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Numerical analysis of a reinforcement learning model with the dynamic aspiration level in the iterated Prisoner's dilemma

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ABSTRACT

Humans and other animals can adapt their social behavior in response to environmental cues including the feedback obtained through experience. Nevertheless, the effects of the experience-based learning of players in evolution and maintenance of cooperation in social dilemma games remain relatively unclear. Some previous literature showed that mutual cooperation of learning players is difficult or requires a sophisticated learning model. In the context of the iterated Prisoner's dilemma, we numerically examine the performance of a reinforcement learning model. Our model modifies those of Karandikar et al. (1998), Posch et al. (1999), and Macy and Flache (2002) in which players satisfice if the obtained payoff is larger than a dynamic threshold. We show that players obeying the modified learning mutually cooperate with high probability if the dynamics of threshold is not too fast and the association between the reinforcement signal and the action in the next round is sufficiently strong. The learning players also perform efficiently against the reactive strategy. In evolutionary dynamics, they can invade a population of players adopting simpler but competitive strategies. Our version of the reinforcement learning model does not complicate the previous model and is sufficiently simple yet flexible. It may serve to explore the relationships between learning and evolution in social dilemma situations.

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1. Introduction

Human beings and other animals often cooperate with each other even in social dilemma situations where to not cooperate is apparently a rational choice. A standard framework in which social dilemma situations are studied is the Prisoner's dilemma game (PD) and its variants. Many theoretical mechanisms for emergence and maintenance of cooperation in social dilemma games have been reported thus far (Axelrod, 1984; Boyd and Richerson, 1985; Nowak, 2006; Sigmund, 2010).

Most of these mechanisms do not deal with the adaptation or learning of individuals. We use the term learning to refer to individual learning (i.e., experience-based adaptation), but not to social learning (i.e., imitation). Learning implies that an individual takes advantage of the history of the games that it has played to perform better in subsequent rounds. A learning individual changes behavior on the basis of some statistics of the game results. Laboratory experiments suggest that humans do learn during sequences of games (Camerer, 2003; Glimcher et al.,

2009). The learning of the social behavior of animals, including humans, has been modeled in various game and non-game situations (Rapoport and Chammah, 1965; Cross, 1983; Boyd and Richerson, 1985; Fudenberg and Levine, 1998; Camerer, 2003).

Learning in a game is relevant only in an iterated game. It is well known that mutual cooperation can be optimal in the iterated PD (Trivers, 1971; Axelrod, 1984). Action rules that have mainly been considered in the context of the iterated PD are those that do not adjust conditional probabilities of cooperation upon experience. A player using a look-up table that relates the next action to the outcome of the game in the current and past few rounds belongs to this class (Axelrod, 1984; Kraines and Kraines, 1989; Nowak and Sigmund, 1989, 1990, 1992, 1993; Nowak, 1990; Lindgren, 1991; Nowak, 2006; Sigmund, 2010). This important class includes well-known strategies such as the tit-for-tat (TFT). However, the flexibility of such a strategy appears to be limited.

Players using reinforcement learning, on which we focus in this study, exploit information about past encounters to adapt the probability of cooperation conditioned by the outcome of the game in a couple of past rounds. Because of their flexibility, such learning players may be strong competitors in the iterated PD. If learning players compete relatively well in a population, the learning

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behavior may spread to become dominant in the population through evolutionary dynamics. Nevertheless, the possible roles of reinforcement learning in the iterated PD, either in favor of or against the promotion of cooperation, are relatively unexplored. In fact, players using reinforcement learning have generally been unsuccessful in the PD and other social dilemma games (Macy, 1996; Sandholm and Crites, 1996; Posch et al., 1999; Taiji and Ikegami, 1999; Macy and Flache, 2002; Masuda and Ohtsuki, 2009). Although an artificial neural network model, for example, enables mutual cooperation (Gutnisky and Zanutto, 2004), such a complicated mechanism may not be implemented by humans or other animals. It seems that the current understanding of social dilemmas is mostly based on studies in the fields of evolutionary biology and economics. Because experience-based learning, and reinforcement learning in particular, is quite evident in humans and other animals, both in terms of behavior and neural activities (Camerer, 2003; Glimcher et al., 2009), clarifying the role of reinforcement learning in the iterated PD may provide an additional understanding of how subjects cope with social dilemmas.

In the present study, we numerically examine a variant of the reinforcement learning model (Karandikar et al., 1998; Posch et al., 1999; Macy and Flache, 2002) in the iterated PD. Following Macy and Flache (2002), we call the original model the Bush–Mosteller (BM) model. A player obeying the BM reinforcement learning (BM player for short) would continue an action (i.e., cooperate or defect) after gaining a relatively large payoff and would switch the action otherwise. If the threshold payoff above which the player satisfices, which is called the aspiration level, is fixed, BM players can mutually cooperate (Rapoport and Chammah, 1965; Macy, 1991, 1996; Posch et al., 1999; Macy and Flache, 2002; Izquierdo et al., 2007, 2008). The BM player with the fixed aspiration level studied in these articles is essentially the same as Pavlov that only uses the information about the immediate past (Kraines and Kraines, 1989; Nowak and Sigmund, 1993). Pavlov is known to be exploited by the unconditional defector and behave too generously to the unconditional cooperator.

Real subjects may adapt the aspiration level in response to the results of the game (Simon, 1959). The BM model with the adaptive aspiration level is not known to yield a large probability of mutual cooperation except in some limited cases (Karandikar et al., 1998; Posch et al., 1999; Macy and Flache, 2002). We remark that performance of other reinforcement learning models with the adaptive aspiration level have also been investigated in the PD and other games (Pazgal, 1997; Kim, 1999; Palomino and Vega-Redondo, 1999; Dixon, 2000; Börgers and Sarin, 2000; Oechssler, 2002; Bendor et al., 2003; Napel, 2003; Cho and Matsui, 2005). In the temporal difference learning, which is a dominant form of reinforcement learning in the brain, dopamine neurons represent the difference between the obtained reward and the dynamic expected reward that changes according to the subject's experience (Schultz et al., 1997; Montague and Berns, 2002; Daw and Doya, 2006; Glimcher et al., 2009). The reinforcement signal in the BM model with the adaptive aspiration level is given by the difference between the obtained reward and the dynamically changing aspiration level such that the BM model with the adaptive aspiration level is at least loosely connected to neural evidence.

