)_{3a}$  doublet. At order  $(X_L)^2$ , they in term produce a new adjoint of  $SU(2)_{3a}$ , which would induce an independent contribution to the transitions between the third generation and the active flavor, and hence correct the relation in Eq. (7). Since these are independent contributions, barring cancellations, this relation would still hold for each of them separately. Furthermore, the extra  $SU(2)_{3a}$  doublet would in general contribute to transitions between the first two generations, and should therefore be strongly constrained. We thus conclude that we expect the expression in Eq. (7) to hold to a good accuracy.

Finally, we comment on the fact that one can constrain third generation alignment with data involving the first two generations. Consider, for instance, an alignment model that saturates the bounds from  $B_d$  mixing, including CP violation. In other words, the TeV-scale new physics contributions are required to be approximately aligned with the down Yukawa. This structure would necessarily contribute to CP violation in  $D-\bar{D}$  mixing, since the real and the imaginary parts cannot be simultaneously eliminated. Such a scenario was investigated in [3], where it was shown that in practice the resulting contributions are still two-to-three orders of magnitude below the present bounds.

#### 4. Examples

#### 4.1. Relating CP violation in hadronic K and D decays

The argument presented in Section 2 allows us to relate the existing constraints in the kaon system to  $SU(3)_Q$  breaking NP contributions to direct CP violation in the charm system, and in particular, to relate  $\Delta a_{CP}$  to  $\epsilon'/\epsilon$ . The relevant  $SU(3)_Q$  breaking NP operators in Eq. (2) induce at low energies contributions of the form  $Q_{1,2,5,6}^q$  defined in Eqs. (8) and (15) of [6]. The weakest bound on any of these operators from  $\epsilon'/\epsilon$  is given by [6]

$$\left| \operatorname{Im}(C_2^{(0)}) \right| \lesssim 4.5 \times 10^{-5} \left( \frac{\Lambda_{\mathrm{NP}}}{350 \, \mathrm{GeV}} \right)^2,$$
 (8)

for  $Q_2^{(0)} = (\bar{d}_{\alpha}s_{\beta})_{V-A} \sum_q (\bar{q}_{\beta}q_{\alpha})_{V-A}$ , where  $\alpha$  and  $\beta$  are color indices, and the sum over q includes the u, d, s, c, b flavors. The contributions of such operators to  $\Delta a_{CP}$  are given by [6]

$$\Delta a_{CP}^{NP} \approx 8.9 \sum_{i} \operatorname{Im}(C_{i}^{NP}) \operatorname{Im}(\Delta R_{i}^{NP}), \qquad (9)$$

where  $\Delta R_i^{\rm NP}$  denotes the ratio of the NP amplitude and the leading SM "tree" contribution. Applying the bound in Eq. (8) to Eq. (9), and assuming  $\Delta R_i^{\rm NP} \sim 1$ , we find

$$\Delta a_{CP}^{\rm NP} \lesssim 4 \times 10^{-4}.\tag{10}$$

We thus learn that in any  $SU(2)_L$  invariant NP model, the contributions of the  $Q_{1,2,5,6}^q$  operators to  $\Delta a_{CP}$  must be negligible.

### 4.2. Semileptonic K and D decays

An important class of rare *K* decays are those involving a pion and a lepton pair. The short distance contributions to  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  or for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are dominantly CP violating. (The  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  decay also receives a non-negligible CP conserving long distance contribution.) So far, only upper bounds on these rates have been set [7]

$$Br(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10}, Br(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}, Br(K_L \to \pi^0 \nu \bar{\nu}) < 2.6 \times 10^{-8},$$
(11)

all at 90% confidence level. These experimental results can be translated into constraints on the Wilson coefficients of the appropriate effective operators:

$$\mathcal{H}_{\Delta s=1}^{\text{eff}} \supset \frac{C_{sd}^{\ell_{R/L}}}{\Lambda_{\text{NP}}^2} (\bar{s}d)_{V-A} (\bar{\ell}\ell)_{V\pm A} + \frac{C_{sd}^{\nu}}{\Lambda_{\text{NP}}^2} (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}, \quad (12)$$

where  $\Lambda_{\text{NP}}$  is the NP scale and the superscripts  $\ell_{R/L}$  distinguish the operators containing the right-handed (V + A) and left-handed (V - A) charged lepton currents (in the standard notation  $(C_{sd}^{\ell_R} \pm C_{sd}^{\ell_L})/2$  are  $C_{9,10}$ ).

