Influence of geometrical defects on the mechanical behaviour of hollow-sphere structures

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ABSTRACT

Hollow-sphere structures could represent an alternative to classical cellular materials, such as metal foams or honeycombs, for various structural applications; such stainless steel random structures are already on the market. One advantage of hollow-sphere structures unlike metal foams ensues from the possibility to stack the spheres regularly, even if in the literature there are only examples of limited size regular stackings for the moment. Higher mechanical properties than those of random cellular structures are expected for such regular structures according to modelling studies. Nevertheless, because of the difficulty in processing perfect regular stackings, it seems to be critical to study the influence of architectural defects on the overall mechanical behaviour of these cellular structures. Emphasis is on geometrical defects by introducing some dispersions on the sphere thickness and the meniscus size. Influences of both the magnitude of dispersion and the distribution of the defects on the mechanical behaviour of hollow-sphere structure are investigated. Especially, collapse mechanisms resulting from plasticity and their inhomogeneous localisation in the structure are studied in details. The case of periodic defects is addressed too in order to compare the mechanical response of infinite stackings to that of finite ones. This work highlights the significant influence of the defects on the effective mechanical behaviour of hollow-sphere structures. Most of the time, geometrical dispersion and defects are detrimental for the stacking behaviour, especially when understructures made of the defective hollow spheres or menisci are observed.

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1. Introduction

For many years cellular metallic hollow-sphere structures (HSSs), or stackings, have been studied because of their interest for aeronautical applications. Similarly to metal foams or honeycombs, high specific mechanical properties and multi-functional materials are expected (Evans et al., 1997; Öchsner and Augustin, 2009), being useful to develop lightweight aeronautical frames. Another main advantage of cellular materials is their ability to dissipate mechanical energy thanks to the plastic collapse of their constitutive cells, hence HSSs could be very interesting in view of impact resistance applications. Nevertheless, according to Daxner et al. (1999) who carried out a very original and rich study about the influence of mesoscopic inhomogeneities on the energy absorption capabilities of metal foams, the more homogeneous the deformation in the foam, the more energy is dissipated. Indeed, due to their processing route, metal foams show a strong heterogeneity of their architecture, in terms of local density and local strength, resulting in an inhomogeneous collapse at the mesoscopic scale of several constitutive cells. Regular HSSs, more than being rather convenient for modelling, could be promising solutions to tackle with this issue. However, the processing of such large perfect regular HSSs seems to be impossible, or at least difficult, to reach. Consequently, the issue of the influence of architectural defects on the effective behaviour of HSSs has to be discussed.

Friedl et al. (2008) performed a very interesting experimental characterisation on 316L stainless steel HSSs elaborated by powder metallurgy for both compressive and tensile loads. Their results provide many pieces of information on the mechanisms that govern the mechanical response of HSSs but also on the sensitivity of their behaviour to different constitutive parameters, like their relative density or the sphere thickness or radius. Caty et al. (2008a,b) also furnished significant results on the same materials, especially in fatigue (Caty et al., 2008a, 2009). They have characterised both the morphology of HSSs and their collapse mechanisms, using X-ray tomography. Fallet et al. (2008) and Lhuissier et al. (2009) also used X-ray tomography to characterise damage mechanisms which govern the collapse of these 316L HSSs under quasi-static compression. Two complementary mechanisms were...
observed: the crushing of the hollow spheres and the creation of new contacts between neighbouring spheres. Different relative densities and geometries, varying both the sphere thickness and the meniscus size (i.e., the ‘meniscus’ defines the joint between two neighbouring spheres), were tested and simulated using the finite-element method (FEM) in order to discuss their influence on the overall mechanical behaviour of HSSs. Thanks to the FEM too, Fiedler et al. (2007) studied adhesively bonded HSSs modelling a bimaterial, in order to characterise the contributions of both the hollow spheres and the adhesive. The model proposed was able to appreciate a densification plateau assuming that the behaviours of the two constitutive materials were elastic perfectly plastic. By varying the geometry of the constitutive spheres and the mechanical properties of the constitutive materials, they also predicted an increase of the effective strength of HSSs with that of adhesive bonds or the sphere thickness (Fiedler et al., 2006, 2010). They also highlighted the disymmetric behaviour of HSSs under tensile and compressive loads (Fiedler et al., 2010). Fiedler and Öchsner (2007) performed a complementary study to characterise the anisotropy of such adhesively bonded HSSs using the FEM and varying the loading direction. According to the authors, the behaviour of a simple cubic-like stacking is close to an isotropic one. As regards loading direction, according to the authors, the behaviour of a simple cubic-like stacking is close to an isotropic one. As regards loading direction, Marcadon and Feyel (2009) recently addressed the issue of the influence of the constitutive material behaviour (i.e., the material of which the stacking walls consist of) on the effective mechanical properties of HSSs. Particularly, they have shown the strong contribution of the localised plasticity on the collapse mechanisms of HSSs, hence on their mechanical response. An accurate knowledge of the in situ constitutive mechanical properties, especially the plastic ones, is a critical issue to propose a relevant modelling of HSSs (Marcadon and coworkers).