We show that a simple modification of the BM model with the adaptive aspiration level drastically changes the behavior of the player. The modified BM player mutually cooperates with a large probability and is competitive in evolutionary dynamics. The modification is done such that the reinforcement signal is reflected to the action selection in the next round fairly strongly. The aspiration level must adapt with a low to intermediate learning rate for sustaining cooperation. It should be noted that our modification to the BM model does not introduce an additional complexity to the

original BM model with the adaptive aspiration level (Karandikar et al., 1998; Posch et al., 1999; Macy and Flache, 2002).

2. Model

We consider the symmetric two-person PD whose payoff matrix is given by

$$\begin{array}{cc} & \begin{array}{c} C \quad D \\ \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \\ \begin{array}{c} C \\ D \end{array} & \end{array} \quad (1)$$

where $T > R > P > S$ and $R > (T+S)/2$. The entries of Eq. (1) represent the payoffs that the row player gains. Each row (column) corresponds to the action of the row (column) player, i.e., cooperation (C) or defection (D). Because $T > R$ and $P > S$, mutual defection is the only Nash equilibrium of the single-shot game. Unless otherwise stated, we assume a standard payoff matrix for the PD given by $R=3$, $T=5$, $S=0$, and $P=1$.

A pair of players play the PD for a predetermined number of rounds denoted by t_{\max} . We denote the round by $t (=1, 2, \dots)$. Although the Nash equilibrium of the iterated PD is defection in all the rounds, which can be derived by backward induction, we assume for simplicity that players do not carry out backward induction. We could avoid this technical subtlety by assuming that a next round occurs with a certain probability such that the last round is not known beforehand (Axelrod, 1984; Nowak, 2006).

To model a learning player, we use a variant of the BM reinforcement learning model adapted to the game situation, pioneered in Rapoport and Chammah (1965). Our model is a variant of the BM model with the adaptive aspiration level (Karandikar et al., 1998; Posch et al., 1999; Macy and Flache, 2002).

In round t , the BM player intends to cooperate with probability p_t . We set the initial condition to $p_1=0.5$. In addition, we assume that the player misimplements the action (i.e., C or D) to play the opposite action with a small probability ε . The payoff that the BM player gains in round t is denoted as $r_t \in \{R, T, S, P\}$. We define the stimulus, or the reinforcement signal, using the sigmoidal function as

$$s_t = \tanh[\beta(r_t - A_t)], \quad (2)$$

where A_t is the aspiration level in round t above which the BM player satisfices. The degree of satisfaction is parametrized by s_t , and $-1 < s_t < 1$ holds true. If $s_t > 0$ ($s_t < 0$), the BM player is motivated to keep (switch) the current action in the next round. The sensitivity of the stimulus to the reinforcement signal $r_t - A_t$ is parametrized by $\beta \geq 0$.

The dynamics of the probability of cooperation are given by

$$p_{t+1} = \begin{cases} p_t + (1-p_t)s_t & (\text{Action in round } t = C, \text{ and } s_t \geq 0), \\ p_t + p_t s_t & (\text{Action in round } t = C, \text{ and } s_t < 0), \\ p_t - p_t s_t & (\text{Action in round } t = D, \text{ and } s_t \geq 0), \\ p_t - (1-p_t)s_t & (\text{Action in round } t = D, \text{ and } s_t < 0). \end{cases} \quad (3)$$

Finally, the dynamics of the aspiration level are given by

$$A_{t+1} = (1-h)A_t + h r_t, \quad (4)$$

where h represents the learning rate of the aspiration level, which is also called habituation (Macy and Flache, 2002). In contrast to previous models in which h decays as t increases (Erev and Roth, 1998; Cho and Matsui, 2005), we assume that h is a fixed constant. Unless otherwise stated, we set the initial value of A_t to $A_1 = (R+T+S+P)/4$, which is equal to the expected payoff when there are an equal number of cooperators and defectors in a population. As a remark, the possibility of cooperation in the iterated PD and other games was examined when the update of A_t is driven

by the average payoff over time (Kim, 1999; Cho and Matsui, 2005), the maximal experienced payoffs (Pazgal, 1997), or the payoff averaged over the population (Palomino and Vega-Redondo, 1999; Oechssler, 2002).

The difference between our model and the Macy–Flache model (Macy and Flache, 2002) lies in Eq. (2). Macy and Flache use $s_t = \ell(r_t - A_t) / \max[T - A_t, A_t - S]$ instead of Eq. (2). As described below, this difference results in a remarkable difference in the behavior of the player. In other words, we show that reacting strongly to the play in the previous round (i.e., large β) is necessary for mutual cooperation. A deterministic decision maker with the adaptive aspiration level used in Posch et al. (1999) corresponds to $\beta = \infty$. We numerically show that β does not have to be extremely large for mutual cooperation. We remark that, if $\beta = \infty$ and the aspiration level is fixed (i.e., $h = 0$), the strategy is a win-stay lose-shift one. In particular, our BM model with $\beta = \infty$ and $h = 0$ is equivalent to the Pavlov strategy (Kraines and Kraines, 1989; Nowak and Sigmund, 1993) if $P < A_t < R$.

3. Results

3.1. BM versus BM

In this section, we examine the performance of a BM player playing against another BM player. We assume that the two players employ the same values of β and h . For a range of β and h , the fraction of the rounds in which the focal BM player cooperates is shown for three values of implementation error, $\varepsilon = 0, 0.01, 0.1$, and two values of the number of rounds, $t_{\max} = 100, 1000$, in Fig. 1. The presented values are averages over 100 trials in this and the following figures unless otherwise stated. The fraction of cooperation is large when h is small and β is large. The results are fairly robust, despite some degradation, even under 10% of the error in the action implementation (Fig. 1(c, f)). Remarkably, a large

fraction of cooperation can be established only after $t_{\max} = 100$ rounds (Fig. 1(d–f)). These results are in contrast to those for other reinforcement learning models for social dilemma games, where the establishment of mutual cooperation requires a large number of rounds (Masuda and Ohtsuki, 2009) or is simply difficult (Macy, 1996; Sandholm and Crites, 1996; Posch et al., 1999; Taiji and Ikegami, 1999; Macy and Flache, 2002; Masuda and Ohtsuki, 2009).