We start by analyzing the process  $K_L \rightarrow \pi^0 \ell^+ \ell^-$ . Following [8, 9], we can neglect the SM contributions, which are of order 10% or less compared to the current experimental limits. Taking the central values for all the parameters entering the theoretical prediction from [8] and comparing with Eq. (11), we obtain the constraints

$$\left|\operatorname{Im} C_{sd}^{\varrho_{R/L}}\right| < 5.5 \times 10^{-4} \left(\frac{\Lambda_{\mathrm{NP}}}{1 \, \mathrm{TeV}}\right)^{2},$$
  
$$\left|\operatorname{Im} C_{sd}^{\mu_{R/L}}\right| < 9.5 \times 10^{-4} \left(\frac{\Lambda_{\mathrm{NP}}}{1 \, \mathrm{TeV}}\right)^{2}.$$
 (13)

Similarly, we consider the decay channel involving neutrinos. Since the class of operators in Eq. (2) conserves lepton flavor, we can use the Grossman–Nir bound (instead of the presently weaker experimental bound), which relates the rates for charged and neutral kaon decays [10]:

$$\operatorname{Br}(K_L \to \pi^0 \nu \bar{\nu}) < 4.4 \operatorname{Br}(K^+ \to \pi^+ \nu \bar{\nu}).$$
(14)

The latter branching ratio is [7]

$$Br(K^+ \to \pi^+ \nu \bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}.$$
 (15)

Taking a 90% confidence level upper bound and comparing it with the theoretical predictions, following [9], we obtain

$$\left| \operatorname{Im} C_{sd}^{\nu} \right| < 2.6 \times 10^{-4} \left( \frac{\Lambda_{\mathrm{NP}}}{1 \, \mathrm{TeV}} \right)^2.$$
 (16)

Due to our CP violation universality argument, the bounds in Eqs. (13) and (16) apply directly to the charm system as well. The appropriate observables are CP asymmetries involving rare *D* semileptonic decays, for example:

$$a_e^D = \frac{\text{Br}(D^+ \to \pi^+ e^+ e^-) - \text{Br}(D^- \to \pi^- e^+ e^-)}{\text{Br}(D^+ \to \pi^+ e^+ e^-) + \text{Br}(D^- \to \pi^- e^+ e^-)},$$
(17)

as well as for neutrinos instead of charged leptons in the final state (see, e.g., [11]). An upper bound on the asymmetry in Eq. (17) can be obtained as follows. We assume that the SM contribution is essentially CP conserving, so that CP violation is dominated by NP, and that the overall decay rate is dominated by long distance SM contributions [12]. Denoting the NP and the SM amplitudes as  $\mathcal{A}_{NP}$  and  $\mathcal{A}_{SM}$  respectively, with  $|\mathcal{A}_{NP}| \ll |\mathcal{A}_{SM}|$ , we can write

$$\left|a_{e}^{D}\right| \lesssim \frac{2\int d\rho \left|\operatorname{Im}(\mathcal{A}_{\mathrm{NP}})\right| |\mathcal{A}_{\mathrm{SM}}|}{\int d\rho \left|\mathcal{A}_{\mathrm{SM}}\right|^{2}} \lesssim 2\sqrt{\frac{\int d\rho \left|\mathcal{A}_{\mathrm{NP}}\right|^{2}}{\int d\rho \left|\mathcal{A}_{\mathrm{SM}}\right|^{2}}},\tag{18}$$

where  $\int d\rho$  denotes the relevant three body phase space integration. We can identify the denominator of the right-hand side

with the square root of the *experimentally determined* rate  $\Gamma(D^+ \rightarrow \pi^+ e^+ e^-)$ , avoiding the theoretically uncertain evaluation of the SM long distance amplitude. On the other hand, the numerator is dominated by short distance physics, and can be computed reliably using the recent lattice QCD calculation of the  $D \rightarrow \pi$  form factor [13], yielding

$$\left|a_{e}^{D}\right| \lesssim \left(\frac{1 \text{ TeV}}{\Lambda_{\text{NP}}}\right)^{2} \frac{0.1 \left|\operatorname{Im} C_{sd}^{e_{R/L}}\right|}{\sqrt{\operatorname{Br}(D^{+} \to \pi^{+}e^{+}e^{-})}} \lesssim 0.02. \tag{19}$$

