1. Introduction

1.1. The local theory of blocks of finite groups was proposed originally by J. Alperin and M. Broué in [1], and developed by L. Puig [12], where the source algebra of a block is introduced as the smallest algebra which carries the local information of the block. One of the classical applications of the theory is the research on nilpotent blocks (see [2,9]). Recently, understanding the fusions of local pointed groups, L. Puig in [7] and [8] introduces the hyperfocal subalgebra in the source algebra of a block, and proves its existence and uniqueness up to conjugation. The local information of nilpotent blocks are the simplest case, and the structure theorem of their source algebras in [9] is the simplest case of the Puig’s work on hyperfocal subalgebras.

Noting that Puig obtains his results in large enough coefficient fields, in this paper we make a research on the hyperfocal subalgebras of source algebras of blocks over small-ground-fields.

1.2. Let $G$ always be a finite group. Let $p$ be a prime number, and $\mathcal{O}$ be a complete discrete valuation ring with a fraction field $K$ of characteristic zero and a perfect residue field $k = \mathcal{O}/J(\mathcal{O})$ of characteristic $p$. All $\mathcal{O}$-algebras considered in this paper are associative and unitary, and $\mathcal{O}$-free of finite rank; but subalgebras of an algebra are not necessarily unitary, i.e., the identity element of a subalgebra may be different from the identity element.
of the algebra. For an algebra $A$, we denote by $J(A)$, $Z(A)$ and $A^*$ the Jacobson radical of $A$, the center of $A$ and the set of all invertible elements of $A$ respectively. A $G$-algebra means an algebra $A$ with a group homomorphism $G \rightarrow \text{Aut}(A)$, where the latter denotes the automorphism group of the algebra $A$. An interior $G$-algebra means an algebra $A$ with a group homomorphism $G \rightarrow A^*$.

For a $G$-algebra $A$ and a subgroup $P$ of $G$, by $A^P$ we denote the unitary subalgebra of $A$ consisting of the $P$-fixed elements of $A$; and denote

$$A(P) = k \otimes_O \left( A^P / \sum_{Q \leq P} A^Q \right),$$

where $Q$ runs on the set of the proper subgroups of $P$ and $A^P$ denotes the image of the relative trace map $\text{Tr}^P_Q : A^Q \rightarrow A^P$; and we call the canonical surjective homomorphism $\text{Br}^A : A^P \rightarrow A(P)$ the Brauer homomorphism associated with $P$. By the way, we remark that for any $O_G$-module $M$, the $O$-submodule $M^Q$, the trace map $\text{Tr}^P_Q : M^Q \rightarrow M^P$, and $M^P : M(P)$ and the Brauer map $\text{Br}^M_P : M^P \rightarrow M(P)$, are defined similarly.

1.3. Recall that a pointed group $H_\alpha$ on a $G$-algebra $A$ means a pair $(H, \alpha)$, where $H$ is a subgroup of $G$ and $\alpha$ is a conjugate class of primitive idempotents of the algebra $A^H$; a pointed group $K_\beta$ is said to be contained in $H_\alpha$, denoted by $K_\beta \leq H_\alpha$, if $K \leq H$ and there exist $i \in \alpha$ and $j \in \beta$ such that $ij = j = ji$. A pointed group $P_\gamma$ is said to be local if $\text{Br}^A_P(\gamma) \neq \{0\}$. Then all the maximal local pointed groups $P_\gamma$ which are contained in a pointed group $H_\alpha$ form exactly one $H$-conjugate class; and they are called defect pointed groups of $H_\alpha$. Thus the stabilizer $N_H(P_\gamma)$ in $H$ of the defect pointed group $P_\gamma$ of $H_\alpha$ is unique up to conjugation. We set $E_H(P_\gamma) = N_H(P_\gamma)/PCH(P)$. And, for $i \in \gamma$, we set $A_\gamma = iAi$, and call it a source algebra of $H_\alpha$, see [12].

1.4. In the following, let $A = O_G$ be the group algebra over $O$ of the finite group $G$. Obviously, the conjugate action of $G$ induces a $G$-algebra structure on $A$. Let $G_{(b)}$ be a pointed group on $A$; then $b$ is called an $O$-block of $G$. Let $P_\gamma$ be a defect pointed group of $G_{(b)}$ and $i \in \gamma$, and set $A_\gamma = iAi$, which admits an obvious $O_P$-interior algebra structure. Since $\text{Br}^A_P(\gamma)$ is a point of $A(P) \cong kCG(P)$, it determines a unique block $\tilde{b}_\gamma$ of $kCG(P)$ such that $b_\gamma \text{ Br}^A_P(\gamma) \neq 0$. Further, the surjective homomorphism $\text{OCG}(P) \rightarrow kCG(P)$ induces a surjective homomorphism $Z(\text{OCG}(P)) \rightarrow Z(kCG(P))$, hence $\tilde{b}_\gamma$ can be lifted to a unique central primitive idempotent $b_\gamma$ of $\text{OCG}(P)$. Set $\tilde{C}_G(P) = C_G(P)/Z(P)$, and let $\tilde{\tilde{b}}_\gamma$ be the image of $b_\gamma$ in $\text{OCG}(P)$. By [6, 4.3], we have that

$$\tilde{O} = Z(\text{OCG}(P)\tilde{\tilde{b}}_\gamma) \quad (1.4.1)$$

is an unramified Galois extension of $O$, that is, the fraction field $\tilde{K}$ of $\tilde{O}$ is a Galois extension of $K$ and the residue filed $\tilde{k}$ of $\tilde{O}$ is a separable Galois extension of $k$, and they have the same Galois group $\Gamma = \text{Gal}(\tilde{K}/K) = \text{Gal}(\tilde{O}/O) = \text{Gal}(\tilde{k}/k)$, which is in fact cyclic (see [4, 2.2.2]). Moreover, by [6, 4.3] again, $\text{OCG}(P)\tilde{\tilde{b}}_\gamma$ is a full matrix algebra over $\tilde{O}$. Since
A_\gamma is embedded into \mathcal{O}G as interior P-algebras and the embedding is compatible with Brauer homomorphisms, we have that \( A_\gamma / J(A_\gamma) \) is embedded into \( k \otimes_{\mathcal{O}} \mathcal{O}C_G(P)\bar{b}_\gamma \), thus \( A_\gamma / J(A_\gamma) \cong \hat{k} \).

1.5. Let \( \hat{\Lambda} = \hat{\mathcal{O}}G = \hat{\mathcal{O}} \otimes_{\mathcal{O}} A \) and \( P_{_{\hat{\gamma}}} \) be a pointed group of \( \hat{\Lambda} \) such that there exists \( \hat{i} \in \hat{\gamma} \) such that \( \hat{i} \hat{i} = \hat{\gamma} = \hat{\gamma} \). Then \( P_{_{\hat{\gamma}}} \) determines a unique \( \hat{\mathcal{O}} \)-block \( \hat{b} \) of \( \Lambda \) such that \( \hat{b} \hat{b} = \hat{b} \) and we set \( \hat{\Lambda}_{_{\hat{\gamma}}} = \hat{\gamma} \hat{\Lambda} \); since the Brauer homomorphisms \( Br_A^b \) and \( Br_A^\hat{b} \) induce an isomorphism \( \hat{k} \otimes_k A(P) \cong \hat{\Lambda}(P) \), it is easily checked that \( P_{_{\hat{\gamma}}} \) is a defect pointed group of \( G_{_{\hat{b}}} \). Because \( \hat{\Lambda}_{_{\hat{\gamma}}} \) is embedded into \( \hat{\mathcal{O}} \otimes_{\mathcal{O}} A_{\gamma} \) as \( \hat{\mathcal{O}} \)-interior algebras and

\[
(\hat{\mathcal{O}} \otimes_{\mathcal{O}} A_{_{\gamma}} / J(\hat{\mathcal{O}} \otimes_{\mathcal{O}} A_{_{\gamma}})) \cong \hat{\mathcal{O}} \otimes_{\mathcal{O}} (A_{_{\gamma}} / J(A_{_{\gamma}})) \cong \hat{\mathcal{O}} \otimes_{\mathcal{O}} \hat{k} \cong \hat{k}[\Gamma]
\]

is a direct sum of \(|\Gamma|\) copies of \( \hat{k} \); hence \( \hat{\Lambda}_{_{\hat{\gamma}}} / J(\hat{\Lambda}_{_{\hat{\gamma}}}) \) is isomorphic to \( \hat{k} \). Similarly, \( Z(\hat{\Lambda}_{_{\hat{\gamma}}}) / J(Z(\hat{\Lambda}_{_{\hat{\gamma}}})) \) is isomorphic to \( \hat{k} \). That is, \( \hat{i} \) and \( \hat{b} \) are absolutely primitive in \( \hat{A}^P \) and in \( \mathcal{O}\gamma \) respectively.

Let \( \hat{k} \) be an algebraic closure of \( \hat{k} \) and \( \hat{\mathcal{O}} \) be an unramified extension of \( \mathcal{O} \) with the residue field \( \hat{k} \). Then from [6, 2.13], we can conclude that \( \hat{b} = \hat{b} \) is an \( \hat{\mathcal{O}} \)-block of \( G \) and \( \hat{\gamma} \) is contained in a point \( \hat{\gamma} \) of \( P \) on \( \hat{\mathcal{O}}G \); moreover \( P_{_{\hat{\gamma}}} \) is a defect pointed group of the block \( \hat{b} \).