Some studies carried out on metal foams also furnish meaningful results to understand the mechanical behaviour of HSSs. The mechanical behaviour of metal foams is governed by the localised plasticity and the collapse of their constitutive cells too, see for instance the work of Brothers and Dunand (2005) on zirconium foams or the ones of Paul and Ramamurty (2000) and Ruan et al. (2002) on aluminium foams. All these studies also agree with the fact that several elastic and plastic mechanical properties of foams strongly vary with their density according to a power law; the higher the density, the higher the mechanical properties. Gibson and Ashby (1982, 1997) proposed an analytical model to predict the mechanical properties (Young's modulus and yield stress essentially) of a foam thanks to the knowledge of both its constitutive material mechanical properties and the densities. This model, developed for both open- and close-cell foams, assumed that foam edges were loaded under pure bending. A similar approach has been proposed by Hodge and Dunand (2003) assuming that foam edges work in compression. Konstantinidis et al. (2005) also proposed a FEM-based model to characterise the influence of constitutive cells shape on mechanical properties of close-cell foams. Various approaches based on the FEM were developed in order to study the influence of both the microstructure and the presence of defects on the effective strength of metal foams, and in a more general point of view of cellular materials. Simone and Gibson (1998) proposed a 3D modelling of the influence of cell walls curvature and corrugations on the effective behaviour of metal foams. They have shown that Young's modulus and the yield stress of the foam significantly falls when either the wall curvature or the frequency of the corrugations increase. But most of these approaches used 2D modelling of planar cellular architectures and were based on a Voronoi algorithm to create cellular patterns of the plane. Such a methodology suits rather for honeycombs than for metal foams. By this way, some interesting results have been provided by Andrews and Gibson (2001) as concerns the influence of notches and cavities in the edges of open-cell foams on their effective strength. Similar approaches have been developed by Silva and Gibson (1997) to study random cellular structures and to characterise damage localisation, but also by Fazekas et al. (2002) and Adjari et al. (2008) to compare the mechanical responses of regular and random cellular structures, and to deal with the issues of missing edges and holes. These works have put in evidence a strong sensitivity of metal foams mechanical behaviour to the presence of such defects. On the contrary, as concerns HSSs, there is a lack of data in the literature about this issue.

In the present work, the influence of architectural defects on the overall behaviour of HSSs is investigated performing FE-calculations on different simple cubic (SC)-like stackings, consisting of 27 hollow spheres linked to each others by solid menisci. After the description of the modelling assumptions in Section 2, Section 3 discusses first the influence of the spatial distribution of the defects on the effective behaviour of HSSs. Whereas the mean sphere thickness and the mean meniscus radius, computed on the unit cells, are kept constant, some geometrical dispersions are introduced varying either the thickness of several hollow spheres or the radius of several menisci in the stacking. Different distributions of the thinner and the thicker hollow spheres, or the smaller and the larger menisci, are considered. Then, Section 4 is dedicated to the comparison between the mechanical responses of infinite and infinite stackings by considering only a central defect in the 3-sphere side cells. Various cases are studied: a thicker or a thinner central sphere, a missing one, and a smaller or a larger central meniscus. The issue of the magnitude of dispersion is also addressed all along this work. One specificity of the present work comes from the fact that, thanks to the modelling of regular and perfect structures, we are able to investigate the effects of geometrical dispersion and defects on the behaviour of HSSs independently of other effects, such as stacking faults or randomness observed in the aforementioned experimental studies on real stackings (Caty et al., 2008a,b; Fallet et al., 2008; Friedel et al., 2008; Lhuissier et al., 2009). Furthermore, unlike the modelling approaches proposed previously to deal with the issue of geometrical defects (Silva and Gibson, 1997; Simone and Gibson, 1998; Andrews and Gibson, 2001; Fazekas et al., 2002; Adjari et al., 2008), and based on 2D Voronoi cells with straight edges modelled with beam elements, here we consider 3D cells with curved edges. Hence, whereas the collapse mechanisms were mainly unstable ones and buckling in the case of the 2D Voronoi cell-based modelling, in our case collapse mechanisms are stable and based on the localised plasticity induced by the curvature changes near meniscus extremities.

2. Modelling assumptions

2.1. Constitutive material behaviour

The constitutive material was assumed to be elasto-plastic and homogeneous in both the menisci and the hollow spheres. The strain tensor \( \varepsilon = \varepsilon_e + \varepsilon_p \) was thus the sum of an elastic strain \( \varepsilon_e \) and a plastic strain \( \varepsilon_p \).
and a plastic one \( \varepsilon_p \). Elasticity was linear isotropic and governed according to Hooke’s law:

\[
\sigma = \frac{E_m}{1 + \frac{r_m}{v_m}} \left( \varepsilon - \frac{v_m}{1 - 2v_m} \text{tr}(\varepsilon) \right)
\]

(1)

where \( \varepsilon \) and \( \dot{\varepsilon} \) denote the stress tensor and the second-order identity tensor, respectively. \( E_m \) and \( v_m \) are Young’s modulus and Poisson’s ratio of the constitutive material.