In Fig. 1, β must be larger than approximately 2.7 for the fraction of cooperation to be large for small h . When β is in this range, Eq. (2) suggests that the reinforcement signal s_t would be typically close to -1 or 1 before a possible equilibrium is reached. This is because $|r_t - A_t|$ is typically about unity or larger when $R = 3, T = 5, S = 0$, and $P = 1$. Then, Eq. (3) implies that p_t is close to 0 or 1 , and the selection of the action tends to be almost deterministic. This deterministic nature of the BM player seems to pave the way to mutual cooperation. This result is consistent with those obtained from other models of reinforcement learning with the adaptive aspiration level (Palomino and Vega-Redondo, 1999; Oechssler, 2002).

Some mutual cooperation also occurs in the Macy–Flache original BM model with the adaptive aspiration level (Macy and Flache, 2002). For the sake of comparison, the fraction of cooperation in the Macy–Flache model with $\varepsilon = 0$ and $t_{\max} = 1000$ is shown in Fig. 2 for various values of h and the sensitivity to the stimulus ℓ . The fraction of cooperation is much smaller than that for our model. We consider that this is because the stimulus s_t with which to update the probability to cooperate in the next round is not sufficiently sensitive to the reinforcement signal $r_t - A_t$ in the Macy–Flache model. To satisfy $-1 \leq s_t \leq 1$ such that Eq. (3) is well defined, we need $\ell \leq 1$. Then, s_t would not be close to -1 or 1 in a considerable number of rounds. Then, the action in the next round is not likely to be very sensitive to the result of the game in the current round. Regardless of the value of ℓ , Macy’s model roughly corresponds to our model with a small value of β .

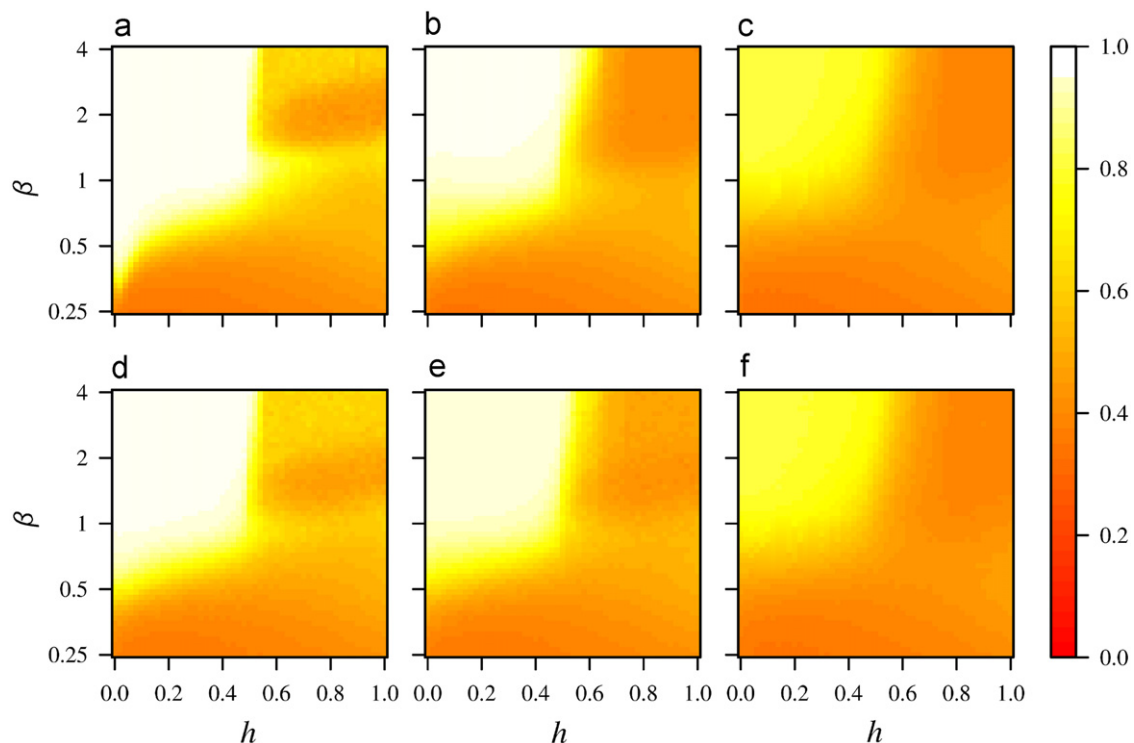


Fig. 1. Fraction of cooperation of the BM player playing against another BM player. The number of rounds is equal to (a–c) $t_{\max} = 1000$ and (d–f) $t_{\max} = 100$. The probability of the misimplementation of the action is equal to (a, d) $\varepsilon = 0$, (b, e) $\varepsilon = 0.01$, and (c, f) $\varepsilon = 0.1$. We set $R = 3, T = 5, S = 0$, and $P = 1$, and vary h and β .

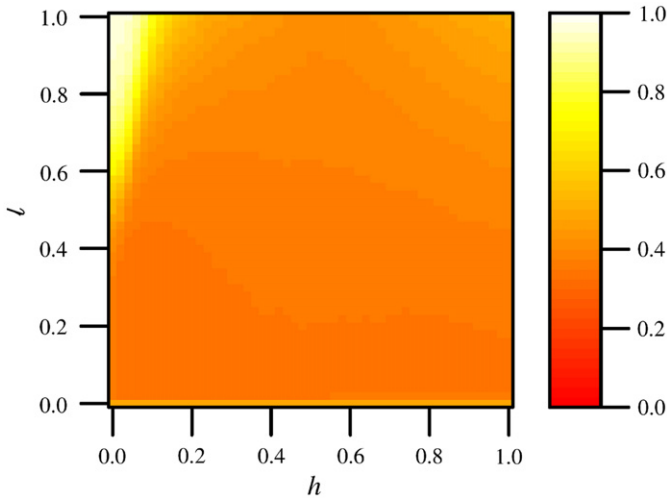


Fig. 2. Fraction of cooperation of the BM player playing against another BM player in the Macy-Flache model. We set $t_{\max}=1000$, $\varepsilon=0$, $R=3$, $T=5$, $S=0$, and $P=1$, and vary h and ε .