1.6. Recall that a self-centralizing pointed group \( Q_{_{\hat{\gamma}}} \) on \( \hat{\mathcal{O}}G \) is a local pointed group on \( \hat{\mathcal{O}}G \) such that \( Z(Q) \) is a defect group of the block \( b_{_{\hat{\gamma}}} \) of \( \mathcal{O}C_G(Q) \) determined by \( \hat{\delta} \) (i.e., \( b_{_{\hat{\gamma}}} Br_Q(\hat{\delta}) \neq \{0\} \)); and, an essential pointed group \( R_{_{\hat{\gamma}}} \) on \( \hat{\mathcal{O}}G \) is a self-centralizing pointed group on \( \hat{\mathcal{O}}G \) such that the quotient \( E_G(R_{_{\hat{\gamma}}} \hat{\mathcal{O}}) \) contains a proper subgroup \( M \) satisfying that \( p \) divides \(|M|\) but does not divide \(|M \cap M^x|\) for any \( x \in E_G(R_{_{\hat{\gamma}}} \hat{\mathcal{O}}) - M \). And recall that the hyperfocal subgroup \( \hat{Q} \) of \( P_{_{\hat{\gamma}}} \) (see [8, 1.3] or [7, 13.2]) is generated by the commutators \( [K, R] \), where \( R_{_{\hat{\gamma}}} \leq P_{_{\hat{\gamma}}} \) is either essential or equal to \( P_{_{\hat{\gamma}}} \) and \( K \) runs over the set of \( p' \)-subgroups of \( N_G(R_{_{\hat{\gamma}}} \hat{\mathcal{O}}) \).

1.7. Let \( Q \) be the normal subgroup of \( P \) generated by \( \hat{Q} \) and the commutators \( [K, \hat{Q}] \) where \( K \) runs over the \( p' \)-subgroups of \( N_G(P_{_{\hat{\gamma}}} \hat{\mathcal{O}}) \).

Our main result is as follows, where \( D \) is called a hyperfocal subalgebra of the \( \mathcal{O} \)-block \( b \).

**Theorem 1.8.** With notation as above, and assume that \( E_G(P_{_{\hat{\gamma}}} \hat{\mathcal{O}}) \) is a \( p' \)-group. Then there exists a \( P \)-stable unitary \( \mathcal{O} \)-subalgebra \( D \) of \( A_{_{\gamma}} \) such that

\[
D \cap P = Qi \quad \text{and} \quad A_{_{\gamma}} = \bigoplus_u Du,
\]

where \( u \) runs on a set of representatives for \( P \mid Q \) in \( P \); and all such subalgebras of \( A_{_{\gamma}} \) are conjugate to each other by \( 1 + J(A_{_{\gamma}}) \).
Remark 1.9. The idempotent \( i \) is the identity element of \( A_Y \); and \( P \equiv Pi \subset (A_Y)^* \) because \( OG \) is a projective \( OP \)-module. The subalgebra \( D \) described in (1.8.1) inherits an \( OQ \)-interior \( P \)-algebra structure from the interior \( P \)-algebra \( A_Y \), so the second equality means that \( A_Y \) is a crossed product of \( P/Q \) by \( D \). More precisely, \( A_Y \cong D \otimes_{OQ} OP \) as \( OP \)-interior algebras, where \( D \otimes_{OQ} OP \) is endowed with multiplication

\[
(d \otimes x)(d' \otimes x') = d(d'^{-1}) \otimes xx', \quad \forall d, d' \in D, \ x, x' \in P;
\]

we denote \( D \otimes_{OQ} P = D \otimes_{OQ} OP \), and call it twisted \( Q \)-group algebra of \( P \) over \( D \). Thus (1.8.1) can be restated as

\[
D \cap Pi = Qi \quad \text{and} \quad A_Y \cong D \otimes_{OQ} P.
\]

(1.9.1)

For details, please see [6, 1.6].

In Section 2 we prove the theorem for the case that \( \hat{O} = \mathcal{O} \); note that \( EG(P_y) \) is always a \( p' \)-group if \( \hat{O} = \mathcal{O} \) (see [6, 4.4.2]). In Section 3 we show some general properties of hyperfocal subalgebras of a block; then we prove the theorem for the case that \( \mathcal{O} < \hat{O} \) in Section 4.

2. Hyperfocal subalgebras in the case that \( \hat{O} = \mathcal{O} \)

2.1. First we mention two general facts; then from 2.3 on we turn to our objects.

Let \( X \) be a group and \( Y \) be a normal subgroup of \( X \) such that \( X/Y \cong G \), i.e., \( X \) is an extension of \( G \) by \( Y \). The conjugation of elements of \( X \) induces a group homomorphism \( G \to \text{Aut}(Y) \) where \( \text{Aut}(Y) \) denotes the outer automorphism group of \( Y \). Such a group \( Y \) which is endowed with a group homomorphism \( G \to \text{Aut}(Y) \) is called a \( G \)-acted group. Recall that a \( G \)-acted group \( Y \) is said to be uniquely split if any extension of \( G \) by \( Y \) splits and all the splittings are pairwise conjugate. Let \( \{Y_n\}_{n \in \mathbb{N}} \) be a normal filtration of \( Y \), i.e., a family of normal subgroups of \( Y \) indexed by the set \( \mathbb{N} \) of the natural numbers such that \( Y_0 = Y \) and \( Y_{n+1} \subset Y_n \) for any \( n \in \mathbb{N} \); then we have a canonical group homomorphism \( c \) from \( Y \) to the projective limit \( \lim \{Y/Y_n\}_{n \in \mathbb{N}} \). We say that \( \{Y_n\}_{n \in \mathbb{N}} \) is a completing filtration of \( Y \) if \( c \) is an isomorphism. A normal filtration \( \{Y_n\}_{n \in \mathbb{N}} \) of \( Y \) is called interior if for any \( n \in \mathbb{N} \) the image of \( Y \) in \( \text{Aut}(Y/Y_{n+1}) \) coincides with the inner automorphism group \( \text{Int}(Y/Y_{n+1}) \) of \( Y_n/Y_{n+1} \). Please see [6, §3] for details.

Lemma 2.2. Let \( \bar{k} \) be an algebraic closure of \( k \) (recall that \( k \) is perfect), and \( \hat{\mathcal{O}} \) be an unramified extension of \( \mathcal{O} \) such that \( \hat{\mathcal{O}}/J(\hat{\mathcal{O}}) = \bar{k} \). If \( \hat{A} \) is a \( \mathcal{O} \)-algebra over \( \hat{\mathcal{O}} \) and \( \bar{B} \) is a \( \mathcal{O} \)-stable subalgebra of \( \hat{A} \), then there are an \( \hat{\mathcal{O}} \subset \hat{\mathcal{O}} \) which is a finite Galois extension over \( \mathcal{O} \) and a \( \mathcal{O} \)-algebra \( \hat{A} \) over \( \hat{\mathcal{O}} \) and a \( \mathcal{O} \)-stable subalgebra \( \bar{B} \) of \( \hat{A} \) such that \( \hat{A} = \hat{\mathcal{O}} \otimes_{\hat{\mathcal{O}}} \hat{A} \) and \( \bar{B} = \hat{\mathcal{O}} \otimes_{\hat{\mathcal{O}}} \bar{B} \).

Proof. Let \( \hat{k} \) be a fraction field of \( \hat{\mathcal{O}} \). Let \( \{a_1, a_2, \ldots, a_n\} \) be an \( \hat{\mathcal{O}} \)-basis of \( \hat{A} \), and \( \{d_1, d_2, \ldots, d_m\} \) be an \( \hat{\mathcal{O}} \)-basis of \( \bar{B} \). Assume that
\[ a_ia_j = \sum_{k=1}^{n} \lambda_{ijk}a_k, \quad d_i = \sum_{k=1}^{n} \mu_{ijk}a_k, \]
\[ d_id_j = \sum_{k=1}^{m} \xi_{ijk}d_k, \quad d_i^x = \sum_{k=1}^{m} \eta_{xik}d_k, \quad x \in G, \]

where all \( \lambda_{ijk}, \mu_{ijk}, \xi_{ijk}, \eta_{xik} \in \hat{\mathcal{O}} \) are algebraic over \( \mathcal{O} \). Let \( \hat{\mathcal{K}} \) be the normal closure of the extension of \( \mathcal{K} \) generated by all the \( \lambda_{ijk}, \mu_{ijk}, \xi_{ijk}, \eta_{xik} \) and let \( \hat{\mathcal{O}} \) be the integral closure of \( \mathcal{O} \) in \( \hat{\mathcal{K}} \). Then \( \hat{\mathcal{K}} \) and \( \hat{\mathcal{O}} \) are finite Galois extensions of \( \mathcal{K} \) and \( \mathcal{O} \) respectively, and \( \hat{\mathcal{A}} = \sum_{i=1}^{n} \hat{\mathcal{O}}a_i \) and \( \hat{\mathcal{B}} = \sum_{i=1}^{m} \hat{\mathcal{O}}d_i \) are desired algebras. \( \square \)

**Remark.** It is clear that the conclusion still holds for finitely many subalgebras of \( \hat{\mathcal{A}} \).

2.3. From now on to the end of this section we keep the notation in 1.2, 1.4, 1.5, and 1.7, and always assume that \( \hat{\mathcal{O}} = \mathcal{O} \); note that in this case \( \mathcal{E}_G(P_i) \) is always a \( p' \)-group (see [6, 4.4.2]). Then for any extension \( \mathcal{O} \subset \hat{\mathcal{O}} \subset \mathcal{O} \), we have that \( b = \hat{b} \) is a block idempotent of \( \hat{\mathcal{O}} \mathcal{G} \), and \( i = \hat{i} \) is a primitive idempotent in \( (\hat{\mathcal{O}} \mathcal{G})^\mathcal{P} \), and \( \hat{\mathcal{A}} = i\hat{\mathcal{O}}\hat{\mathcal{G}}i \) is a source algebra of the \( \hat{\mathcal{O}} \)-block \( b \), where \( \hat{\mathcal{P}}_\gamma \) is a pointed group on \( \hat{\mathcal{O}} \mathcal{G} \) such that \( i \in \hat{\gamma} \).