The plastic hardening was linear isotropic and a von Mises criterion was used to define the yield surface. Therefore \( R(p) = \sigma_{ym} + H_m p \), where \( \sigma_{ym} \) and \( H_m \) denote respectively the constitutive material initial yield stress and hardening modulus. \( R(p) \) is the instantaneous yield stress which results from the constitutive material hardening. \( p \) denotes the cumulated plastic strain.

The activation of the plasticity was governed through the following yield condition:

\[
\mathcal{F} = \sqrt{\frac{3}{2} \text{tr}(\varepsilon s^2) - R(p)}
\]

(2)

where \( s \) denotes the deviatoric part of the stress tensor: \( s = \varepsilon - \frac{1}{3} \text{tr}(\varepsilon) I \).

To model a standard metallic material, the different numerical values were \( E_m = 200 \text{ GPa}, \ v_m = 0.31, \ \sigma_{ym} = 60 \text{ MPa} \) and \( H_m = 10 \text{ GPa} \). Both compression and simple shear tests were simulated on HSSs using the finite-element suite Z-set (www.zset-software.com). Uniaxial compressive loads were applied along crystallographic direction [001] of the stackings whereas shear loads were applied in their plane (101).

Boundary conditions are detailed further.

2.2. Introduction of a geometrical dispersion

The three different geometrical parameters which characterise a hollow-sphere stacking are: (i) the outer radius of the hollow spheres \( R_m \), (ii) their thickness \( t_s \), and (iii) the radius of the menisci \( R_m \) (see Fig. 1). In order to account for a dispersion on the geometrical parameters, stackings of 3-sphere sides were simulated (see Fig. 2a and b). In all cases, the mean values of ratios \( t_s/R_m \) and \( R_m/R_s \) computed on the 3-sphere side cells, equaled respectively 0.18 and 0.30. The sphere outer radius was fixed (\( R_m = 1 \text{ mm} \)) and a dispersion either on the sphere thickness or on the meniscus radius was introduced.

For the sake of simplicity, only SC-like HSSs were considered to deal with the issue of the dispersion on the geometrical parameters. Since the coordination number is the smallest for this stacking type, for given geometry and number of hollow spheres, meshes were smaller for the SC-like HSS than for the other regular stacking types (body centred cubic- and face centred cubic-like ones for instance) because there were less menisci to be meshed. Nevertheless, the number of degrees of freedom resulting from the meshing of such structures was around 2,500,000. Meshes were obtained with BLSURF (Laug and Borouchaki, 1999) and TetMesh-GHS3D (2006), using quadratic tetrahedrons. It has been previously shown that two quadratic tetrahedrons in the sphere thickness represents a good compromise for the convergence to be considered as achieved and for meshes not to be too big (see Marcadon and Feyel, 2009), mainly thanks to the refinement of the meshes in the neighbourhood of meniscus extremities that are areas in which stress concentration phenomena and localised plasticity occur.

Different distributions of the thinner and the thicker hollow spheres (or the smaller and the larger menisci) towards the load direction were considered. Each 3-sphere side cell consisted of nine hollow spheres (menisci resp.) having a thickness (radius resp.) of \( t_s < t_s \) (\( R_m < R_m \) resp.), nine having a thickness (radius resp.) of \( t_s > t_s \) (\( R_m > R_m \) resp.) and nine having a thickness (radius resp.) of \( t_s = t_s \) (\( R_m = R_m \) resp.). The different distributions simulated to tackle with a dispersion on the hollow sphere thickness are shown in Figs. 3 and 4. The corresponding figures for a dispersion on the meniscus radius are not supplied here for the sake of brevity.

For distributions labelled ‘vertical dispersion’ and ‘horizontal dispersion’ the hollow spheres (menisci resp.) were distributed in layers of spheres (menisci resp.) having the same thickness (radius resp.) vertically or horizontally towards the load direction, respectively. For the distribution labelled ‘diagonal dispersion’, whereas the crystallographic plane (01T) consisted of the spheres (menisci resp.) of thickness \( t_s \) (radius \( R_m \) resp.), the spheres (menisci resp.) contained in the upper planes had a thickness \( t_s < t_s \) (radius \( R_m < R_m \) resp.) and those contained in the lower planes had a thickness \( t_s > t_s \) (radius \( R_m > R_m \) resp.). The distribution labelled ‘random dispersion’ was actually not totally random since the spheres (menisci resp.) were placed according to the following rule: only one sphere (meniscus resp.) of each thickness (radius resp.) per row, per column and per diagonal. A fifth distribution labelled ‘no dispersion’ was simulated consisting of spheres and menisci having all the same thickness and radius, which equalled the mean values \( t_s \) and \( R_m \) obviously. It is worth noting that two different stackings with no dispersion were required: one for the dispersion on the sphere thickness and one for the dispersion on the meniscus size. Indeed, due to the fact that the considered stackings are finite ones, the stackings illustrated in Fig. 2a and b are not equivalent. These last distributions refer to the reference solutions as the perfect finite stackings.