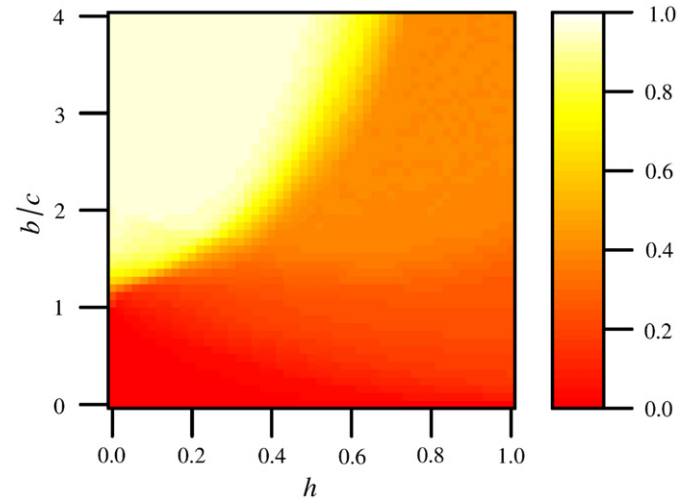


Fig. 3. Fraction of cooperation of the BM player playing against another BM player. We set $t_{\max}=1000$, $\varepsilon=0.02$, $\beta=3$, $c=1$, $R=b-c$, $T=b$, $S=-c$, and $P=0$, and vary h and b/c .

This interpretation is consistent with the result that a small β yields a small fraction of cooperation in our model (Fig. 1).

Our results are also consistent with those in Posch et al. (1999), in which the authors analytically showed that mutual cooperation is difficult when $h=1$ and $\beta=\infty$ (their YESTERDAY strategy) and that mutual cooperation is established if the temptation payoff T is not too large when h is tiny and $\beta=\infty$ (their FARAWAY strategy). A sufficiently small h combined with slight stochasticity in the dynamics of h also leads to mutual cooperation (Karandikar et al., 1998). Our numerical results extend their analytical results in showing that the BM players mutually cooperate up to an intermediate value of h if β is sufficiently large.

To test the robustness of the results against changes in the payoff matrix, we set $R=b-c$, $T=b$, $S=-c$, and $P=0$, and measure the fraction of cooperation as a function of h and the benefit-to-cost ratio b/c . We set $\beta=3$, for which the BM player mutually cooperates when $R=3$, $T=5$, $S=0$, $P=1$, and h is small (Fig. 1). The results for $t_{\max}=1000$, $\varepsilon=0.02$, and $c=1$ are shown in Fig. 3. The cooperation decreases with an increase in h . Nevertheless, the threshold value of b/c above which the BM players mutually cooperate with a probability close to unity differs only slightly up to $h \approx 0.25$.

A small h requires a relatively large number of rounds before the cooperative equilibrium is reached, even if the parameter values are set to yield a cooperative equilibrium. The fraction of cooperation when $t_{\max}=1000$, $\varepsilon=0.02$, $\beta=3$, $R=3$, $T=5$, $S=0$, and $P=1$ is shown in Fig. 4 for various values of h and initial aspiration level A_1 . Fig. 4 suggests that $h > 0.03$ is necessary for h to relax to an equilibrium value within $t_{\max}=1000$ rounds. When h is too small, the fraction of cooperation strongly depends on A_1 . If $P < A_1 < R$, the BM player is essentially the same as Pavlov for such a small h . In this case, mutual cooperation is realized, reflecting the fact that Pavlov cooperates against itself (Kraines and Kraines, 1989; Nowak and Sigmund, 1993). However, if we start from a different A_1 , the fraction of cooperation would be small for a small value of h .

Two BM players may have different parameter values. Because Fig. 4 suggests that the value of A_1 is irrelevant to the fraction of cooperation unless h is too small, we set $A_1=(R+T+S+P)/4$ and examine the case in which two players have different values of h and β . We set $h=0.3$ and $\beta=3$ for a focal BM player. For the opponent BM player with the identical

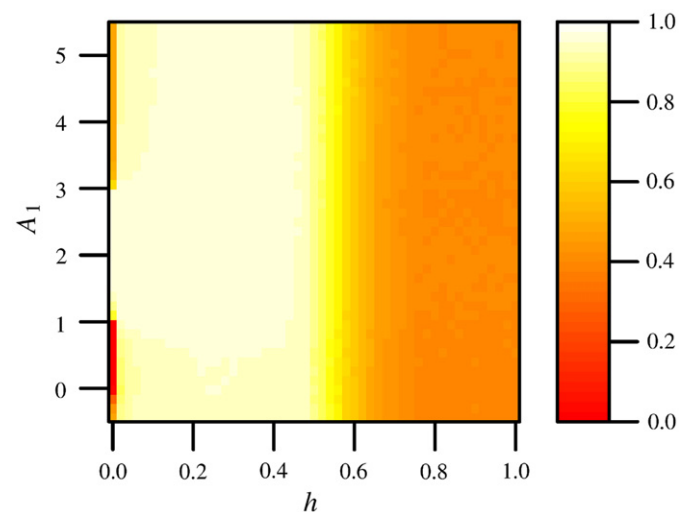


Fig. 4. Fraction of cooperation of the BM player playing against another BM player. We set $t_{\max}=1000$, $\varepsilon=0.02$, $\beta=3$, $R=3$, $T=5$, $S=0$, and $P=1$, and vary h and A_1 .

values of h and β , Figs. 1 and 3 guarantee that the two players mutually cooperate. With $t_{\max}=1000$ and $\varepsilon=0.02$, the fraction of cooperation and the mean payoff for the focal player when the opponent has different values of h and β are shown in Fig. 5(a) and (b), respectively. Fig. 5(a) indicates that the focal BM player mostly cooperates with the opponent with similar values of h and β . Although the fraction of cooperation is small when the opponent has small β , the focal BM player avoids being exploited by the opponent in this way (Fig. 5(b)). In both cases, the focal BM player performs well against the BM opponent.