**Lemma 2.4.** With notation as above, assume that \( \hat{\mathcal{O}} \) is a finite extension of \( \mathcal{O} \) and that \( \hat{\mathcal{D}} \) is a \( P \)-stable \( \hat{\mathcal{O}} \)-subalgebra of \( \hat{\mathcal{A}}_\gamma = \hat{\mathcal{O}} \mathcal{O} \mathcal{A}_\gamma \) fulfilling that \( D \cap P_i = Q_i \) and \( \hat{D} \mathcal{O} \mathcal{Q}_i P = \hat{A}_\gamma \). Then \( N_{1+1}((\hat{\mathcal{A}}_\gamma^\mathcal{P})^{(D)}) = (1 + J(\mathcal{Z}(\hat{\mathcal{A}}_\gamma)))(1 + J(D^\mathcal{P})) \).

**Proof.** The proof is inspired by [7]. Let \( \hat{k} \) be an algebraic closure of \( k \) and \( \hat{\mathcal{O}} \) be a corresponding unramified extension of \( \mathcal{O} \) such that \( \hat{\mathcal{O}}/J(\hat{\mathcal{O}}) = \hat{k} \). Then, by in [6, 2.13.5], \( P_i \) determines a defect pointed group \( P_i \) of the \( \hat{\mathcal{O}} \)-block \( b \); then by [14, 38.10], \( (\hat{\mathcal{A}}_\gamma^\mathcal{P})(P) \cong \hat{k}\mathcal{Z}(P) \), and further we have that \( (\hat{\mathcal{A}}_\gamma^\mathcal{P})(P) \cong \hat{k}\mathcal{Z}(P) \), where \( \hat{k} = \hat{\mathcal{O}}/J(\hat{\mathcal{O}}) \). Moreover, \( \hat{\mathcal{D}}(P) \) is a direct summand of \( (\hat{\mathcal{A}}_\gamma^\mathcal{P})(P) \) as \( k\mathcal{C}_Q(P) \)-modules, and for any \( u \in \mathcal{Z}(P) \), we have \( (\hat{\mathcal{D}}u)(P) \cong \hat{\mathcal{D}}(P) \); consequently \( \hat{\mathcal{D}}(P) \cong k\mathcal{C}_Q(P) \). Let \( U \) be a set of representatives of \( P/\mathcal{Q} \) in \( P \). For any \( a \in N_{1+1}((\hat{\mathcal{A}}_\gamma^\mathcal{P})^{(\hat{\mathcal{D}})}) \), we can write \( a = \sum_{u \in U} a_u \), where \( a_u \in \hat{\mathcal{D}}u \); then \( \sum_{u \in U \cap QZ(P)} Br_P(a_u) = Br_P(i) + J((\hat{\mathcal{A}}_\gamma^\mathcal{P})(P)) \), and thus there exists a suitable \( z \in U \cap QZ(P) \) such that \( Br_P(a_z) \) is not contained in \( J((\hat{\mathcal{A}}_\gamma^\mathcal{P})(P)) \). In particular, there exists \( \lambda \in \hat{\mathcal{O}}^* \) such that \( \lambda a_zz^{-1} = i + J(\hat{\mathcal{D}}^\mathcal{P}) \).

Set \( c = \lambda^{-1}z(a_z)^{-1}a_u \); then \( c \in N_{1+1}((\hat{\mathcal{A}}_\gamma^\mathcal{P})^{(\hat{\mathcal{D}})}) \). Write \( c = i + \sum_{u \in U \cap Q} c_u \), where \( c_u \in \hat{\mathcal{D}}u \); for any \( \tilde{d} \in \hat{\mathcal{D}} \), there exists \( \tilde{d}' \in \hat{\mathcal{D}} \) such that
\[
(\tilde{d} \otimes 1)(i + \sum_{u \in U \cap Q} c_u) = (i + \sum_{u \in U \cap Q} c_u)(\tilde{d}' \otimes 1),
\]
thus \( \tilde{d} = \tilde{d}' \) and further we have \( (\tilde{d} \otimes 1)c_u = c_u(\tilde{d} \otimes 1) \) for any \( u \in U \cap Q \). In conclusion, \( c \in i + J(\mathcal{Z}(\hat{\mathcal{A}}_\gamma)) \). \[ \square \]
\textbf{Lemma 2.5.} With notation as above, assume that \( \hat{O} \) is a unramified Galois extension of \( O \) with the Galois group \( \hat{\Gamma} \) and that \( \hat{D} \) is a \( P \)-stable \( \hat{O} \)-subalgebra of \( \hat{A}_\gamma = \hat{O} \otimes_O A_\gamma \) fulfilling that \( \hat{D} \cap Pi = Qi \) and \( \hat{D} \otimes_Q P = \tilde{A}_\gamma \). Then \( N_{1+J(\hat{A}_\gamma)}(\hat{D}) \) is a uniquely split \( \hat{\Gamma} \)-acted group.

\textbf{Proof.} In fact, the family \( \{1 + J(\hat{D}^P)^{n+1}\}_{n \in \mathbb{N}} \) is clearly an interior completing filtration of \( 1 + J(\hat{D}^P) \), and for any \( n \geq 1 \) the map \( r \mapsto 1 + r \) induces a group isomorphism

\[ J(\hat{D}^P)^n / J(\hat{D}^P)^{n+1} \cong (1 + J(\hat{D}^P)^n) / (1 + J(\hat{D}^P)^{n+1}). \]

by \([6, 3.8]\), \( 1 + J(\hat{D}^P) \) is a uniquely split \( \hat{\Gamma} \)-acted group. Applying \([6, \text{Theorem 3.11}] \) to the case that \( Y = N_{1+J(\hat{A}_\gamma)}(\hat{D}) \) and \( X = N_{1+J(\hat{A}_\gamma)}(\hat{D}) \times \hat{\Gamma} \) and \( \hat{G} = \hat{\Gamma} \) and \( \hat{O} \)-algebra \( \hat{D}^P \) and \( M = J(Z(\hat{A}_\gamma)) \) and \( N = J(Z(D)^P) \), we have that \( (1+M)/(1+N) \) is a uniquely split \( \hat{\Gamma} \)-act group. By Lemma 2.4 and \([6, \text{Corollary 3.6}] \), we conclude that \( N_{1+J(\hat{A}_\gamma)}(\hat{D}) \) is a uniquely split \( \hat{\Gamma} \)-act group. \( \square \)

2.6. A proof of the existence of Theorem 1.8

Set \( \hat{A} = \hat{O}G = \hat{O} \otimes_O A \) and consider the source algebra \( \hat{A}_\gamma = \hat{O} \hat{G} i \) of the block \( \hat{O} G b \) with a defect pointed group \( P_\gamma \) where \( i \in \hat{\gamma} \). By \([7, \text{Theorem 15.10}] \) or \([8, \text{Theorem 1.8}] \), there exists a \( P \)-stable \( \hat{O} \)-algebra \( \hat{D} \) of \( \hat{A}_\gamma \) such that \( \hat{D} \cap Pi = Qi \) and \( \hat{A}_\gamma = \bigoplus_u \hat{D} u \) with \( u \) running on a set of representatives for \( P/Q \) in \( P \). By Lemma 2.2, there are an \( \hat{O} \subseteq \hat{O} \) which is an unramified finite Galois extension of \( O \) and a \( P \)-stable subalgebra \( \hat{D} \) of \( \hat{O} \otimes_O A_\gamma \) such that \( \hat{D} = \hat{O} \otimes_O \hat{D} \). In particular, we also have that

\[ \hat{D} \cap Pi = Qi \quad \text{and} \quad \hat{O} \otimes_O A_\gamma = \bigoplus_u \hat{D} u \]

with \( u \) running on a set of representatives for \( P/Q \) in \( P \). Let \( \hat{\Gamma} \) be the Galois group of \( \hat{O} \) over \( O \), then \( \hat{\Gamma} \) acts on \( \hat{O} \otimes_O A_\gamma \) in a natural way, and \( D = \hat{D} \hat{\Gamma} \) is the desired \( P \)-stable \( \hat{O} \)-algebra of \( A_\gamma \).