On the right-hand side we used Eq. (13) and the experimental upper bound on the branching ratio [7], given that it is close to the estimated long distance contributions [12]. If the above bound would be experimentally violated, the source of the required CP violation could not be of the form of Eq. (2). Finally, we note that this constraint may be refined in the future with improved experimental bounds and theoretical estimates of the relevant processes in either the *K* or the *D* systems.

# 4.3. Semileptonic B decays

Rare semileptonic *B* decays  $B \to X_s \ell^+ \ell^-$ ,  $B \to K^{(*)} \ell^+ \ell^-$ ,  $B_s \to \mu^+ \mu^-$  and  $B \to K^{(*)} \nu \bar{\nu}$  offer direct probes of NP contributions of the form of Eq. (2). At the moment, the most sensitive probe of this kind of NP contribution is the partial branching ratio of the inclusive decay  $B \to X_s \ell^+ \ell^-$  in the so-called "low- $q^2$  region",  $q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2 \in [1, 6]$  GeV<sup>2</sup>. <sup>1</sup> The operator in Eq. (2) contributes to the effective weak Hamiltonian, similar to Eq. (12),

$$\mathcal{H}_{\Delta b=1}^{\text{eff}} \supset \frac{C_{bs}^{\ell_{R/L}}}{\Lambda_{\text{NP}}^2} (\bar{b}s)_{V-A} (\bar{\ell}\ell)_{V\pm A} + \frac{C_{bs}^{\nu}}{\Lambda_{\text{NP}}^2} (\bar{b}s)_{V-A} (\bar{\nu}\nu)_{V-A}.$$
 (20)

Employing the relevant semi-analytic NP formulae for both the electron and muon channels [18], we can derive bounds on  $C_{\text{NP}}^i$ . The experimental results are presented averaged over the electron and muon channels [19], resulting in [16]

$$Br(B \to X_s \ell^+ \ell^-)_{low} = (1.60 \pm 0.50) \times 10^{-6}.$$
 (21)

To bound the operators in Eq. (20), we require that the NP contribution to the particular leptonic channel should be consistent with the above averaged value. In order to extract robust bounds on  $\text{Im}(C_{\text{NP}}^i)$  from  $\text{Br}(B \to X_s \ell^+ \ell^-)_{\text{low}}$ , we marginalize over the corresponding real parts as well as the SM theoretical uncertainties as given in [18]. In this way we obtain at 95% C.L<sup>2</sup>

$$\left|\operatorname{Im}(C_{bs}^{\ell_{L}})\right| < 1.6 \times 10^{-3} \left(\frac{\Lambda_{\mathrm{NP}}}{1 \,\mathrm{TeV}}\right)^{2}, \quad \text{for } \ell = e, \, \mu,$$
  
$$\left|\operatorname{Im}(C_{bs}^{\ell_{R}})\right| < 8.5 \times 10^{-4} \left(\frac{\Lambda_{\mathrm{NP}}}{1 \,\mathrm{TeV}}\right)^{2}, \quad \text{for } \ell = e, \, \mu.$$
(22)

Finally,  $C_{\rm NP}^{\nu}$  can be bounded directly from the experimental searches for the  $B \to K^{(*)} \bar{\nu} \nu$  decays [21], which yield [22]

$$\left| \operatorname{Im}(C_{bs}^{\nu}) \right| < 7.5 \times 10^{-3} \left( \frac{\Lambda_{\rm NP}}{1 \, {\rm TeV}} \right)^2, \quad \text{for all } \nu.$$
(23)

A similar analysis could in principle be performed also for the  $b \rightarrow d$  transitions. However, at present, the associated experimental constraints are much weaker [7], and no interesting bounds can be obtained.