For a given distribution, two magnitudes of dispersion were simulated: either \( (t_s/R_m = 0.14, \ t_s/R_s = 0.22) \) or \( (R_m/R_s = 0.25, \ R_m/R_s = 0.35) \), and either \( (t_s/R_m = 0.10, \ t_s/R_s = 0.26) \) or \( (R_m/R_s = 0.20, \ R_m/R_s = 0.40) \), respectively.

3. Geometrical dispersion and effective behaviour of hollow-sphere structures

3.1. Effects of geometrical dispersion in the case of a compressive loading

Compression behaviour was studied first applying a uniaxial compressive load along the \( \hat{Z} \) direction of the simulated stackings, which corresponded to their crystallographic direction [001]. Because of geometrical dispersion the 3-sphere side stackings cannot be considered here as the unit cells of periodic media. Thus, it would have been meaningless to assume these cells as representative volume elements by applying different planar or periodic boundary conditions on their faces, especially for the diagonal dispersion. Finite stackings were considered by applying free bound-

**Fig. 1.** Median cross-section of two half hollow spheres linked by a meniscus: definition of HSSs characteristic geometrical parameters.
ary conditions on the lateral faces of the stackings (faces with normals $\tilde{x}$, $-\tilde{x}$, $\tilde{y}$ and $-\tilde{y}$), whereas their lower face (normal $-\tilde{z}$) was fixed (i.e., the displacement $u_3 = 0$ along $\tilde{z}$, plus $u_1 = 0$, the displacement along $\tilde{x}$ on one edge orthogonal to $\tilde{x}$, and $u_2 = 0$, the displacement along $\tilde{y}$, on one other adjacent edge, thus orthogonal to $\tilde{y}$, to avoid rigid modes). The displacement $u_3$ was imposed at the upper face of the stackings (normal $\tilde{z}$) up to $-0.06$ mm ($-1\%$ in strain), and at a constant rate.

Except for few calculations which were conducted up to the occurrence of the internal self-contact between neighbouring spheres to further investigate collapse mechanisms (see Section 3.4), most of the calculations were stopped at a strain level of $-1\%$ because of calculation costs. Indeed, each calculation took about a week and a half despite of having been multithreaded on four-core processors and a 16 Gbites memory. Nevertheless, this was not detrimental for the discussion since the observed collapse mechanisms based on localised plasticity are stable ones. The activation of the internal self-contact might call into question this conclusion, but it occurs for significantly larger strain levels of about 15–20% and over on stackings without any geometrical defects, depending on the geometrical parameters (see Marcadon and Feyel, 2009). Despite of extending calculations to larger strain levels and accounting for internal self-contact might be a very challenging and interesting issue, it is beyond the scope of the present paper.

Figs. 5 and 6 show the stress–strain curves obtained for the two magnitudes of dispersion, both on the sphere thickness and the meniscus radius, and for all the considered distributions. The overall strain and stress, $E_{33}$ and $\Sigma_{33}$ respectively, are the classical nominal ones. The strain was computed from the total imposed displacement $u_3$ and the initial thickness of the stackings whereas the stress was computed from the average local stress in the cell walls, $\langle \sigma_{33} \rangle$, divided by the relative densities of the stackings, i.e., the ratios between the volume of matter and the volume of the box containing the stacking. In each figure the reference curve obtained for the stacking ‘no dispersion’ is plotted. Stackings effective
behaviours have been determined by fitting the mechanical response of a homogeneous equivalent medium (HEM) to the stress–strain curves, using a classical Levenberg–Marquardt method. Effective Young’s moduli $E_{\text{eff}}$ have been determined first from both stress–strain curves between 0 and 0.05% of global strain, in the elastic regime. They have then been introduced in the fitting procedure to improve the estimation of the different plastic moduli. According to Marcadon and Feyel (2009), a non-linear exponential hardening term has been added in order to correctly describe the smooth transition between both the elastic and the plastic parts of stress–strain curves. Therefore, regarding the effective behaviour $R(p)$ is replaced by $R_{\text{eff}}(p) = \sigma_{\text{eff}}^p + H_{\text{eff}}^p \cdot p + Q_{\text{eff}}^p \cdot (1 - \exp(-b_{\text{eff}}^p \cdot p))$ in Eq. 2, with $\sigma_{\text{eff}}^p$, $H_{\text{eff}}^p$, $Q_{\text{eff}}^p$ and $b_{\text{eff}}^p$ denoting the stacking effective yield stress, hardening modulus and coefficients of the non-linear hardening term, respectively. The different moduli are supplied in Tables 1 and 2 for the dispersions on the sphere thickness and on the meniscus radius respectively. Stacking relative densities $f$ are also given in these tables.