2.7. A proof of the uniqueness of Theorem 1.8

With notation as above, assume that both \( D \) and \( D' \) are two \( P \)-stable \( \hat{O} \)-subalgebras of \( A_\gamma \) fulfilling (1.8.1). Then (1.8.1) also holds in \( \hat{A}_\gamma \) for both \( \hat{O} \otimes_O D \) and \( \hat{O} \otimes_O D' \), i.e.,

\[ (\hat{O} \otimes_O D) \cap Pi = Qi \quad \text{and} \quad \hat{A}_\gamma = (\hat{O} \otimes_O D) \otimes_Q P; \]
\[ (\hat{O} \otimes_O D') \cap Pi = Qi \quad \text{and} \quad \hat{A}_\gamma = (\hat{O} \otimes_O D') \otimes_Q P. \]

By \([7, 14.7] \) or \([8, 1.8] \), there is an \( \tilde{a} \in 1 + J(\hat{A}_\gamma^P) \) such that \( (\hat{O} \otimes_O D)^{\tilde{a}} = (\hat{O} \otimes_O D')^{\tilde{a}} \). By Lemma 2.2, there are an \( \hat{O} \subseteq \hat{O} \) which is an unramified finite Galois extension of \( O \) and an \( \tilde{a} \in i + J((\hat{O} \otimes_O A_\gamma)^P) \) such that \( (\hat{O} \otimes_O D)^{\tilde{a}} = (\hat{O} \otimes_O D')^{\tilde{a}} \). Considering the action on
\(\tilde{\mathcal{O}} \otimes_{\mathcal{O}} A_{\gamma}\) of the Galois group \(\tilde{\Gamma}\) of \(\tilde{\mathcal{O}}\) over \(\mathcal{O}\), for any \(t \in \tilde{\Gamma}\) we have \((\tilde{\mathcal{O}} \otimes_{\mathcal{O}} D)^{t(y)} = \tilde{\mathcal{O}} \otimes_{\mathcal{O}} D^t\).

Set \(\Delta_y = \tilde{\mathcal{O}} \otimes_{\mathcal{O}} A_y\), \(\tilde{\mathcal{D}} = \tilde{\mathcal{O}} \otimes_{\mathcal{O}} D\), and \(\tilde{D}' = \tilde{\mathcal{O}} \otimes_{\mathcal{O}} D'\), and let \(S\) be the set of all the elements \(\bar{a}\) of \(1 + J(\tilde{A}_y^p)\) such that \(\tilde{D}^\bar{a} = \tilde{D}'\). Then it is easily checked that the group \(N_{1+J(\tilde{A}_y)}(\tilde{D}) \times \tilde{\Gamma}\) acts on the set \(S\) and \(N_{1+J(\tilde{A}_y)}(\tilde{D})\) acts regularly on \(S\). Since \(N_{1+J(\tilde{A}_y)}(\tilde{D})\) acts regularly on \(S\), the stabilizer of any element of \(S\) in \(N_{1+J(\tilde{A}_y)}(\tilde{D}) \times \tilde{\Gamma}\) is isomorphic to \(\tilde{\Gamma}\); then, by Lemma 2.5 and [6, 3.3], \(S^\tilde{\Gamma}\) is non-empty; moreover, there exists \(a \in (1 + J(\tilde{A}_y^p))\tilde{\Gamma} = 1 + J(\tilde{A}_y^p)\) such that \(\tilde{D}^a = \tilde{D}'\); consequently, \(D^a = (\tilde{D}^\bar{a})^{\tilde{\Gamma}} = (\tilde{D}')^{\tilde{\Gamma}} = D'\).

3. The local structure of hyperfocal subalgebras

3.1. Keep the notation in 1.2 throughout this section. First we recall a general notation, then turn to show some general properties of hyperfocal subalgebras.

Let \(H\) be a normal subgroup of \(G\). Assume that \(A\) is an \(H\)-interior \(G\)-algebra, i.e., \(A\) is a \(G\)-algebra with an \(H\)-interior \(G\)-algebra structure compatible with the \(G\)-action, cf. [6, 1.6]. Let \(K_{\gamma}\) and \(L_{\delta}\) be pointed groups on \(A\), and assume that \(K \subseteq H \cdot L\). Recall that a group epimorphism from \(K\) to \(L\) is an orbit on the set of the injective group homomorphisms from \(K\) to \(L\) under the natural action of the product \(\text{Int}(K) \times \text{Int}(L)\) of the inner automorphism groups \(\text{Int}(K)\) and \(\text{Int}(L)\) of \(K\) and of \(L\) respectively. We say that a group epimorphism determined by an injective group homomorphism \(\phi : K \rightarrow L\), fulfilling \(\phi(y) \in yH\) for all \(y \in K\), is an \(A\)-fusion from \(K_{\gamma}\) to \(L_{\delta}\) if, for some \(i \in \gamma\) and some \(j \in \delta\), there exists \(a \in A^*\) such that \(iai \subseteq (jA_j)^a\) and

\[(ai)^y = (y^{-1}\phi(y))ai \quad \text{and} \quad (ia^{-1})^y = ia^{-1}(y^{-1}\phi(y))^{-1}, \quad \forall y \in K.\]  

By \(F_A(K_{\gamma}, L_{\delta})\) we denote the set of the \(A\)-fusions from \(K_{\gamma}\) to \(L_{\delta}\), and write \(F_A(K_{\gamma})\) instead of \(F_A(K_{\gamma}, K_{\gamma})\). Further, suppose \(\phi(K) = L\) and let \(\Delta_{\phi}(K) = \{(\phi(x), x)\}_{x \in K}\) be a subgroup of \(L \times K\); then \(jA_i\) admits an \(\mathcal{O}\Delta_{\phi}(K)\)-module structure defined by \((\phi(x), x)a = \phi(x)ax^{-1}\) for any \(x \in K\) and \(a \in jA_i\). Note that, if \(\phi(K) = L\), \(\phi^{-1}\) also determines an \(A\)-fusion from \(L_{\delta}\) to \(K_{\gamma}\).

**Lemma 3.2.** With notation as above, a group isomorphism \(\phi : K \cong L\) such that \(\phi(x) \in xH\) for all \(x \in K\) determines an \(A\)-fusion from \(K_{\gamma}\) to \(L_{\delta}\) if and only if

\[(iA_j)^{A_{\phi^{-1}(L)}}(jA_i)^{\Delta_{\phi}(K)} = iA^K_i.\]  

**Proof.** The essential materials of the proof are from [7]. In any case, it is easily checked that the left side of the equality is contained in the right one and it is a two-sided ideal of the right one. If \(\bar{\phi} \in F_A(K_{\gamma}, L_{\delta})\) and \(a \in A^*\) fulfills equality (3.1.1), then \(ai\) and \(ia^{-1}\) belong to \((jA_i)^{\Delta_{\phi}(K)}\) and \((iA_j)^{\Delta_{\phi^{-1}(L)}}\) respectively, thus the equality (3.2.1) holds. Conversely, since \(iA^K_i\) is a local algebra, the equality (3.2.1) implies that we can choose
c \in (iA^j)^{A_{\phi^{-1}}(L)} \text{ and } d \in (jA^i)^{A_{\phi}(K)} \text{ such that } cd \text{ is invertible in } iA^ki; \text{ modifying our choice, we may assume that } cd = i; \text{ then } dc \text{ is a non-zero idempotent of } jA^j; \text{ hence } dc = j. \text{ In particular, } i \text{ and } j \text{ are conjugate in } A, \text{ i.e., } i = j^b \text{ for a } b \in A^*\text{. We claim that } a = d + (1 - j)b(1 - i) \text{ is invertible in } A \text{ and fulfills equality (3.1.1); indeed, it is easily checked that } c + (1 - i)b^{-1}(1 - j) \text{ is the inverse of } a, \text{ and, since } ai = d \text{ and } ia^{-1} = c, \text{ the equality follows from the fact that } \Delta_{\phi}(K) \text{ fixes } ai \text{ and } \Delta_{\phi^{-1}}(L) \text{ fixes } ia^{-1}. \quad \square

3.3. From now on we turn to the notation 1.4, 1.5, and 1.7, and always assume that \( D \) is a \( P \)-stable unitary subalgebra of \( A_{\gamma} \) fulfilling (1.8.1). Then \( A_{\gamma} \) is an interior \( P \)-algebra, while \( D \) is an \( O_{Q} \)-interior \( P \)-algebra. Note that \( \gamma \cap A_{\gamma} = \gamma \cap D = \{ i \} \text{, so } P_{[i]} \text{ is a local pointed group on both } A_{\gamma} \text{ and } D; \text{ we denote the both by } P_{\gamma} \text{ again for convenience. Further, we identify } Pi \subseteq A_{\gamma} \text{ with } P, \text{ and identify } u_{i} \in Pi \text{ with } u \in P \text{ for convenience.}

Let \( \phi : P \rightarrow P \) determine a \( D \)-fusion of \( P_{\gamma} \), i.e., \( \bar{\phi} \in FD(P_{\gamma}) \), and assume that \( a \in D \) makes (3.1.1) holds; then in \( A_{\gamma} \) (not in \( D \)) (3.1.1) is rewritten as \( a^{-1}ya = \phi(y), \forall y \in P \).