3.2. BM against reactive strategies

We examine the behavior of the BM player against players adopting the reactive strategy. A reactive strategy is an often used non-learning strategy, and it is specified by two parameters p and q ($p, q \in [0, 1]$) and the initial condition. The reactive player cooperates with probabilities p and q when the opponent cooperates and defects in the previous round, respectively (Nowak and Sigmund, 1989, 1992; Nowak, 2006). Unconditional cooperation

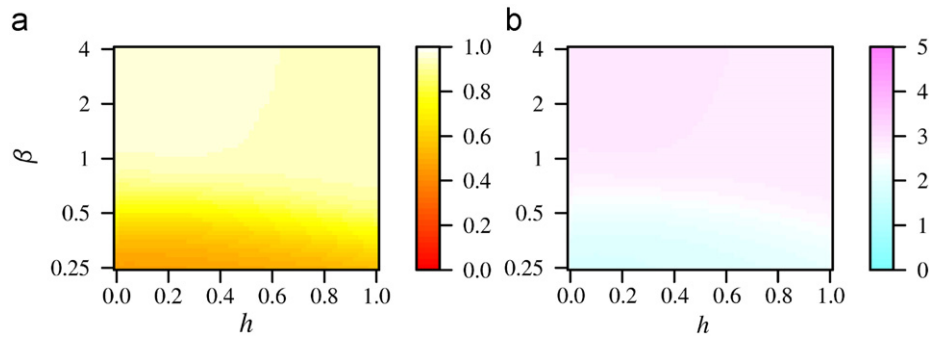


Fig. 5. Behavior of a focal BM player with $h=0.3$ and $\beta=3$ against a BM opponent with different values of h and β . (a) Fraction of cooperation and (b) mean payoff of the focal BM player. We set $t_{\max}=1000$, $\varepsilon=0.02$, $R=3$, $S=0$, $T=5$, and $P=1$.

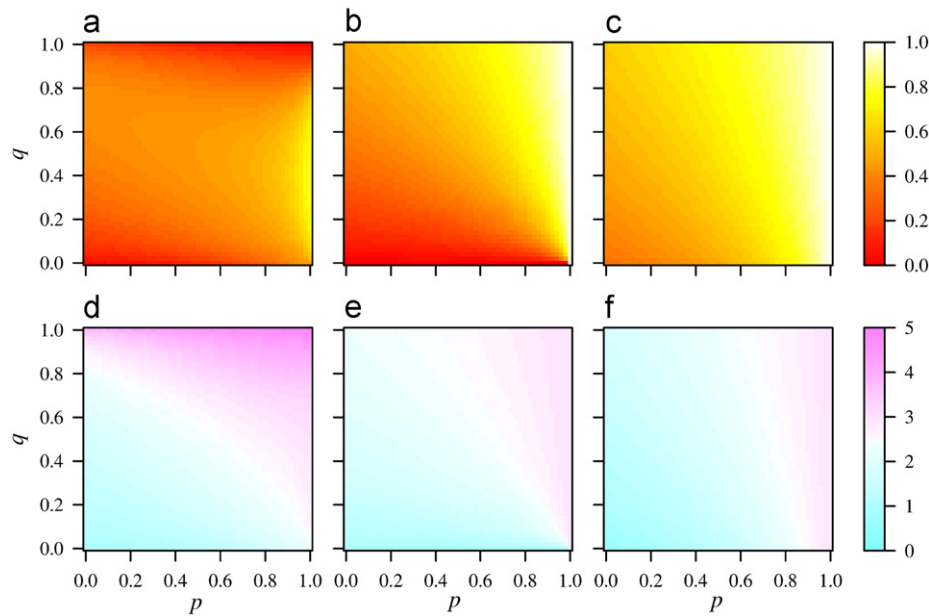


Fig. 6. Fraction of cooperation of (a) BM player, (b) TFT, and (c) GTFT against reactive strategies. We set $t_{\max}=1000$, $\varepsilon=0.02$, $h=0.3$, $\beta=3$, $R=3$, $S=0$, $T=5$, and $P=1$. Mean payoff of (d) BM player, (e) TFT, and (f) GTFT against reactive strategies.

(ALLC), unconditional defection (ALLD), and TFT correspond to $(p,q)=(1,1)$, $(0,0)$, and $(1,0)$, respectively. We assume that a player with the reactive strategy cooperates in the first round.

The fraction of cooperation of the BM player against various reactive strategies is shown in Fig. 6(a) for $t_{\max}=1000$, $\varepsilon=0.02$, $h=0.3$, and $\beta=3$. The BM player rarely cooperates with ALLC and ALLD. To never cooperate is the optimal action against these two strategies. The BM player cooperates with TFT in approximately half the rounds. This is not an optimal behavior; perpetual cooperation is optimal when the opponent is TFT (Axelrod, 1984). The mean payoff for the BM player against the reactive strategy is shown in Fig. 6(d). For all the values of p and q , the mean aspiration level of the BM player is indistinguishable from the mean payoff shown in Fig. 6(d). As already implied in Fig. 6(a), the BM player exploits ALLC and gains more than 4.5 per round. The BM is not exploited by ALLD and gains approximately $P=1$ per round. The BM player gains approximately 2.5 per round against TFT. This value is smaller than but not too far from $R=3$ per round, which would be obtained by mutual cooperation with TFT. Fig. 6(a) shows that the BM player cooperates with a large probability with generous tit-for-tat (GTFT) defined by $p=1$ and $q=1/3$ for the current payoff matrix. GTFT is a strong competitor in the iterated PD (Nowak and Sigmund, 1992). Although the BM player

occasionally defects against GTFT, the BM player gains $\approx R=3$ per round, which would be obtained by mutual cooperation.

The BM player does not play optimally against TFT. However, the BM player is generally strong against reactive strategies, as compared to TFT and GTFT. To support this, we plot the fraction of cooperation and the mean payoff for TFT against the reactive strategy in Fig. 6(b) and (e), respectively. The plotted values are analytical solutions obtained by Nowak and Sigmund (Nowak and Sigmund, 1989, 1990; Nowak, 1990, 2006), which are summarized in Appendix for completeness. As shown in Fig. 6(b), TFT does not cooperate with itself because TFT is intolerant to haphazard defection of the opponent (Nowak and Sigmund, 1992). In addition, TFT does not exploit ALLC. This is why TFT is eventually invaded by ALLD in evolutionary simulations in which ALLC, ALLD, and TFT coexist (Nowak and Sigmund, 1992, 1993). The payoff for TFT against the reactive strategy (Fig. 6(e)) is smaller than that for the BM player (Fig. 6(d)) for a wide range of p and q . This is particularly true for large values of p , which encompass TFT, GTFT, and ALLC.