Since free boundary conditions were applied on the lateral faces of the different simulated stackings it would have been meaningless to determine effective Poisson’s ratios because of Poisson’s effect resulting in the curvature of the lateral faces. Thus the transversal strain in that case is no longer linked to the longitudinal one through Poisson’s ratio. But this was not a real issue since whatever the value of Poisson’s ratio the other moduli fitted remained unchanged in the case of a uniaxial compressive (or tensile) load. Results are discussed in Section 3.3.

### 3.2. Effects of geometrical dispersion in the case of a shear loading

A simple shear loading in the plane (101) of the stackings was simulated applying a displacement $u_1$, up to $-0.06$ mm and at a constant rate, at the top face of the different 27-sphere stackings along their crystallographic direction [100] (x direction), and with a planar boundary condition (i.e., the displacement $u_3$ was the same for all the nodes at the top face). The bottom face of the cells was fixed and free boundary conditions were applied on their lateral faces, as for the aforementioned case of compression. In the case of finite stackings only a simple shear loading could be simulated, contrarily to infinite stackings for which a pure shear loading could be simulated (see Section 4.2). The aim of investigating shear behaviour was to understand more precisely the contribution of the menisci to the effective strength of stackings.

Fig. 7 and 8 show stress–strain curves obtained for the two magnitudes of dispersion, both on the sphere thickness and the meniscus radius, and for all the considered distributions. $\Sigma_{31}$ and $E_{31} = \Gamma_{31}/2$ denote the components of the global stress and strain tensors in the plane ($\bar{x}, \bar{z}$). They were obtained in a similar way as the aforementioned compression components. Effective shear moduli $l_{\text{eff}}$ have been
estimated between 0 and 0.05% of global strain on stress–strain curves for all the considered distributions and dispersions (see Tables 1 and 2). Results are detailed with those for compression in the following section.

### 3.3. Discussion

All the calculations previously described show that the dispersion on the sphere thickness influences the stacking effective behaviour more strongly than that on the meniscus radius. The effective shear modulus remains slightly sensitive to geometrical dispersion, whereas effective Young’s modulus decreases with an increasing magnitude of dispersion. The same conclusion can be drawn for the effective yield stress which is the plastic property the most sensitive to geometrical dispersion. Such a decrease in the effective strength when increasing the magnitude of dispersion is in agreement with the trends predicted by Grenestedt and Bassinet (2000), modelling flat-faced Kelvin’s structures whose cell wall thicknesses randomly varied.

The effect of a dispersion on the sphere thickness can be simply explained and was expected: having a dispersion on the sphere thickness results in a fall of the effective strength of the stacking

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**Fig. 6.** Effect of geometrical dispersion on the meniscus radius for a compressive loading (a) smaller dispersion (b) larger dispersion.

**Fig. 7.** Effect of geometrical dispersion on the sphere thickness for a shear loading (a) smaller dispersion (b) larger dispersion.

**Fig. 8.** Effect of geometrical dispersion on the meniscus radius for a shear loading (a) smaller dispersion (b) larger dispersion.
and, the larger the magnitude of dispersion, the more drastic the fall of the stacking strength. This is true for both compressive and shear loads. Thinner hollow spheres appear like elements weakening all the structure. On the contrary, the effect of a dispersion on the meniscus radius is more complicated. Whereas in the case of compression having a dispersion on the meniscus radius entails a decrease of the stacking strength, in the case of shear the stacking strength raises by adding a dispersion on $R_m$. This could be explained by the fact that menisci have a stiffening effect on the structure. They are rigid bodies that act against the collapse of the hollow spheres by reducing the thinning of the cell walls between neighbouring menisci. In the case of compression, menisci are less stressed contrarily to the case of shear, explaining why their stiffening effect is only observed in the second case. The understanding of the role of the menisci on the overall behaviour of HSSs seems to be critical in view of their potential use, especially as core of sandwich structures they could be stressed under shear.

To investigate deeper this issue, let us suppose that the stacking strength depends on the quantities of matter. If the sphere thickness is significantly smaller than the sphere radius, the volume of a sphere can be approached by $V_s = 4\pi r_s^2$. Similarly, a meniscus can be approximated by a cylinder of radius $R_m$ and height $2t_m$, and its volume equals approximately $V_m = 2\pi R_m t_m$. Consequently, whereas the stacking strength varies according to $t_s$ only, it varies according to $R_m$ raised to a power of two. Complementary calculations have been carried out on a stacking having no geometrical dispersion but with a meniscus radius $R_m$, such as the total section of menisci, computed on the 27 ones on which a dispersion was introduced, equalled the one obtained in the case of the larger dispersion on the meniscus size: $27\pi R_m^2 = 9\pi (R_m^2 + R_m^2 + R_m^2)$. Thus $R_m^2/R_s^2 = 0.31$, whereas $t_s/R_s$ still equaled 0.18. Corresponding stress–strain curves, obtained for compressive and shear loads, are plotted in Figs. 6b and 8b for comparison with those obtained for the stackings having the larger dispersion on the meniscus size. These curves confirm that now having a dispersion on the meniscus size is detrimental for the stacking effective strength, similarly to the trend already observed in the case of a dispersion on the sphere thickness. The effective moduli determined thanks to the fitting procedure described previously were $E_{eff} = 7.2 \text{ GPa}$, $\mu_{eff} = 3.5 \text{ GPa}$, $\sigma_{eff} = 4.3 \text{ GPa}$, $H_{eff} = 348.1 \text{ MPa}$, $Q_{eff} = 2.2 \text{ MPa}$ and $b_{eff} = 523.2$ for this particular stacking. A significant contribution of the localised plasticity on the stacking mechanical response is dismissed, hence a higher effective yield stress. Actually, the section of the menisci transversally to their revolution axis seems to be a more relevant parameter than the meniscus radius for the dimensioning of HSSs, especially when addressing their shear behaviour.