In other words,

\[
FD(P_{\gamma}) = ND^*(P) / (ND^*(P) \cap (A_{\gamma}^P)^*P). \tag{3.3.1}
\]

where \( ND^*(P) = \{ a \in D^* | P^a = P \}. \) On the other hand, it is known from [11, 2.13 and 3.1] that

\[
FA_{\gamma}(P_{\gamma}) = N_{A_{\gamma}^*}(P) / (A_{\gamma}^P)^*P = E_G(P_{\gamma}). \tag{3.3.2}
\]

Since it is shown in the end of 1.4 that \( (A_{\gamma}^P)^*/(i + J(A_{\gamma}^P)) \cong \hat{k} \), by [13, Chapter II, Proposition 8] we get

\[
(A_{\gamma}^P)^* \cong (i + J(A_{\gamma}^P)) \rtimes \hat{k}^*; \tag{3.3.3}
\]

with a suitable identification we regard \( \hat{k}^* \subseteq (A_{\gamma}^P)^* \text{ and } (A_{\gamma}^P)^* = (i + J(A_{\gamma}^P)) \rtimes \hat{k}^*\). And

\[
E_G(P_{\gamma}) = N_{A_{\gamma}^*}(P) / (i + J(A_{\gamma}^P))P \tag{3.3.4}
\]

is an extension of \( E_G(P_{\gamma}) \) by \( \hat{k}^* \), we call it a \( \hat{k}^* \)-group with \( \hat{k}^* \)-quotient \( E_G(P_{\gamma}) \).

Lemma 3.4. Notation as above. Then \( FD(P_{\gamma}) = FA_{\gamma}(P_{\gamma}) \).

Proof. The essential materials of the proof come from [7]. It is clear that \( FD(P_{\gamma}) \subseteq FA_{\gamma}(P_{\gamma}) \). Let \( \phi \in FD(P_{\gamma}) \) and \( \phi \) be a suitable representative of the \( A_{\gamma} \)-fusion. It follows from Lemma 3.2 that

\[
A_{\gamma}^{A_{\phi}(P)} A_{\gamma}^{A_{\phi^{-1}}(P)} = A_{\gamma}^{P},
\]

since \( P_{\gamma} \) is local, this equality implies that the \( k \)-linear map

\[
(A_{\gamma})(\Delta_{\phi}(P)) \otimes k (A_{\gamma})(\Delta_{\phi^{-1}}(P)) \rightarrow (A_{\gamma})(P)
\]
induced by the multiplication in $A_\gamma$ is surjective. Let $T$ be a set of representatives for $P/Q$ in $P$, and $U$ be the set of $t \in T$ such that $\phi(y)t^{-1}y^{-1} \in Qt^{-1}$ for any $y \in P$, we have

$$(A_\gamma)(\Delta_\phi(P)) = \bigoplus_{u \in U} (D \otimes_u u^{-1})(\Delta_\phi(P))$$

and

$$(A_\gamma)(\Delta_{\phi^{-1}}(P)) = \bigoplus_{u \in U} (D \otimes_u (\Delta_{\phi^{-1}}(P))).$$

Consequently, there exist $u, v \in U$ and $c, d \in D$ such that $\Delta_\phi(P)$ fixes $c \otimes u^{-1}$, $\Delta_{\phi^{-1}}(P)$ fixes $d \otimes v$ and the product $(c \otimes u^{-1})(d \otimes v)$ is invertible in $A_\gamma^P$; thus, modifying the choice of the second factor, we can assume that $v = u$ and $cd^u = i$. In particular, we get $\phi(Q) = Q$. Since $\Delta_\phi(P)$ fixes $c \otimes u^{-1}$ and $\Delta_{\phi^{-1}}(P)$ fixes $d \otimes v$, it is easily checked that $c \in D_{\Delta_\phi}(P)$ and $d^u \in D_{\Delta_{\phi^{-1}}}(P)$; then, since $cd^u = i$, we get

$$D_{\Delta_\phi}(P)D_{\Delta_{\phi^{-1}}}(P) = D_P.$$

Thus, by Lemma 3.2 again, we have that $\hat{\phi} \in F_D(P_\gamma)$. 

**Lemma 3.5.** $D_P/J(D_P) \cong A_\gamma^P/J(A_\gamma^P)$.

**Proof.** Assume that $T/Q = Z(P/Q)$ and $U$ is a set of the representatives of $T$ in $P$. Then

$$A_\gamma(P) = \bigoplus_{u \in U} (D \otimes_Q u)(P).$$

It is easy to check that $(D \otimes_Q u)(P)(D \otimes_Q v)(P) \subset (D \otimes_Q uv)(P)$ for any $u, v \in P$; i.e., $(A_\gamma)(P)$ is a $T$-graded $k$-algebra. Set $I = (A_\gamma)(P)J(D(P))(A_\gamma)(P)$. By the computation similar to the first and second paragraphs of the proof of [7, Lemma 7.3], we have that $J(D(P)) \subset J(A(P))$ and that $I$ is a $T$-graded proper ideal of $(A_\gamma)(P)$ with the $t$-component

$$I_t = \sum_{y \in T} ((A_\gamma)(P)J(D(P))(A_\gamma)(P))_{y-1},$$

thus $(A_\gamma)(P)/I$ is a $T$-graded $k$-algebra with 1-component isomorphic to $D_P/J(D_P)$. Set

$$T' = \{ t \in T \mid ((A_\gamma)(P)/I)_{t}((A_\gamma)(P)/I)_{t-1} = ((A_\gamma)(P))_{1} \},$$

then it is easily checked that $T'$ is a subgroup of $T$ (see [5, Lemma 8]), and by [5, Lemma 9], $\bigoplus_{t \in T'}((A_\gamma)(P)/I)_t$ is a crossed product and $\bigoplus_{t \in T'}((A_\gamma)(P)/I)_t$ is a nilpotent ideal of $(A_\gamma)(P)/I$. Since $D_P/J(D_P)$ is a perfect field, $\bigoplus_{t \in T'}((A_\gamma)(P)/I)_t$ is isomorphic to the group algebra of $T'$ over $D_P/J(D_P)$; thus $D_P/J(D_P) \cong A_\gamma^P/J(A_\gamma^P)$. 

$\square$
Remark. By the lemma and 1.4 and [13, Chapter II, Proposition 8], we can lift it to an algebra injection \( \hat{O} \to D^p \); on the other hand, a choice of the subgroup \( \hat{k}^* \) of \( (A^p_\gamma)^* \) in 3.3 also determines an algebra injection \( \hat{O} \to A^p_\gamma \). But by [3, Lemma 2.3], these two algebra injections are conjugate by \( i + J(A^p_\gamma) \); so with a suitable choice, we can assume that they coincide with each other. So, in the following we assume that \( \hat{O} \subseteq D^p \); and, since \( \hat{O}^* = (1 + J(\hat{O})) \times \hat{k}^* \), we have

\[
(D^p)^* = (i + J(D^p)) \times \hat{k}^*.
\]  

(3.5.1)

3.6. From now on, we further always assume that \( E_G(P_\gamma) \) is a \( p' \)-group.

Since \( N_G(P_\gamma) \) stabilizes both \( C_G(P) \) and the block \( b_\gamma \) of \( \hat{O}C_G(P) \), we see that \( E_G(P_\gamma) \) acts on \( \hat{O} \) by the equality (1.4.1). Then the actions of \( N_G(P_\gamma) \) on \( P \) and \( \hat{O} \) determine a group homomorphism

\[
E_G(P_\gamma) \to \tilde{\text{Aut}}(\hat{O}, \hat{O}P),
\]

(3.6.1)

where \( \text{Aut}(\hat{O}, \hat{O}P) \) denotes the group of the \( \hat{O} \)-semi-linear automorphisms of \( \hat{O}P \), and \( \tilde{\text{Aut}}(\hat{O}, \hat{O}P) \) denotes the quotient group of \( \text{Aut}(\hat{O}, \hat{O}P) \) by the inner automorphism group \( \text{Int}(\hat{O}P) \) of \( \hat{O}P \) induced by all the invertible elements of \( \hat{O}P \).

Because the kernel of the surjective homomorphism \( N_G(P)/C_G(P) \to E_G(P_\gamma) \) is a \( p \)-group, we can lift it to an injective group homomorphism \( E_G(P_\gamma) \to \text{Aut}(P) \). Thus, the actions of \( E_G(P_\gamma) \) on both \( P \) and \( \hat{O} \) determine a group homomorphism

\[
\theta : E_G(P_\gamma) \to \text{Aut}(\hat{O}, \hat{O}P)
\]

(3.6.2)

such that \( \theta(E_G(P_\gamma)) \) stabilizes both \( \hat{O} \) and \( P \), and for any \( \tilde{x} \in E_G(P_\gamma) \) there is a \( p' \)-element \( s \in N_G(P_\gamma) \) fulfilling

\[
\theta(\tilde{x})(u) = u^s, \quad \forall u \in P.
\]

(3.6.3)

In the following we fix such a group homomorphism \( \theta \) in (3.6.2); and note that by the definition 1.7 of \( Q \) and (3.6.3) we have the following conclusion:

\[
Q \text{ is stabilized by the } E_G(P_\gamma)\text{-action on } P \text{ through } \theta.
\]

(3.6.4)

We remark that the (3.6.1) can always be lifted to a unique \( \text{Int}(\hat{O}P) \)-conjugate class of homomorphisms \( E_G(P_\gamma) \to \text{Aut}(\hat{O}, \hat{O}P) \), but the lifting which stabilizes \( P \) may not exist if \( E_G(P_\gamma) \) is not a \( p' \)-group, cf [6, 1.14 and 1.15].