The fraction of cooperation and the mean payoff for GTFT player against the reactive strategy are shown in Fig. 6(c) and (f), respectively (see Appendix for derivation). The GTFT player

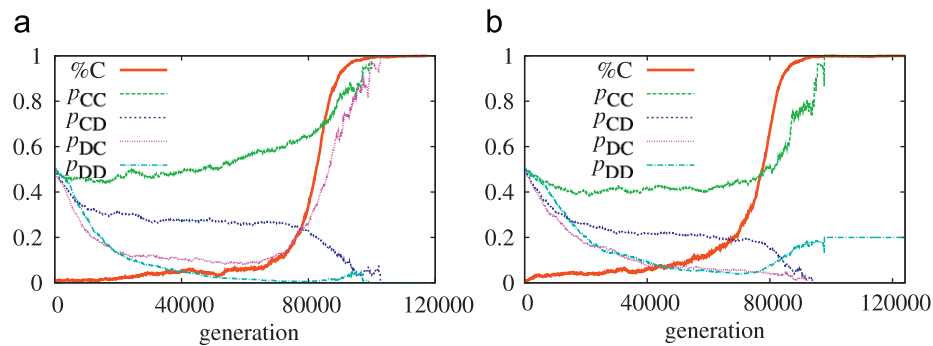


Fig. 7. Time courses of evolutionary dynamics. We set $t_{\max}=1000$, $\varepsilon=0.02$, $h=0.3$, $\beta=3$, $R=3$, $T=5$, $S=0$, and $P=1$. (a) Results for $N=2585$, $m=3$, and 4^4 types of memory-one strategies. (b) Results for $N=2617$, $m=5$, and 6^4 types of memory-one strategies.

performs better than the BM player and TFT for large p . However, GTFT is too generous to ALLC and ALLD. When p and q are both small or both large, the payoff for GTFT (Fig. 6(f)) is smaller than that for the BM player (Fig. 6(d)). In addition, the payoff for GTFT and that for the BM player are comparable when $q > p$ and when (p, q) is close to that of GTFT. On the basis of these numerical results, we conclude that the performance of the BM player against the reactive strategy is comparable to that of GTFT.

3.3. Evolutionary simulations

If the BM player is a strong competitor in the iterated PD, it should be able to evolve in a population in which different strategies coexist. To examine this point, we simulate evolutionary dynamics of populations where BM players and non-learners coexist in the beginning. We model non-learners by the stochastic memory-one strategy with which the player determines an action based on its own action and that of the opponent in the previous round (Nowak et al., 1995).

There are four types of outcomes of the pairwise interaction in a round, i.e., CC, CD, DC, and DD. The first and second letters (i.e., C or D) represent the actions of the focal player obeying the memory-one strategy (memory-one player for short) and the opponent, respectively. The memory-one player is parametrized by the action in the first round and four probabilities p_{CC} , p_{CD} , p_{DC} , and p_{DD} . The probability corresponding to the outcome of the present round is used as the probability that the memory-one player cooperates in the next round. For example, if both players cooperate, the memory-one player cooperates with probability p_{CC} in the next round. Initially, the memory-one player is assumed to cooperate with probability p_{CC} . The memory-one strategy includes many important strategies such as the reactive strategy and Pavlov (Nowak et al., 1995). We assume $p_{CC}, p_{CD}, p_{DC}, p_{DD} \in \{0, 1/m, 2/m, \dots, (m-1)/m, 1\}$ and that there are initially an equal number of memory-one players of each type. The case $m=3$ with a slight modification is employed in a previous study (Hauert and Stenull, 2002). To be realistic, we assume that both the memory-one player and the BM player misimplement the intended action with probability $\varepsilon=0.02$.

We denote the number of players in the population by N . Each player in the population plays against each of the other $N-1$ players iteratively for $t_{\max}=1000$ rounds in a single generation. The results shown in the following are qualitatively the same if t_{\max} is reduced to 500. We normalize the payoff of each player by dividing it by $(N-1)t_{\max}$ such that the payoff per generation falls between S and T . We update the strategy of the players during evolutionary dynamics according to the Fermi rule (Szabó and Tóke, 1998; Traulsen et al., 2006). At the end of each generation, we pick a pair of players i and j from the population with equal probability. We denote their single-generation payoffs as $r^{(i)}$ and $r^{(j)}$. With probability

$1/[1 + \exp(\tilde{\beta}(r^{(i)} - r^{(j)}))]$, player i copies the strategy of player j . With the remaining probability, player j copies the strategy of player i . We set $\tilde{\beta}=1$. If the parent (i.e., player whose strategy is copied) is the BM player, the child (i.e., player copying the strategy of the parent) becomes the BM learner. In this case, both the parent and the child start with $p_{t=1}=0.5$ and $A_{t=1}=(R+T+S+P)/4$ in the next generation. If the parent is a memory-one player, the child inherits the parent's parameter values p_{CC} , p_{CD} , p_{DC} , and p_{DD} . For simplicity, we do not consider mutations.

To examine the possibility that the BM player invades a population of players with various memory-one strategies, we start evolutionary simulations with 1% of the BM players in a population. Two time courses of typical runs when $h=0.3$ and $\beta=3$ are shown in Fig. 7. In Fig. 7(a), we set $m=3$ and prepare 10 memory-one players of each of the $4^4=256$ types and 25 BM players in the beginning. Therefore, $N=2560+25=2585$. In Fig. 7(b), we set $m=5$ and prepare two memory-one players of each of the $6^6=1296$ types and 25 BM players in the beginning. Therefore, $N=2592+25=2617$. In both cases, the BM players can invade the population of memory-one players to eventually become dominant. Within the memory-one players, those with large p_{CC} tend to survive at early stages of evolutionary dynamics before they are overwhelmed by the BM player.