### 3.4. Understructures and collapse mechanisms

A quantitative comparison with experimental results from the literature would have been debatable since the overall behaviour of cellular structures strongly depends on their constitutive material properties and processing routes (Amsterdam et al., 2008; Friedl et al., 2008; Mangipudi et al., 2010; Marcadon et al., 2012). Nevertheless, there are many studies in the literature which addressed the influence of geometrical parameters, without any dispersion on them, on the effective behaviour of HSSs (Fallet et al., 2008, 2007, 2008; Lhuissier et al., 2009; Marcadon and Feyel, 2009; Sanders and Gibson, 2003a,b). Their conclusions are helpful here to discuss the specific contribution of the geometrical dispersion on collapse mechanisms, despite some care has to be taken when comparing the results obtained on our small and finite HSSs and those obtained on larger and often random structures.

Results obtained highlighted that certain distributions of the weaker and the stiffer spheres, or menisci, are more detrimental for the stacking effective behaviour. Indeed, if the weaker parts of the structure are distributed by the way to form a weaker understructure, this understructure will collapse the first and thus weaken all the structure (see Fig. 9a). Such a localised collapse of the defective hollow spheres first during the compaction of HSSs has

### Table 1

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<th>Dispersion type</th>
<th>$f$ (%)</th>
<th>$E_{eff}$ (GPa)</th>
<th>$\mu_{eff}$ (GPa)</th>
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<th>$H_{eff}$ (MPa)</th>
<th>$Q_{eff}$ (MPa)</th>
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### Table 2

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<td>3.8</td>
<td>341.4</td>
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<td>492.3</td>
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<tr>
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<td>3.6</td>
<td>329.2</td>
<td>2.1</td>
<td>493.3</td>
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</table>
already been observed experimentally by Yamada et al. (2005), studying 5-sphere side SC-like aluminium HSSs with some thinner spheres. Fig. 9b clearly shows that the thinner sphere is indented by its thicker neighbouring sphere. This mechanism of localised damage totally agree with also the in situ X-ray tomography characterisations performed by Fallet et al. (2008) and Lhuissier et al. (2009) on stainless steel HSSs, for instance. As illustrated in Fig. 9a and b, in some cases the simulations were performed up to larger compaction levels and stopped just before the occurrence of the internal self-contact between neighbouring spheres. These complementary calculations aimed at confirming the stability of the collapse mechanisms captured by our modelling approach.

In the case considered here, the vertical dispersion is obviously the most detrimental for the stacking overall behaviour since all the thinnest spheres (or the smallest menisci) are stressed first and in the same time. On the contrary, the horizontal dispersion is the least detrimental one because it is impossible to stress a thinner sphere (or a smaller meniscus) without stressing simultaneously a thicker sphere (or a larger meniscus).

Such an architectural collapse mechanism is the one usually involved in the stress plateau classically observed during the compaction of cellular structures, but it is not the only one. Indeed, the slope of the compression curve part associated with the cellular material densification not only depends on architectural dispersion (geometrical defects, misalignment of cells, etc.), as observed here, but also on constitutive material plastic properties (Marcadon and Feyel, 2009). Whereas the slope decreases when introducing an architectural dispersion, it increases with the constitutive material hardening capabilities. Such a competition between a constitutive material-induced hardening and a geometry-induced one has already been described in the literature (Amsterdam et al., 2008; Mangipudi et al., 2010).

4. Central defects and infinite stackings

4.1. Influence of a central defect

As mentioned before, due to the 3D distribution of the geometrical defects considered first it was impossible to rigorously apply periodic boundary conditions on the simulated stackings to model infinite ones, the meshes not being symmetric. To tackle with this issue, 3-sphere side SC-like HSSs were considered again, but this time only the central hollow sphere or the central meniscus had a specific thickness or radius, respectively. Meshes were obtained from the eighth of the stackings using their planes of symmetry in order to be sure that, for each node of the mesh on a face, there was a corresponding one on the opposite face. Stackings were meshed using quadratic tetrahedrons again (see Fig. 10a–c). Concerning the mechanical assumptions for the constitutive material behaviour they remained unchanged (see Section 2.1).