In the long run, the strongest constraints on NP contributions with a new weak phase to the  $C_{bs}^{\ell_{R/L}}$  Wilson coefficients in Eq. (20) (again  $(C_{bs}^{\ell_R} \pm C_{bs}^{\ell_L})/2$  are  $C_{9,10}$  in the rare *b* decay literature) may come from CP violation studies in  $b \rightarrow s\ell^+\ell^-$  mediated decays. These operators dominate in the large- $q^2$  region, while the electromagnetic penguin operator,  $O_7$ , is also important at small  $q^2$ . The  $B \to K^* \ell^+ \ell^-$  mode is particularly promising, since the distribution of the  $K^* \rightarrow K\pi$  decay products allows to extract information about the polarization of the  $K^*$ . When combined with the angular distributions of the two charged leptons, it is possible to construct observables probing directly CP violating contributions to the relevant short-distance Wilson coefficients [23]. Such observables could potentially be measured at LHCb and SuperB [24]. On the other hand, the direct CP asymmetries depend on strong phases, which are small in the inclusive  $B \to X_{\rm s} \ell^+ \ell^-$  decay (outside the resonance region), and are poorly known in the exclusive  $B \to K^{(*)} \ell^+ \ell^-$  case. Another probe of this physics could be the study of time-dependent CP asymmetries in these modes. While these are challenging experimentally, the interpretation of the results would be theoretically cleaner. The SM predicts that the timedependent CP asymmetry vanishes, as it does in  $B_s \rightarrow \phi \phi$ , to an even better accuracy than in  $B_s \rightarrow \psi \phi$ , due to a  $2\beta_s - 2\beta_s$  cancellation between the mixing and decay phases. The same cancellation occurs in NP models in which the mixing amplitude is modified as  $M_{12}^{\rm SM} \times R^2$  and the decay amplitude is modified as  $A^{\rm SM} \times R$ . While this is the case in most supersymmetric models, it is not generic, and is violated, for example, by models containing a Z' which has a flavor-changing coupling to quarks and non-universal couplings to quarks and leptons. (With very large data sets at the upgraded LHCb, a time-dependent  $B_s \rightarrow \mu^+ \mu^-$  analysis would also be worth pursuing.)

To analyze the connection between  $t \to cZ$  and FCNC  $b \to s$  decays, we need to consider the NP operators before the *Z* is integrated out [25]. For example, the operator  $(\bar{b}s)_{V-A}(H^{\dagger}DH)$  contributes to Eq. (20), since after electroweak symmetry breaking  $H^{\dagger}D_{\mu}H \to gv^{2}Z_{\mu}$ . Thus the relevant Wilson coefficient,  $C_{bs}^{H}$ , is constrained from  $B \to X_{s}\ell^{+}\ell^{-}$ , similar to Eq. (22), as  $|\text{Im}(C_{bs}^{H})| < 8.7 \times 10^{-3}(\Lambda_{\text{NP}}/\text{TeV})^{2}$ . Top decays into final states with a jet and a pair of charged leptons offer a probe of the related  $(X_{L}^{u})_{tc}$  and  $(X_{L}^{u})_{tu}$  contributions [26]. The expected sensitivity of this mode with 100 fb<sup>-1</sup> at the 14 TeV LHC is  $|C_{tc(u)}^{H}| \leq 0.2(\Lambda_{\text{NP}}/\text{TeV})^{2}$  [27, 25], where the relevant operator is defined as  $(\bar{t}c(u))_{V-A}(H^{\dagger}DH)$ . According to Eq. (7), we can conclude that barring cancellations, any experimental signal of CP violation in this channel would have to be due to  $SU(3)_{U}$  breaking NP.

#### 5. Implications for SUSY models

In SUSY models the left-handed squark mass-squared matrix,  $\tilde{m}_Q^2$ , is the only source of  $SU(3)_Q$  breaking, and is approximately  $SU(2)_L$  invariant (see, e.g., [28] and references therein). In the following we discuss a universal constraint on  $\tilde{m}_Q^2$  from  $\Delta F = 1$  CP violation. In addition, we consider an example of  $\Delta F = 2$  constraints in relation to alignment models, where our argument about universality of the CP phase also plays a role. In all cases the bounds can be directly applied on the corresponding mass insertion parameters.

First we analyze the constraint from  $\epsilon'/\epsilon$ . In the super-CKM basis, the neutral gaugino couplings are flavor diagonal, while the mass matrices of the squarks are not diagonal in general.