4. Discussion

We numerically analyzed the behavior of a BM model in the iterated PD. Our model is a modification of the BM model used by Macy and Flache (2002) such that the probability of cooperation in the next round is made sensitive to the reinforcement signal obtained in the current round. Our model is also a close variant of the models used by Karandikar et al. (1998) and Posch et al. (1999). When the adaptation of the aspiration level is not too fast, the modified BM player mutually cooperates with a large probability. The BM player also performs efficiently against reactive strategies and in evolutionary dynamics in a population comprising various memory-one strategies. Up to our numerical efforts, the results are robust against the error in the action implementation and the change in the payoff matrix describing the PD.

The BM player performs at least comparably to memory-one players such as GTFT and Pavlov, which are strong competitors in the iterated PD. Although the BM player is inferior to these strategies when playing against TFT, it performs better than GTFT against other strategies including ALLC and ALLD. In an evolutionary context, naively cooperating with ALLC allows it to prosper by a neutral drift, which eventually invites the invasion of malicious players such as ALLD. Therefore, it is important to be able to exploit ALLC for a strategy to survive in evolutionary

dynamics (Nowak and Sigmund, 1993). This property is not satisfied by TFT (Axelrod, 1984), GTFT (Nowak and Sigmund, 1992), Pavlov (Kraines and Kraines, 1989; Nowak and Sigmund, 1993), and BM model with a fixed aspiration level (Macy, 1991, 1996; Posch et al., 1999; Macy and Flache, 2002). In contrast, our BM player as well as the temporal difference learner (Masuda and Ohtsuki, 2009) are capable of exploiting ALLC.

As a model of humans and other animals in iterated games, a learning strategy may be generally disadvantageous as compared to simpler learning strategies and non-learning strategies in at least two aspects. First, the number of rounds before learning is established may be large. For example, the temporal difference learning, a type of reinforcement learning, must be implemented with a very small learning rate to realize mutual cooperation (Masuda and Ohtsuki, 2009). In contrast, the learning of the modified BM model is completed in one to some hundreds of rounds. This speed of learning is comparable to that of other learning models in which mutual cooperation is obtained within ten to hundred rounds (Macy, 1991, 1996; Erev and Roth, 1998, 2001; Hauert and Stenull, 2002; Macy and Flache, 2002). Second, humans or other animals subjected to social dilemma situations may not implement a complex learning strategy. In this aspect, the BM model with the adaptive aspiration level, both the original ones (Karandikar et al., 1998; Posch et al., 1999; Macy and Flache, 2002) and ours, has a clear advantage. The BM model is simpler than many learning models including the temporal difference learning (Sandholm and Crites, 1996; Masuda and Ohtsuki, 2009), fictitious play (Erev and Roth, 1998; Fudenberg and Levine, 1998; Camerer, 2003), genetic algorithms (Macy, 1996), and artificial neural networks (Macy, 1996; Sandholm and Crites, 1996; Taiji and Ikegami, 1999; Gutnisky and Zanutto, 2004).

The memory-one strategy, for example, can be regarded as a reinforcement learning because the probability of cooperation is a function of the outcome in the previous round. This is also the case for analogous strategies with longer memory (Lindgren, 1991). Nevertheless, in this study, we are concerned with the cases where the probabilities of cooperation conditioned by the recent results of the game adapt over time. In the case of the BM model, adaptation is realized by the dynamic aspiration level. Learning players in this restricted sense cope with various types of opponents more flexibly than the memory-one strategy or its extension with longer memory. For example, we showed that the mean aspiration level of the BM player is almost equal to the mean payoff against different reactive strategies (Section 3.2). This result indicates that the BM player flexibly behaves as different types of win-stay lose-shift strategists depending on the opponent. Learning in games is a recent outstanding issue involving interdisciplinary research fields such as behavioral game theory and neuroeconomics (Fudenberg and Levine, 1998; Camerer, 2003; Glimcher et al., 2009). Because the effect of learning is evident in laboratory experiments (Camerer, 2003; Glimcher et al., 2009), it may be important to consider individual learning in addition to evolution to understand the behavior of agents, particularly that of humans, in social dilemma situations. Our model, which is simple yet competitive in the PD, may be used for examining various problems with regard to relationships between learning and cooperation in social dilemma situations.

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Appendix A. Reactive strategy against itself

When the focal player obeys the reactive strategy with parameters \bar{p} and \bar{q} , the long-term behavior of the focal player against the reactive strategy with parameters p and q can be analytically calculated (Nowak and Sigmund, 1989, 1990; Nowak, 1990, 2006). The probability that the focal player cooperates is given by