Different central defects were simulated: either a thinner or a thicker central hollow sphere (a smaller or a larger meniscus resp.) was introduced in the stacking. The thickness values chosen for the defective central sphere were \( t_{th}/R_s = 0.10, 0.14, 0.22 \) and 0.26, whereas \( t_{ch}/R_s \) equalled 0.18 for the standard hollow spheres. In the case of a dispersion on the meniscus size, the radius of the defective meniscus was \( R_{sn}/R_t = 0.2, 0.25, 0.35 \) and 0.40, compared to a radius of \( R_{sn}/R_t = 0.30 \) for the other menisci. Furthermore, in the case of a defective central hollow sphere, the particular case of a missing sphere was studied too. The case of a missing meniscus was not simulated because of a too large calculation cost involved by the introduction of some contact conditions between the two neighbouring spheres without a meniscus.

Both finite and infinite stackings could be considered now. The case of infinite stackings is addressed below (see Section 4.2). As regards finite stackings, similarly to the case of the 3D geometrical dispersion, uniaxial compressive and simple shear loads were simulated applying a displacement at the top of the HSSs, whereas the bottom was fixed and the four lateral faces were let free of load. Displacements were applied along crystallographic directions [001] and [100], up to \(-0.06 \text{ mm} \) (\(-1\% \) in strain), in the case of compressive and shear loads, respectively. Boundary conditions were exactly the same as those previously described in the case of the 3D geometrical dispersion (see Section 3). Corresponding mechanical responses are plotted in Figs. 11 and 12. Effective mechanical properties were identified using the same fitting procedure as before. Their numerical values and the relative densities are given in Table 3. Results are discussed in details in Section 4.3.
4.2. Infinite stackings

To model infinite HSSs, periodic boundary conditions were applied on the faces of the 3-sphere side HSSs, which could be now considered as the Representative Volume Elements (RVEs) of the different simulated stackings. Thus the total displacement field \( \vec{u} \) reads:

\[
\vec{u} = \varepsilon \cdot \vec{X} + \vec{v}
\]

in each point \( \vec{X} \) of the RVE, with \( \varepsilon \) denoting the overall strain tensor and \( \vec{v} \) the periodic part of the displacement field. Multipoint constraints were applied on the different faces of the RVE to be sure that the components of the periodic displacement field on a face remained the same as those on the opposite face. Rigid modes...
were blocked by fixing one node at the centre of the RVE ($u_1 = u_2 = u_3 = 0$). The components $E_{33}$ and $E_{31}$ of the macroscopic strain tensor $\mathbf{E}$ were imposed separately up to $-1$ and $-0.5\%$ ($\Sigma_{31} = -1\%$) at constant rates, respectively, to simulate the compressive and shear loads, the other components being stress free. The corresponding overall stress components $R_{33}$ and $R_{31}$ were then computed from the average stress in the stacking walls and the relative densities, as it was the case for finite stackings. In the case of a compressive loading there is no difference between periodic, planar or symmetric boundary conditions and the deformation of the RVE is the same. Contrarily, in the case of a shear loading, now a pure shear one, periodic boundary conditions result in a specific deformation of the RVE. Some corrugations of the faces are observed because of the heterogeneity of the local strength in the RVE (Fig. 13a and b). These periodic boundary conditions are absolutely needed to capture the right effective mechanical response of infinite stackings.

It can be noticed that now there was only one reference stacking with no geometrical dispersion, neither on the sphere thickness nor on the meniscus radius. The two different 3-sphere side cells, the one for a defective central sphere and the one translated for a defective central meniscus, were two RVEs of the same stacking. Similarly to those of finite stackings, the effective mechanical properties of infinite stackings were identified thanks to the same aforementioned fitting procedure. But this time effective Poisson’s ratios $\nu_{eff}$ could be determined from strain maps between 0 and 0.05% of global strain. Corresponding numerical values are supplied in Table 3 and the different mechanical responses are plotted in Figs. 14 and 15. Results are discussed below.

### 4.3. Finite vs. infinite stackings

If the results on finite stackings are considered first, it is worth noting that there is only a slight effect of introducing a geometrical defect on the stacking central sphere. Obvious trends are only visible in the case of a defective central sphere on Young’s modulus and in a lesser extent on the shear one. These moduli raise with the defective sphere thickness. In the case of a defective central

<table>
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<th>Defect type</th>
<th>$f$ (%)</th>
<th>$E_{eff}$ (GPa)</th>
<th>$\nu_{eff}$ (GPa)</th>
<th>$\nu_{eff}$</th>
<th>$\sigma_{eff}^{\mu}$ (MPa)</th>
<th>$H_{eff}$ (MPa)</th>
<th>$Q_{eff}$ (MPa)</th>
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<td>334.4</td>
<td>2.2</td>
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<td>511.5</td>
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<td>2.4</td>
<td>–</td>
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<td>314.2</td>
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<td>3.3</td>
<td>–</td>
<td>4.0</td>
<td>337.4</td>
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<td>$t_s/R_s = 0.26$</td>
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<td>$R_m/R_s = 0.40$</td>
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<td>370.9</td>
<td>2.2</td>
<td>516.4</td>
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Fig. 13. Von Mises stress maps for a shear loading for −0.5% of global strain (a) for the infinite reference stacking (stress values vary between 0.9 and 389.0 MPa), and (b) for the infinite stacking with a missing central hollow sphere (stress values vary between 0.2 and 440.3 MPa).
meniscus, a tenuous trend seems also exist on effective Young’s modulus, that increases with the size of the defective meniscus. All the other effective moduli do not depend on the presence of defects. Such results assert the strong effect of having defective understructures rather than isolated defects on the effective strength of HSSs.