<sup>&</sup>lt;sup>1</sup> We checked that other related presently measured and theoretically clean observables like the low- $q^2$  forward-backward asymmetry ( $A_{FB}$ ) in  $B \to K^* \ell^+ \ell^-$  [14], the high- $q^2$  region in  $B \to X_s \ell^+ \ell^-$  [15,16], or the leptonic decay  $B_s \to \mu^+ \mu^-$  [17] do not yield competitive bounds on the NP contributions that we consider here.

<sup>&</sup>lt;sup>2</sup> Recent analyses [20] of NP in semileptonic  $b \rightarrow s$  transitions obtained somewhat stronger bounds by relying on high- $q^2 A_{FB}$  and partial branching ratio measurements of exclusive  $B \rightarrow K^{(*)}\ell^+\ell^-$  decays. We do not consider these observables, since they are subject to substantial theoretical (form factor) uncertainties.



**Fig. 1.** The bound on  $\delta_Q^{12}$  as a function of the angle  $\alpha$  (see text). The angle  $\alpha$  is plotted on a log scale in the basis  $\lambda_C = 0.23$ , so that a value of 1 on the *x* axis corresponds to  $\alpha = \lambda_C$  (large angle), while a value of 5 gives  $\alpha = \lambda_C^2$  (small angle – down alignment). The vertical doted line shows the angle of optimal alignment (weakest bound). The light (dark) shaded region corresponds to a gluino mass  $m_{\tilde{g}}$  of 1 (1.5) TeV, and inside each region the average squark mass  $\tilde{m}_{\tilde{Q}}$  is varied in the range  $[0.8m_{\tilde{g}}, 1.2m_{\tilde{g}}]$ . The upper edge of each region (weakest bound) comes from the lowest  $\tilde{m}_{\tilde{Q}}$ .

New contributions to CP violation in  $\Delta F = 1$  processes involving left-handed quarks are induced by the imaginary off-diagonal elements of  $\tilde{m}_Q^2$ , and can be parameterized in terms of the ratios  $\delta_{LL}^{ij} \equiv (\tilde{m}_Q^2)^{ij}/\tilde{m}_{\bar{Q}}^2$ , where i, j = 1, 2 are flavor indices and  $\tilde{m}_{\bar{Q}} \equiv (m_{\bar{Q}_1} + m_{\bar{Q}_2})/2$  is the average squark mass (this choice is consistent to linear order with the convention of [29]). The experimental constraint on new contributions to  $\epsilon'/\epsilon$  is translated to the following bound on the left-handed mass insertion parameter [29] Im  $\delta_{LL}^{12} \leq 0.5$  for  $\tilde{m}_{\bar{Q}} = m_{\bar{g}} = 500$  GeV. This can be straightforwardly rephrased as a robust constraint on the level of degeneracy

$$\delta_Q^{12} \equiv \frac{m_{\tilde{Q}_2} - m_{\tilde{Q}_1}}{m_{\tilde{Q}_2} + m_{\tilde{Q}_1}} \leqslant 0.25 \left(\frac{500 \text{ GeV}}{\bar{m}_{\tilde{Q}}}\right). \tag{24}$$

This bound is weaker than the one obtained by combining the bounds from  $\epsilon_K$  and  $D-\bar{D}$  mixing [1]. Yet, interestingly, it could have constrained degeneracy without the need for any additional measurements involving D mesons, more than 20 years ago already, when the experimental uncertainty of  $\epsilon'/\epsilon$  approached the  $10^{-3}$  level [30].