**Proposition 3.7.**

1. There is a subgroup \( \hat{E} \) of \( N_{A^p_\gamma}(P) \) such that \( \hat{E} \supseteq \hat{k}^* \) (recall \( \hat{k}^* \subseteq (D^p)^* \subseteq (A^p_\gamma)^* \), see (3.5.1)) and (3.3.4) induces an isomorphism \( \hat{E} \cong \hat{E}_G(P_\gamma)^* \); and all such subgroups of \( N_{A^p_\gamma}(P) \) are conjugate by \( N_{A^p_\gamma}(P \times \hat{k}^*) \cap (i + J(A^p_\gamma))P \).
(2) There is a subgroup $\hat{E}$ of $N_{D^*}(P)$ such that $\hat{E} \supseteq \hat{k}^*$ and (3.3.4) induces an isomorphism $\hat{E} \cong \hat{E}_G(P_\gamma)^\circ$; and all such subgroups of $N_{D^*}(P)$ are conjugate by $N_{D^*}(P \times \hat{k}^*) \cap (i + J(A^P_{\gamma})) P$.

**Proof.** (1) Denote by $V$ the centralizer of $\hat{k}^*$ in $J(A^P_{\gamma})$; it is clear that $V$ is an $O$-submodule of $J(A^P_{\gamma})$ satisfying that $V, V \subset V$, thus $i + V$ is a subgroup of $i + J(A^P_{\gamma})$. Then, by (3.3.2) and (3.3.3), we have $N_{A^P_{\gamma}}(P \times \hat{k}^*)/(i + V) \times \hat{k}^*) P \cong E_G(P_\gamma)$, thus we have a short exact sequence

$$1 \to (i + V) P \hat{k}^*/\hat{k}^* \xrightarrow{incl} N_{A^P_{\gamma}}(P \times \hat{k}^*)/\hat{k}^* \xrightarrow{\rho} E_G(P_\gamma) \to 1, \quad (3.7.1)$$

where “incl” is the inclusion map and $\rho$ is induced by (3.3.4). However, $P(i + V) \hat{k}^*/(i + V)\hat{k}^*$ is a finite $p'$-group and $E_G(P_\gamma)$ is a finite $p'$-group, $P(i + V) \hat{k}^*/(i + V)\hat{k}^*$ is a uniquely split $E_G(P_\gamma)$-acted group. On the other hand, since $i + V$ is equal to the subgroup of $y \in i + J(A^P_{\gamma})$ such that $\tilde{O}^\circ = \tilde{O}$, by [10, Lemma 4.10] and [6, Proposition 3.5], $i + V$ is a uniquely split $E_G(P_\gamma)$-acted group. Further, $P$ and $i + V$ centralize each other, by [6, 3.6] we have that $(i + V) P \hat{k}^*/\hat{k}^*$ is a uniquely split $E_G(P_\gamma)$-acted group. Therefore the sequence (3.7.1) is uniquely split, that is, there is a subgroup $\tilde{E}/\tilde{k}^*$ such that the restriction map $\rho|_{\tilde{k}^*}: \tilde{E}/\tilde{k}^* \to E_G(P_\gamma)$ is an isomorphism; and all such subgroups of $N_{A^P_{\gamma}}(P)/\hat{k}^*$ are conjugate to each other by $(i + V) P \hat{k}^*/\hat{k}^*$.

(2) By Lemma 3.4 we have $F_D(P_\gamma) = E_G(P_\gamma)$, thus by (3.3.1) we have an exact sequence

$$1 \to (N_{D^*}(P \times \hat{k}^*) \cap P(A^P_{\gamma})^*/\hat{k}^* \xrightarrow{incl} N_{D^*}(P \times \hat{k}^*)/\hat{k}^* \xrightarrow{\rho} E_G(P_\gamma) \to 1. \quad (3.7.2)$$

Set $W = V \cap J(D^P)$; then it is clear that $W$ is the centralizer of $\hat{k}^*$ in $J(D^P)$ and that $W$ is an $O$-submodule of $J(D^P)$ such that $W, W \subset W$, thus $i + W$ is a subgroup of $i + J(D^P)$. Then similar to the proof below (3.7.1), we also can obtain that

$$N_{D^*}(P \times \hat{k}^*)/i + J(A^P_{\gamma}) P \text{ is a uniquely split } E_G(P_\gamma)-\text{acted group}. \quad (3.7.3)$$

thus we get the conclusions of (2). □

**Remark.** Recall that $D$ is $P$-stable, from the proposition we have the following conclusion:

*If $\hat{k}^* \subseteq \hat{E} \subseteq A^P_{\gamma}$ such that (3.3.4) induces an isomorphism $\hat{E} \cong \hat{E}_G(P_\gamma)$, then there is an $a \in i + J(A^P_{\gamma})$ such that $\hat{E} \subseteq D^a$. \quad (3.7.4)*

3.8. Now we follow the idea of [6, §4] to choose $\hat{i}$ and $\hat{b}$ in 3.5 suitably. Let $\hat{j}$ be the primitive idempotent of $\hat{O} \otimes_O \hat{O}$ which is mapped non-zero by the homomorphism
\[ \hat{O} \otimes \hat{O} \to \hat{O}, \lambda \otimes \mu \mapsto \lambda \mu. \]

By [6, Proposition 4.10], there exists an injective unitary homomorphism from \( \hat{O} \) to \( A_\gamma \), hence we have an injective homomorphism

\[ \hat{O} \otimes \hat{O} \to \hat{O} \otimes A_\gamma. \tag{3.8.1} \]

By [6, 4.13.2], \( \hat{j} \) determines a primitive idempotent \( \hat{i} \) of \( A_\gamma \) through the above homomorphism (3.8.1), and there exists a unique local point \( \hat{\gamma} \) of \( P \) on \( \hat{O} \hat{G} \) such that \( \hat{\gamma} \in \hat{\gamma} \). Let \( \hat{b} \) be the \( \hat{O} \)-block of \( G \) such that \( b \hat{\gamma} = \hat{\gamma} \); by [6, 2.13.5], \( P \hat{\gamma} \) is a defect pointed group of \( G \{ \hat{b} \} \).

Set \( \hat{A}_\gamma = \hat{G} \hat{A}_\gamma \hat{G} \); then \( \hat{A}_\gamma \) is a source algebra of \( \hat{O} \hat{G} \). Then, by [6, 1.19.1], the usual trace map \( \text{Tr}_1 \) on \( \hat{O} \otimes A_\gamma \) induces a \( \hat{k} \)-group homomorphism \( \hat{E}_G(P \hat{\gamma})^\circ \to \hat{E}_G(P \gamma)^\circ \) which is a lifting of the inclusion map \( E_G(P \gamma) \subset E_G(P \gamma) \). Thus by [6, 1.20], \( \hat{A}_\gamma \) admits an \( \hat{O} \hat{E}_G(P \gamma)^\circ \)-interior \( \hat{E}_G(P \gamma)^\circ \)-algebra structure, unique up to \( \hat{A}_\gamma \)-conjugation, such that the action of \( \hat{E}_G(P \gamma)^\circ \) stabilizes the image of \( \hat{O} \hat{P} \) and induces the group homomorphism (3.6.2); and there exists an \( OP \)-interior algebra isomorphism

\[ \eta: A_\gamma \cong \hat{A}_\gamma \otimes \hat{E}_G(P \gamma)^\circ \hat{E}_G(P \gamma)^\circ. \tag{3.8.2} \]

Moreover, by our choice of the group homomorphism (3.6.2), \( \hat{E}_G(P \gamma)^\circ \) stabilizes \( P \) and \( \hat{A}_\gamma \) also admits an \( \hat{O}\hat{E}_G(P \gamma)^\circ \)-interior \( P \times \hat{E}_G(P \gamma)^\circ \)-algebra structure, which extends the usual interior \( \hat{O} \hat{P} \)-algebra structure on \( \hat{A}_\gamma \); and the isomorphism (3.8.2) becomes an \( O \hat{P} \)-interior algebra isomorphism. In particular, \( \eta^{-1} \) induces an injection

\[ P \times \hat{E}_G(P \gamma)^\circ \to A_\gamma^\circ. \tag{3.8.3} \]

**Theorem 3.9.** Notation as above. If \( D \) is a hyperfocal subalgebra of \( A_\gamma \) (i.e., (1.8.1) holds for \( A_\gamma \) and \( D \)), then there are an \( a \in i + J((A_\gamma^\circ)) \), and a hyperfocal subalgebra \( \hat{D} \) of \( \hat{A}_\gamma \) (i.e., (1.8.1) holds for \( \hat{A}_\gamma \) and \( \hat{D} \)) which inherits from \( \hat{A}_\gamma \) an \( \hat{O} \hat{E}_G(P \gamma)^\circ \)-interior \( \hat{E}_G(P \gamma)^\circ \)-algebra structure, and an \( O \hat{Q} \)-interior \( P \)-algebra isomorphism \( \eta': D^a \cong \hat{D} \otimes \hat{E}_G(P \gamma)^\circ \hat{E}_G(P \gamma)^\circ \) such that the following diagram is commutative:

\[ \begin{array}{ccc}
A_\gamma & \xrightarrow{\eta} & \hat{A}_\gamma \otimes \hat{E}_G(P \gamma)^\circ \hat{E}_G(P \gamma)^\circ \\
\text{incl} & & \text{incl} \otimes \text{id}
\end{array} \]

\[ \begin{array}{ccc}
D^a & \xrightarrow{\eta'} & \hat{D} \otimes \hat{E}_G(P \gamma)^\circ \hat{E}_G(P \gamma)^\circ \\
\text{incl} & & \text{incl} \otimes \text{id}
\end{array} \]

where “incl” and “id” denote the inclusion map and the identity map, respectively.
Proof. We trace the construction of the isomorphism (3.8.2) in [6, 4.11–4.14].