$$s_1 = \frac{q(\bar{p}-\bar{q})+\bar{q}}{1-(\bar{p}-\bar{q})(p-q)}. \quad (5)$$

The mean payoff of the focal player is given by

$$Rs_1s_2 + Ss_1(1-s_2) + T(1-s_1)s_2 + P(1-s_1)(1-s_2), \quad (6)$$

where s_1 is given by Eq. (5) and

$$s_2 = \frac{\bar{q}(p-q)+q}{1-(\bar{p}-\bar{q})(p-q)}. \quad (7)$$

References

- Axelrod, R., 1984. Evolution of Cooperation. Basic Books, NY.
- Bendor, J., Diermeier, D., Ting, M., 2003. A behavioral model of turnout. *Am. Political Sci. Rev.* 97 (2), 261–280.
- Börgers, T., Sarin, R., 2000. Naive reinforcement learning with endogenous aspirations. *Int. Econ. Rev.* 41 (4), 921–950.
- Boyd, R., Richerson, P.J., 1985. Culture and the Evolutionary Process. The University of Chicago Press, Chicago.
- Camerer, C.F., 2003. Behavioral Game Theory. Princeton University Press, NJ.
- Cho, I.K., Matsui, A., 2005. Learning aspiration in repeated games. *J. Econ. Theory* 124 (2), 171–201.
- Cross, J.G., 1983. A Theory of Adaptive Economic Behavior. Cambridge University Press, Cambridge.
- Daw, N.D., Doya, K., 2006. The computational neurobiology of learning and reward. *Curr. Opin. Neurobiol.* 16, 199–204.
- Dixon, H.D., 2000. Keeping up with the joneses: competition and the evolution of collusion. *J. Econ. Behav. Organ.* 43 (2), 223–238.
- Erev, I., 1998. Predicting how people play games: reinforcement learning in experimental games with unique, mixed strategy equilibria. *Am. Econ. Rev.* 88, 848–881.
- Erev, I., Roth, A.E., 2001. Simple reinforcement learning models and reciprocity in the Prisoner's dilemma game. In: Gigerenzer, G., Selten, R. (Eds.), *The Adaptive Toolbox*, pp. 215–231.
- Fudenberg, D., Levine, D.K., 1998. *The Theory of Learning in Games*. MIT Press, MA.
- Glimcher, P.W., Camerer, C.F., Fehr, E., Poldrack, R.A., 2009. Neuroeconomics—Decision Making and the Brain. Academic Press, London.
- Gutnisky, D.A., Zanutto, B.S., 2004. Cooperation in the iterated Prisoner's dilemma is learned by operant conditioning mechanisms. *Artif. Life* 10 (4), 433–461.
- Hauert, C., Stenull, O., 2002. Simple adaptive strategy wins the Prisoner's dilemma. *J. Theor. Biol.* 218 (3), 261–272.
- Izquierdo, L.R., Izquierdo, S.S., Gots, N.M., Polhill, J.G., 2007. Transient and asymptotic dynamics of reinforcement learning in games. *Games Econ. Behav.* 61 (2), 259–276.
- Izquierdo, S.S., Izquierdo, L.R., Gots, N.M., 2008. Reinforcement learning dynamics in social dilemmas. *J. Artif. Soc. Simul.* 11 (2), 1.
- Karandikar, R., Mookherjee, D., Ray, D., Vega-Redondo, F., 1998. Evolving aspirations and cooperation. *J. Econ. Theory* 80 (2), 292–331.
- Kim, Y., 1999. Satisficing and optimality in 2×2 common interest games. *Econ. Theory* 13 (2), 365–375.
- Kraines, D., Kraines, V., 1989. Pavlov and the Prisoner's dilemma. *Theory Decis.* 26 (1), 47–79.
- Lindgren, K., 1991. Evolutionary phenomena in simple dynamics. In: *Proceedings of Artificial Life II*, pp. 295–312.
- Macy, M., 1996. Natural selection and social learning in Prisoner's dilemma. *Sociol. Meth. Res.* 25 (1), 103–137.
- Macy, M.W., 1991. Learning to cooperate: stochastic and tacit collusion in social exchange. *Am. J. Sociol.* 97 (3), 808–843.
- Macy, M.W., Flache, A., 2002. Learning dynamics in social dilemmas. *Proc. Natl. Acad. Sci. USA* 99, 7229–7236.
- Masuda, N., Ohtsuki, H., 2009. A theoretical analysis of temporal difference learning in the iterated Prisoner's dilemma game. *Bull. Math. Biol.* 71, 1818–1850.
- Montague, P.R., Berns, G.S., 2002. Neural economics and the biological substrates of valuation. *Neuron* 36, 265–284.

- Napel, S., 2003. Aspiration adaptation in the ultimatum minigame. *Games Econ. Behav.* 43 (1), 86–106.
- Nowak, M., 1990. Stochastic strategies in the Prisoner's dilemma. *Theor. Popul. Biol.* 38, 93–112.
- Nowak, M., Sigmund, K., 1989. Game-dynamical aspects of the Prisoner's dilemma. *Appl. Math. Comput.* 30, 191–213.
- Nowak, M., Sigmund, K., 1990. The evolution of stochastic strategies in the Prisoner's dilemma. *Acta Applicandae Math.* 20, 247–265.
- Nowak, M., Sigmund, K., 1993. A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's dilemma game. *Nature* 364, 56–58.
- Nowak, M.A., 2006. *Evolutionary Dynamics*. The Belknap Press of Harvard University Press, MA.
- Nowak, M.A., Sigmund, K., 1992. Tit for tat in heterogeneous populations. *Nature* 355, 250–253.
- Nowak, M.A., Sigmund, K., El-Sedy, E., 1995. Automata, repeated games and noise. *J. Math. Biol.* 33 (7), 703–722.
- Oechssler, J., 2002. Cooperation as a result of learning with aspiration levels. *J. Econ. Behav. Organ.* 49 (3), 405–409.
- Palomino, F., Vega-Redondo, F., 1999. Convergence of aspirations and (partial) cooperation in the Prisoner's dilemma. *Int. J. Game Theory* 28 (4), 465–488.
- Pazgal, A., 1997. Satisficing leads to cooperation in mutual interests games. *Int. J. Game Theory* 26 (4), 439–453.
- Posch, M., Pichler, A., Sigmund, K., 1999. The efficiency of adapting aspiration levels. *Proc. R. Soc. London B* 266 (1427), 1427–1435.
- Rapoport, A., Chammah, A.M., 1965. *Prisoner's Dilemma: A Study in Conflict and Cooperation*. Michigan University Press, Ann Arbor, MI.
- Sandholm, T.W., Crites, R.H., 1996. Multiagent reinforcement learning in the iterated Prisoner's dilemma. *Biosystems* 37 (1–2), 147–166.
- Schultz, W., Dayan, P., Montague, P.R., 1997. A neural substrate of prediction and reward. *Science* 275, 1593–1599.
- Sigmund, K., 2010. *The Calculus of Selfishness*. Princeton University Press, Princeton, NJ.
- Simon, H.A., 1959. Theories of decision-making in economics and behavioral science. *Am. Econ. Rev.* 49 (3), 253–283.
- Szabó, G., Tóke, C., 1998. Evolutionary Prisoner's dilemma game on a square lattice. *Phys. Rev. E* 58 (1), 69–73.
- Tajji, M., Ikegami, T., 1999. Dynamics of internal models in game players. *Physica D* 134 (2), 253–266.
- Traulsen, A., Nowak, M.A., Pacheco, J.M., 2006. Stochastic dynamics of invasion and fixation. *Phys. Rev. E* 74 (1), 011909.
- Trivers, R.L., 1971. The evolution of reciprocal altruism. *Q. Rev. Biol.* 46, 35–57.