Constraints on alignment models that balance the bounds from mixing and CP violation in the K and D systems have been analyzed in [1]. Here we comment on their results for supersymmetric models based on our CP universality argument. According to the parameterization employed in [1],  $\sin \alpha (\sin 2\gamma)$  is proportional to the real (imaginary) part of the off-diagonal element of the NP flavor violating source in the down mass basis. CP universality implies that in the up mass basis,  $\sin 2\gamma$  still corresponds to the imaginary part, while the real part is rotated by twice the Cabibbo angle. Eq. (31) in [1] gives the bounds on squark mass degeneracy for the cases of vanishing  $(\sin 2\gamma = 0)$  and maximal  $(\sin 2\gamma \sim 1)$ phase. We argue that the latter case is irrelevant, since it violates the assumption of alignment. In contrast, while realistic models of alignment generically do not control the fundamental CP violating phases, they force both  $\sin \alpha$  and  $\sin 2\gamma$  to be small, and should therefore be taken to be comparable [31]. This leads to a much weaker bound than the more stringent one in [1]. In particular, the bound on  $\delta_0^{12}$  from  $\epsilon_K$  and  $\Delta m_K$  for  $\sin \alpha \sim \sin 2\gamma$  is shown in Fig. 1 as a function of the angle  $\alpha$ , for various ranges of the relevant SUSY parameters (see the caption). It can be seen that on the right-hand side of the plot, where the angle is very small (down alignment), the strongest constraint comes from  $\Delta m_D$ , while on the left-hand side, where the angle is large,  $\epsilon_K$  is the dominant constraint. The vertical dashed line marks the transition point, where the alignment is optimal, yet as evident from the plot, making the angle smaller only mildly affects the bound on  $\delta_Q^{12}$ . For the case where the gluino mass and the average squark mass are both 1 TeV, the weakest bound is  $\delta_Q^{12} \lesssim 0.13$ . This occurs around  $\log_{\lambda} \alpha \sim 2.5$ , so the universal CP violating phase is of order  $\lambda_C^{2.5}$ . This implies an upper bound on CP violation in  $D-\bar{D}$  mixing of order 0.2, around the current experimental limit on ||q/p| - 1| [32], which is expected to be improved significantly in the near future.

It is interesting that a modest level of degeneracy can be obtained only from the renormalization group equation (RGE) flow, when starting from anarchy at the SUSY breaking mediation scale [33]. Moreover, in order to satisfy the bounds on degeneracy from optimal alignment models, as presented in Fig. 1, the mediation scale does not have to be very high. To show this, we use the SUSY RGE for the diagonal squark mass entries, which is dominated by the gluino contribution. Neglecting the other gaugino contributions, we can solve the relevant equations at one loop analytically

$$\frac{1}{\alpha_s(M_S)} = \frac{1}{\alpha_s(\Lambda)} + \frac{b_3}{2\pi} \ln \frac{\Lambda}{M_S},$$
(25)

$$\frac{m_{\tilde{g}}(\Lambda)}{m_{\tilde{g}}(M_{S})} = 1 + \alpha_{s}(\Lambda) \frac{b_{3}}{2\pi} \ln \frac{\Lambda}{M_{S}},$$
(26)

$$m_{\tilde{Q}_{1,2}}^2(M_S) - m_{\tilde{Q}_{1,2}}^2(\Lambda) = \frac{8}{3b_3} \big[ m_{\tilde{g}}(\Lambda)^2 - m_{\tilde{g}}(M_S)^2 \big],$$
(27)

where  $\Lambda$  is the typical scale of the new supersymmetric particles (taken to be 1 TeV),  $M_S$  is the SUSY breaking mediation scale,  $b_3 = -3$  is the MSSM QCD beta function and the last equation is written in the squark mass basis. In addition, we define  $\sum m_{\tilde{Q}}^2(\mu) = m_{\tilde{Q}_1}^2(\mu) + m_{\tilde{Q}_2}^2(\mu)$  and  $\Delta m_{\tilde{Q}}^2(\mu) = m_{\tilde{Q}_2}^2(\mu) - m_{\tilde{Q}_1}^2(\mu)$ . Then in our approximation, only  $\sum m^2$  has a nontrivial RGE evolution, while  $\Delta m^2$  is invariant. Writing

$$\delta_Q^{12}(\mu) = \frac{\Delta m_{\tilde{Q}}^2(\mu)}{\sum m_{\tilde{Q}}^2(\mu) [1 + \sqrt{1 - (\Delta m_{\tilde{Q}}^2(\mu) / \sum m_{\tilde{Q}}^2(\mu))^2}]},$$
 (28)

we observe that only the denominator has a nontrivial RGE evolution. Furthermore, in the IR,  $\delta_Q^{12}$  approaches the limit  $\delta_Q^{12} \approx \Delta m_{\tilde{O}}^2 / 2 \sum m_{\tilde{O}}^2$ .