A more drastic decrease of the stacking effective strength is observed in the case of a missing central hollow sphere; all the effective moduli are significantly lower in that case compared to those of the stacking with no defect. Such an effect of missing constitutive cells has been already described in the literature (Fazekas et al., 2002; Adjari et al., 2008). It vanishes when increasing the mean distance between two neighbouring defects and rapidly becomes negligible.

Now, if we consider the case of the infinite stackings, similar conclusions can be drawn. The effect of having only a central defective sphere or meniscus is barely visible on the effective mechanical responses of stackings, except for the case of a missing central hollow sphere inducing a significant decrease of the stacking strength. A slight trend is observed on effective Young’s modulus

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**Fig. 14.** Effect of a central defect on the infinite stacking strength in the case of a compressive loading for (a) a defective hollow sphere (b) a defective meniscus.

**Fig. 15.** Effect of a central defect on the infinite stacking strength in the case of a shear loading for (a) a defective hollow sphere (b) a defective meniscus.

**Fig. 16.** Comparison between effective mechanical responses of the finite and the infinite stackings (a) for a compressive loading, and (b) for a shear loading.
which increases with the central hollow sphere thickness and the central meniscus radius. Moreover, similarly to the case of a 3D geometrical dispersion, having a geometrical dispersion on the sphere thickness seems to be more detrimental for the stacking effective behaviour than having one on the meniscus size.

The comparison between the effective mechanical responses of finite and infinite stackings is more interesting. Stress–strain curves obtained for both cases for the simulated stackings are plotted in Fig. 16a and b, in the case of compressive and shear loads respectively. For the sake of clarity, only the curves obtained for the stackings with no central defect and with a missing central hollow sphere are represented because all the other mechanical responses are very close to those of the reference stackings. It is worth noting that the strength of the infinite stacking is systematically higher than that of the finite stacking. Whereas for a compressive loading periodic boundary conditions induce only a minor increase of the stacking strength, which can be explained by side effects observed on the finite stackings, for a shear loading the influence of the periodic boundary conditions on the effective stacking strength is considerable. The underestimation of the stacking strength observed here when there were no periodic boundary conditions totally agree with classical results of the periodic homogenisation. In composite materials, the absence of periodic boundary conditions results in an overestimation of the material behaviour when the inclusions are stiffer than the matrix, and in an underestimation in the case of softer inclusions. Here, the pores of the stackings act as soft inclusions.

5. Conclusions

The present work addressed the influence of geometrical dispersion and defects on the effective behaviour of HSSs introducing some dispersions on the geometrical parameters of their constitutive hollow spheres and menisci. Simulated stackings were loaded under uniaxial compression and shear loadings. One important aim here was to investigate the effect of geometrical dispersion specifically, and without any combined contributions of either stackings faults or variations of the geometrical parameters.

The case of a 3D distribution of the defects in finite 3-sphere side stackings was studied first. Results showed that for a given distribution of the defective hollow spheres or menisci the stacking effective strength decreases with an increasing magnitude of the geometrical dispersion. It has been also observed that having a geometrical dispersion on the hollow sphere thickness is more detrimental for the stacking behaviour than having one on the meniscus radius. The most sensitive effective mechanical properties to the geometrical dispersion were Young’s modulus, the yield stress and in a lesser extent the shear modulus. It is worth noting that the section of the menisci seems to be a more relevant parameter than their radius for the dimensioning of HSSs, especially under shear loading. Calculations have also confirmed the significant influence of having weaker understructures on the localisation of collapse mechanisms in the structure. These architecture-scale mechanisms contribute with the constitutive material work-hardening to the specific stress plateau, or more generally the effective hardening, commonly observed experimentally for such cellular structures.

The effect of having only one isolated defect in the structure on its effective behaviour was rather slight, except for a major defect like a missing hollow sphere. Effective mechanical properties tend to increase with the defective sphere thickness or the defective meniscus radius. The comparison between the mechanical responses of both finite and infinite stackings in the case of an isolated defect has shown that the effective strength of the finite stackings is significantly lower than the one of the infinite stackings. This last trend is particularly sizeable in the case of a shear loading.

The main limitation of our modelling approach ensues from the fact that, because of calculation costs, reference could be made only to small finite stackings. Except for the case of the periodic structures that could refer to infinite stackings thanks to the use of appropriate periodic boundary conditions, in all the other cases some particular 3-sphere side finite stackings were considered. Hence, some care has to be taken when extending the observed trends to larger and more realistic structures. As mentioned before, the extension of the modelling approach to larger compaction levels accounting for the internal self-contact between neighbouring spheres should be a great challenge and an interesting issue.

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