Obviously the subgroup \(^\hat{k}^*\) of \(\hat{E}_G(P_\gamma)\) determines a subgroup \(\hat{k}^*\) of \((A_{P_\gamma}^\circ)^*\) through the isomorphism (3.8.2); now we fix the later subgroup \(\hat{k}^*\). By [3, Lemma 2.3], we can assume without loss of the generality that \(D\) contains \(\hat{k}^*\), thus by (3.7.4), we also can assume that \(D\) contains the image of \(\hat{E}_G(P_\gamma)^\circ\) in \(A_\gamma\) and the homomorphism from \(\hat{O}\) to \(A_\gamma\) induces an injective unitary homomorphism of \(\hat{E}_G(P_\gamma)^\circ\)-algebras from \(\hat{O}\) to \(D\).

Let \(\Gamma\) be the Galois group of \(\hat{O}\) over \(O\). We can regard \(\hat{O} \otimes_O D\) as an \(\hat{O} \otimes_O \hat{O}\)-interior \(\Gamma \times \hat{E}_G(P_\gamma)^\circ\)-algebra (cf. [6, 1.6]). The formula (3.8.1) can be rewritten as

\[
\hat{O} \otimes_O \hat{O} \to \hat{O} \otimes_O D \subset \hat{O} \otimes_O A_\gamma,
\]

which is a homomorphism of \(\Gamma \times \hat{E}_G(P_\gamma)^\circ\)-algebras over \(\hat{O}\).

Let \(\hat{J}\) be the set of primitive idempotents of \(\hat{O} \otimes_O \hat{O}\), and \(\hat{j}\) be the element of \(\hat{J}\) which does not vanish through the product map \(\hat{O} \otimes_O \hat{O} \to \hat{O}\). Through (3.9.2), by \(\hat{i}\) and \(\hat{i}'\) we denote the image of \(\hat{j}\) in \(\hat{O} \otimes_O A_\gamma^\circ\) respectively. Since the group \(\Gamma \times \hat{E}_G(P_\gamma)^\circ\) stabilizes on \(\hat{I}\), it also stabilizes \(\hat{J}\). And both \(\hat{j}\) and \(\hat{i}'\) have the same stabilizer, denoted by \(\hat{H}\), in \(\Gamma \times \hat{E}_G(P_\gamma)^\circ\). Since \(\Gamma\) acts regularly on \(\hat{I}\) and \(\hat{J}\), the second projection map

\[
\hat{H} \to \hat{E}_G(P_\gamma)^\circ
\]

induces a group homomorphism

\[
\varphi : \hat{H} \to \hat{E}_G(P_\gamma)^\circ.
\]

Thus there is a suitable group homomorphism \(\hat{\tau} : \hat{E}_G(P_\gamma)^\circ \to \Gamma\) such that

\[
\hat{H} = \left\{ (\hat{\tau}(\hat{x}), \hat{x}) \mid \hat{x} \in \hat{E}_G(P_\gamma)^\circ \right\}.
\]

It is easily checked that in \(\hat{O} \otimes_O \hat{O}\) the action of \(E_G(P_\gamma)\) on \(\hat{O}\) induced by \(\hat{\tau}\) coincides with the action of \(\hat{E}_G(P_\gamma)^\circ\) in (3.8.2) (cf. [6, 4.12]), so the stabilizer of \(\hat{j}\) and \(\hat{i}'\) in \(\hat{E}_G(P_\gamma)^\circ\) (identified with \(1 \times \hat{E}_G(P_\gamma)^\circ\)) coincides with the converse image \(\hat{K} \subseteq \hat{E}_G(P_\gamma)^\circ\) of the kernel \(K\) of the homomorphism \(E_G(P_\gamma) \to \text{Aut}_O(\hat{O})\).

Considering the corresponding action of \(\Gamma\) on \(\hat{O} = \hat{O} \otimes_O OG\), by [6, 4.13.2], we have that \(\hat{i}\) belongs to a local point \(\hat{y}\) of \(P\) on \(\hat{O}G\), and \(\hat{y}'' \neq \hat{y}\) for any nontrivial element \(\sigma\) of \(\Gamma\). In particular, \(E_G(P_\gamma) = K\). Let \(\hat{a} = [\hat{b}]\) be the point of \(G\) on \(\hat{O}G\) such that \(P_\gamma \subset G_{\hat{a}}\); similarly to [6, 4.13.4], we have

\[
The\ stabilizer\ \Gamma^{\hat{a}}\ of\ \hat{a}\ in\ \Gamma\ coincides\ with\ the\ image\ of\ E_G(P_\gamma)\ in\ \Gamma;
\]

and \(\text{Tr}_{\hat{1}}^{\hat{a}}(\hat{i})\ belongs\ to\ Z(\hat{O} \otimes_O D)^\circ\).

\[
(3.9.4)
\]
It is similar to [6, 4.14] that \( \hat{D} = \hat{i}(\hat{\mathcal{O}} \otimes \mathcal{O} \hat{D}) \hat{i} \) inherits from \( \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \) the \( \hat{\mathcal{O}} \hat{H} \)-algebra structure, and \( \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \hat{b} \) inherits the \( \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-interior \( \Gamma^\sigma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-algebra structure. Hence the characterization [6, 2.7.4] applies to \( \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \hat{b} \) in \( \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \); whereas, since \( \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-interior \( \Gamma^\sigma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-algebra structure. Hence the characterization [6, 2.7.4] applies to \( \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \). Similar to the isomorphism [6, 4.14.1] which is written as \( \zeta \) in the first row of diagram (3.9.5) below, we get an \( \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-interior \( \Gamma^\sigma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma \)-algebra isomorphism \( \zeta' \) shown in the second row of the diagram

\[
\hat{\mathcal{O}} \otimes \mathcal{O} A_{\gamma} \xrightarrow{\cong} \hat{\mathcal{O}}(\Gamma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma) \otimes_{\hat{\mathcal{O}}(\Gamma^\sigma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma)} \text{Ind}_{\hat{\mathcal{K}}}^{\hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma} (\hat{A}_{\gamma})
\]

\[
\hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \xrightarrow{\cong} \hat{\mathcal{O}}(\Gamma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma) \otimes_{\hat{\mathcal{O}}(\Gamma^\sigma \times \hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma)} \text{Ind}_{\hat{\mathcal{K}}}^{\hat{\mathcal{O}} \hat{E}_G(\mathcal{P}_\gamma)^\sigma} (\hat{D})
\]

and \( \zeta'(\hat{d}) = 1 \otimes (1 \otimes \hat{d} \otimes 1) \) for \( \hat{d} \in \hat{D} = \hat{i}(\hat{\mathcal{O}} \otimes \mathcal{O} \hat{D}) \hat{i} \subseteq \hat{\mathcal{O}} \otimes \mathcal{O} \hat{D} \). Comparing with [6, 4.14], we see that the diagram (3.9.5) is commutative.

Since \( (\hat{\mathcal{O}} \otimes \mathcal{O} A_{\gamma})^\Gamma = A_{\gamma} \) and \( (\hat{\mathcal{O}} \otimes \mathcal{O} \hat{D})^\Gamma = \hat{D} \), by [6, 2.8 and 2.10] we have the following isomorphism (\( \psi \) is the isomorphism (3.9.3)):

\[
\text{Res}_{\psi}(A_{\gamma}) \cong \hat{A}_{\gamma} \otimes_{\hat{\mathcal{K}}} \hat{H} \quad \text{and} \quad \text{Res}_{\psi}(\hat{D}) \cong \hat{D} \otimes_{\hat{\mathcal{K}}} \hat{H}
\]

where the first one is just [6, 4.14.2] and the second one is compatible with the first one. In addition, it is not difficult to check that \( \hat{D} = \hat{i}(\hat{\mathcal{O}} \otimes \mathcal{O} \hat{D}) \hat{i} \) is a \( \hat{\mathcal{O}} \)-stable \( \hat{\mathcal{O}} \)-subalgebra of \( \hat{A}_{\gamma} \) satisfying that

\[
\hat{D} \otimes \mathcal{O} \hat{P} = \hat{A}_{\gamma} \quad \text{and} \quad \hat{D} \cap \hat{P} = \hat{Q}. \]

In a word, taking the \( \Gamma \)-fixed algebras of the terms of the diagram (3.9.5), we get the desired commutative diagram (3.9.1). □

4. Hyperfocal subalgebras in the case that \( \mathcal{O} < \hat{\mathcal{O}} \)

4.1. Throughout this section we keep the notation in 1.4, 1.5 and 1.7, and always assume that \( \hat{\mathcal{E}}_G(\mathcal{P}_\gamma) \) is a \( \hat{\mathcal{O}} \)-group, and fix the choice of \( \theta \) in (3.6.2) and \( \hat{i} \), \( \hat{b} \) in 3.8. In particular, in (3.8.2) we have the isomorphism

\[
\eta: A_{\gamma} \xrightarrow{\cong} \hat{A}_{\gamma} \otimes_{\hat{\mathcal{E}}_G(\mathcal{P}_\gamma)} \hat{\mathcal{E}}_G(\mathcal{P}_\gamma)^\sigma.
\]

Lemma 4.2. Notation as above. Then there is a \( \hat{\mathcal{O}} \times \hat{\mathcal{E}}_G(\mathcal{P}_\gamma)^\sigma \)-stable subalgebra \( \hat{D} \) of \( \hat{A}_{\gamma} \) such that

\[
\hat{D} \cap \hat{P} = \hat{Q} \quad \text{and} \quad \hat{D} \otimes \mathcal{O} \hat{P} = \hat{A}_{\gamma},
\]
and any two such subalgebras are conjugate by \( \hat{i} + J(\hat{A}_\gamma^o) \). Moreover, such a subalgebra \( \hat{D} \) contains the image of \( \hat{E}_G(P_\gamma)^0 \) in \( \hat{A}_\gamma \).