In Fig. 2 we show the contours of  $M_S$  values that yield the optimal  $\delta_Q^{12}(\Lambda)$  as a function of the gluino and average squark masses. Since  $M_S$  is sensitive to the level of anarchy assumed for  $\delta_Q^{12}(M_S)$ , we choose conservatively to take  $\delta_Q^{12}(M_S) = 2$ , which actually corresponds to the extreme hierarchy case  $m_{\tilde{Q}_2}^2 \gg m_{\tilde{Q}_1}^2$ . Any finite ratio between the masses would lead to a lower mediation scale than in Fig. 2. We find that, quite remarkably, a large portion of the parameter space, with TeV superpartner masses, is consistent with a fully anarchic spectrum at a moderate mediation scale. Furthermore, we even find a non-negligible region where  $\delta_Q^{12}(\Lambda) \sim 0.3$  is allowed. For instance for a gluino mass of 1.3 TeV we find that the first two generation squark masses can be 550 GeV and 950 GeV respectively at the TeV scale, which can hardly be considered as a degenerate spectrum.



**Fig. 2.** Contours for various values of the SUSY breaking mediation scale  $M_S$  in the parameter space defined by  $m_{\tilde{g}}$  and  $\bar{m}_Q$ , assuming  $\delta_Q^{12}(M_S) = 1$ . Darker shaded regions correspond to higher  $M_S$ . Also shown are contours for  $\delta_Q^{12}(\Lambda) = 0.1, 0.2, 0.3$  in solid, dashed and dotted blue lines, respectively, where  $\delta_Q^{12}(\Lambda) > 0.3$  between the two dotted lines.

#### 6. Conclusions

We have shown that NP that breaks the left-handed  $SU(3)_0$ quark flavor symmetry induces approximately universal contributions to CP violation in  $\Delta F = 1$  processes, in that they are not affected by flavor rotations between the up and the down mass bases. Therefore, these sources cannot be aligned, and can be constrained by the strongest bound coming either from the up or the down sectors. We have used this result to show that the bound from  $\epsilon'/\epsilon$  prohibits an SU(3)<sub>0</sub> breaking explanation of the recent LHCb evidence for CP violation in charm decays. A consequence of this CP universality is that SUSY alignment models, even with a low SUSY breaking mediation scale, are consistent with current data, since the universal CP phase tends to be suppressed. Therefore, fundamentally squarks need not be degenerate. We note in this respect that the current direct experimental searches for squarks are assuming degeneracy in the first two generations, and therefore their lower bounds do not strictly apply in the context of alignment models which could have a significant splitting between the first two generations. Finally, in this framework CP violation in  $D-\bar{D}$  mixing is bounded from above with a maximal value which is close to the current experiment sensitivity. Other types of models of alignment (see [34], for example), as in the case of the SUSY example, also tend to yield more anarchy in the up sector. Hence, they are expected to be constrained by flavor transition measurements in the up sector, with the contributions to CP violation somewhat suppressed.

We have also discussed the universality of CP violation involving the third generation, and established a linkage between CP violation in rare bottom and top guark decays, which might be tested in the far future. It is interesting to note that the combination of the current direct constraints on the superpartner spectrum and naturalness implies the possibility that the first two generation squarks are rather heavy, while the third generation left-handed squarks are approximately degenerate [35]. In such a case, flavor violation involving the third generation would approximately satisfy our universality condition. In this setup there is no generic reason to expect the entries of the left-handed squark matrices to be real. Thus, since the spectrum is hierarchal, the experimental bound on the level of flavor violation can be applied directly as constraints on the phases, which should be of order 0.05 (0.15) for  $\Delta b = 2$  processes in the  $B_d$  ( $B_s$ ) systems for the third generation doublet and gluino of around 500 GeV [36]. This further implies, within this framework, a strong suppression of CP violating processes involving only left-handed squarks, in either the down or the up sectors.

#### Acknowledgements

We thank Yuval Grossman, Gino Isidori, Yossi Nir and Michele Papucci for useful discussions. We thank David Straub and Wolfgang Altmannshofer for pointing out a numerical error in Eq. (22) in the first version of the manuscript. The work of J.F.K. was supported in part by the Slovenian Research Agency. G.P. is supported by the GIF, Gruber foundation, IRG, ISF and Minerva. Z.L. was supported in part by the Office of High Energy Physics of the US Department of Energy under contract DE-AC02-05CH11231.

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