**Proof.** Since we have proved in Section 2 that Theorem 1.8 holds for \( \hat{A}_\gamma \), there exists a \( P \)-stable \( \hat{O} \)-subalgebra \( \hat{D} \) satisfying (4.2.1), and \( \hat{i} + J(\hat{A}_\gamma^o) \) acts transitively on the set \( \hat{D} \) of all the \( P \)-stable \( \hat{O} \)-subalgebras \( \hat{D} \) satisfying (4.2.1). By (3.6.4), \( \hat{E}_G(P_\gamma)^0 \) not only stabilizes \( P \), and stabilizes \( Q \) as well; so \( \hat{E}_G(P_\gamma) \) also acts on \( \hat{D} \). Thus \( (\hat{i} + J(\hat{A}_\gamma^o))\hat{E}_G(P_\gamma) \) acts on \( \hat{D} \). For any \( \hat{D} \in \hat{D} \), by Lemma 2.4, we have

\[
N_{i + J(\hat{A}_\gamma^o)}(\hat{D}) = (\hat{i} + J(Z(\hat{A}_\gamma)))(\hat{i} + J(\hat{D}^o)),
\]

which is a \( E_G(P_\gamma) \)-acted group. By [6, 4.3 and 3.11], \( N_{i + J(\hat{A}_\gamma^o)}(\hat{D}) \) is a uniquely split \( E_G(P_\gamma) \)-acted group; moreover by [10, 4.6], \( \hat{i} + J(\hat{A}_\gamma^o) \) is a uniquely split \( E_G(P_\gamma) \)-acted group. So, by [6, 3.3], \( \hat{D}E_G(P_\gamma) \) is nonempty and \( (\hat{i} + J(\hat{A}_\gamma^o))E_G(P_\gamma) \) acts transitively on \( \hat{D}E_G(P_\gamma) \).

Let \( \hat{D} \) be a \( \mathcal{P} \times \hat{E}_G(P_\gamma)^0 \)-stable \( \hat{O} \)-subalgebra of \( \hat{A}_\gamma \) such that (4.2.1) holds. Then Proposition 3.7 applies to the case \( \mathcal{O} = \hat{O} \), and we get a subgroup \( \hat{F} \) of \( D^* \) such that \( \hat{k}^* \subseteq \hat{F} \subseteq N_{D^r}(P_\gamma) \) and \( \hat{F} \cong \hat{E}_G(P_\gamma)^o \). Let \( \hat{F} \) be the set of all such subgroups \( \hat{F} \) of \( D^* \), then \( N_{D^r}(P_\gamma) \cap (\hat{i} + J(\hat{A}_\gamma^o))P_\gamma \) acts by conjugation on \( \hat{F} \) transitively. Hence \( (\hat{i} + J(\hat{A}_\gamma^o))P_\gamma \) acts on \( \hat{F} \) transitively. However, by (3.7.3), \( N_{D_\gamma}(P_\gamma) \cap (\hat{i} + J(\hat{A}_\gamma^o))P_\gamma \) is a uniquely split \( E_G(P_\gamma) \)-acted group; hence, by [6, 3.3], \( \hat{F}E_G(P_\gamma)^0 \neq \emptyset \). That is, \( E_G(P_\gamma) \) stabilizes a subgroup \( F \) of \( N_{D^r}(\hat{i}) \) with a group isomorphism \( \sigma : E_G(P_\gamma)^0 \cong F \).

For convenience, we identify the image of \( \hat{E}_G(P_\gamma)^0 \) in \( \hat{A}_\gamma^o \) with \( \hat{E}_G(P_\gamma)^0 \). Then it is easily checked that the set \( \{ \sigma(\hat{\gamma})\hat{x}^{-1} \mid \hat{x} \in \hat{E}_G(P_\gamma)^0 \} \) is a \( \mathcal{P} \)-subgroup of \( \hat{A}_\gamma^o \); however, \( (\hat{A}_\gamma^o)^* \cong \hat{k}^* \times J(\hat{A}_\gamma^o) \). By [13, Chapter II, Proposition 8] and \( \hat{i} + J(\hat{A}_\gamma^o) \) is a \( \mathcal{P} \)-divisible group, \( \{ \sigma(\hat{\gamma})\hat{x}^{-1} \mid \hat{x} \in \hat{E}_G(P_\gamma)^0 \} \subset \hat{k}^* \). That is, we have proved the equality \( F = \hat{E}_G(P_\gamma)^0 \). \( \square \)

4.3. A proof of the existence of Theorem 1.8

By Lemma 4.2, there exists a \( P \)-stable \( \hat{O} \)-subalgebra \( \hat{D} \) of \( \hat{A}_\gamma \) which satisfies (4.2.1) and contains the image of \( \hat{E}_G(P_\gamma)^0 \) in \( \hat{A}_\gamma \) and is stabilized by \( \hat{E}_G(P_\gamma)^0 \). Then we have the following \( \mathcal{P} \times \hat{E}_G(P_\gamma)^0 \)-interior algebra isomorphisms

\[
A_\gamma \cong \hat{A}_\gamma \otimes \hat{E}_G(P_\gamma)^0 \hat{E}_G(P_\gamma)^0
\]

\[
\cong \hat{A}_\gamma \otimes_{\mathcal{P} \times \hat{E}_G(P_\gamma)^0} (\mathcal{P} \times \hat{E}_G(P_\gamma)^0) \]
\[
\cong (\hat{\mathcal{D}} \otimes Q P) \otimes_{P \times \hat{\mathfrak{E}}_G(P_\gamma)^0} (P \times \hat{\mathfrak{E}}_G(P_\gamma)^0)
\]
\[
\cong (\hat{\mathcal{D}} \otimes Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0) (P \times \hat{\mathfrak{E}}_G(P_\gamma)^0) \otimes_{P \times \hat{\mathfrak{E}}_G(P_\gamma)^0} (P \times \hat{\mathfrak{E}}_G(P_\gamma)^0)
\]
\[
\cong (\hat{\mathcal{D}} \otimes Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0) (Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0) \otimes_{Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0} (P \times \hat{\mathfrak{E}}_G(P_\gamma)^0)
\]
\[
\cong (\hat{\mathcal{D}} \otimes Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0 \hat{\mathfrak{E}}_G(P_\gamma)^0) \otimes_{Q \times \hat{\mathfrak{E}}_G(P_\gamma)^0} (P \times \hat{\mathfrak{E}}_G(P_\gamma)^0)
\]

Thus, set \(D\) to be the image in \(A_\gamma\) of the crossed product \(\hat{\mathcal{D}} \otimes \hat{\mathfrak{E}}_G(P_\gamma)^0\) through the isomorphism (4.1.1); then \(D\) is a \(P\)-stable unitary \(O\)-subalgebra \(D\) of \(A_\gamma\) and satisfies the condition

\[
D \cap P_i = Q_i \quad \text{and} \quad D \otimes_Q P = A_\gamma.
\]

4.4. A proof of the uniqueness of Theorem 1.8

Let \(D\) be as above, and assume that \(D'\) is also a \(P\)-stable \(O\)-subalgebra of \(A_\gamma\) which satisfies

\[
D' \cap P_i = Q_i \quad \text{and} \quad D' \otimes_Q P = A_\gamma.
\]

By Theorem 3.9, there are an \(a \in i + J(A_\gamma^0)\) and a hyperfocal subalgebra \(\hat{D}'\) of \(\hat{A}_\gamma\) such that \(D''\) is the image in \(A_\gamma\) of \(\hat{D}' \otimes \hat{\mathfrak{E}}_G(P_\gamma)^0\) through the isomorphism (4.1.1). Since it is proved in Section 2 that Theorem 1.8 holds for \(\hat{A}_\gamma\), there is an \(\hat{a} \in \hat{i} + J(\hat{A}_\gamma^0)\hat{\mathfrak{E}}_G(P_\gamma)^0\) such that \(\hat{D}' \hat{a} = \hat{D}\); therefore, there exists \(a' \in i + J(A_\gamma^0)\) such that \(D''\) is the image of \(\hat{D} \otimes \hat{\mathfrak{E}}_G(P_\gamma)^0 \hat{\mathfrak{E}}_G(P_\gamma)^0\) through the isomorphism (4.1.1); that is, \(D''a = D\).

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References


