Stress analysis of adhesively bonded sandwich pipe joints subjected to torsional loading

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Abstract

Composite pipes are becoming popular in the offshore oil and gas industry. These pipes are connected to one-another by various configurations of joints. The joints are usually the weakest link in the system. In this investigation we examine the response of various joint configurations subjected to torsion, one of the most common loading conditions in piping systems. Specifically, the theoretical analysis used to evaluate the stress field in the adhesive layers of tubular and socket type bonded sandwich lap joints is presented here. The two adherends of the joints may have different thickness and materials, and the adhesive layer may be flexible or brittle. The analysis is based on the general composite shell theory. The stress concentrations at and near the end of the joints as functions of various parameters, such as the overlap length, and thickness of the adhesive layer are studied. The effects of different adherend thickness ratios, adhesive thickness and overlap length are also studied. Results obtained from the proposed analytical solutions agree well with the results obtained from finite element analysis and those obtained by other workers.

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1. Introduction

Composite materials such as fiberglass reinforced thermoset plastics have been used in piping systems for more than 40 years. This application of composite materials was developed in response to significant corrosion problems associated with metallic pipes in the chemical process and pulp and paper industries. Composite pipes have also been utilized in wastewater treatment, power, and petroleum productions. Most recently, composite tubes have been used in forming truss structures of space launch vehicles to reduce their weight. With properly developed manufacturing process for composite pipes (such as centrifugal casting and computer-controlled filament winding, within the past decade, the mechanical properties of such pipes have been dramatically improved. Ideally, a piping system would be designed without joints, since joints could be a source of weakness and/or excess weight. However, limitations on component size imposed by manufacturing process
and the requirement of inspection, accessibility, repair, and transportation/assembly necessitates some load-carrying joints in most piping systems. The rule of thumb states that one joint should be installed for every four feet of composite pipe in marine application, thus further demanding the development of reliable composite pipe joints.

The most commonly used joining methods for composite pipes are (i) adhesive-bonded socket joints, (ii) tubular lap joints, (iii) heat-activated coupling joints, and (iv) flanged joints. The first three configurations are considered as permanent joints, while the flanged joints provide the opportunity and ease of quick assembly/disassembly for installation, inspection, and repair. Nevertheless, most composite flanges are connected to composite pipes with one of the aforementioned three permanent joining methods. The same joint mechanism is found in adhesive-bonded socket joints, butt-and-strap joints, and heat-activated coupling joints. In all these joints there are essentially two pieces of composite pipes to be joined, a coupling to carry the load at the connection, and a medium to transfer the load from the pipes to the coupling.

While many articles are available in the literature on adhesive-bonded composite joints with “flat” configuration, much less work has been performed on investigating the characteristics of joints in composite piping systems.

Among the investigations carried out on pipe joints, the problem of torsional stress in tubular lap joints was first investigated by Volkersen (1965). In Volkersen’s analysis, the two tubular adherends of the joint were treated by mechanics of material approach, in which the presence of the circumferential shear stress was ignored, and the adhesive layer was treated as a sort of “shearing spring” acting between the two adherends. Following Volkersen’s work, Adams and Peppiatt (1977) improved Volkersen’s analysis by taking the thickness of the adhesive layer into account. Chen and Cheng (1992) based on the variational principle of complementary energy method, presented a stress distribution formulation for the adhesively bonded tubular lap joint under torsion.

All the above models considered only the isotropic adherends. Chon (1982) applied the two-dimensional polar theory for the analysis of tubular joints, by which the unknown parameters were related to the composite layers; the more layers, the more unknown parameters. Graves and Adams (1981) applied the finite element method for the analysis of the tubular lap joint composed of steel tube adhesively bonded to a composite tube. More recently, Yang (2000), Yang et al. (2002) used a one-dimensional model to simulate the system response of composite pipe joints under tensile loading by employing the two-dimensional theory for analyzing composite pipe joints under bending loading, by an orthotropic methodology.

In the present study, an analytical model for treatment of sandwich pipe joints subjected to torsional loading was developed. Sandwich laminated cylindrical shell theory was utilized to describe the kinematics and constitutive relations of the composite pipe and coupling. The model is capable of treating adherends with different thickness and/or different materials and layups, and the adhesive layer may be flexible or brittle. The solution is applicable for treating different pipe system joints, such as tubular lap joints, socket joints and heat-activated pipe joints. The stress concentrations at the end regions of the joints were investigated as functions of various parameters (such as the overlap length, and thickness of adhesive layer).

2. Model development

As described earlier, a generalized model was developed to cover all three types of composite pipe joints. The three components used in the model derivations and their representations in the actual joint systems are listed in Table 1.

<table>
<thead>
<tr>
<th>Model representation</th>
<th>Adhesive-bonded socket joint</th>
<th>Tubular lap joint</th>
<th>Heat-activated coupling joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupling</td>
<td>Socket</td>
<td>Fiber reinforcement</td>
<td>Epoxy prepreg</td>
</tr>
<tr>
<td>Adhesive</td>
<td>Adhesive</td>
<td>Resin matrix</td>
<td>Epoxy resin</td>
</tr>
<tr>
<td>Pipe</td>
<td>Pipe</td>
<td>Pipe</td>
<td>Pipe</td>
</tr>
</tbody>
</table>
2.1. Kinematics relations

The assumed displacement functions describing the behaviour of the cylindrical geometry shown in Fig. 1 can be stated based on Vinson (1993) and Seide (1975) by

\[
\begin{align*}
    u_x &= 0 \\
    w &= 0 \\
    u_s &= u_s(x)
\end{align*}
\]

which leads to the following displacement–strain relationships (Seide, 1975; Koiter, 1960) (see Fig. 1)

\[
\begin{align*}
    \varepsilon_x &= e_x = \kappa_x = \kappa_s = 0 \\
    \gamma_{xs} &= \frac{du_x}{dx} \\
    \kappa_{xs} &= \frac{3}{4R} \frac{du_x}{dx}
\end{align*}
\]

where \(u_x, w, u_s\) are the displacements in axial, radial and tangential directions, respectively; \(\varepsilon_x, e_s, \kappa_x, \kappa_s\) are the in-plane strains and bending curvatures in \(x\) and \(s\) directions, respectively. \(\gamma_{xs}\) is the shear strain and \(\kappa_{xs}\) is the modified twist. \(R\) is the radius of the mid-plane of the pipe wall.

2.2. Constitutive relations

The components of force and moments in the pipe subject to torsion can be defined by Seide (1975):

\[
\begin{align*}
    N_x &= N_s = M_x = M_s = 0 \\
    \overline{N}_{xs} &= N_{xs} + \frac{1}{2R} \overline{M}_{xs} = \int_{-h/2}^{h/2} \tau_{xs} \left[ 1 + \frac{z}{2R} \right] dz \\
    \overline{M}_{xs} &= \frac{1}{2} (M_{xs} + M_{sx}) = \int_{-h/2}^{h/2} \tau_{xs} \left[ 1 + \frac{z}{2R} \right] dz
\end{align*}
\]

where the forces and moment are defined as in Fig. 2(a) and (b).

The constitutive relations for the composite pipe, with reference to Fig. 3, can be written as:

\[
\begin{align*}
    \begin{cases}
    \overline{N}_{xs} &= \left[ K_{xs} + \frac{3}{4R^2} D_{xs} \right] \gamma_{xs} - 2A_{xs} \kappa_{xs} \\
    \overline{M}_{xs} &= -A_{xs} \gamma_{xs} + 2D_{xs} \kappa_{xs}
    \end{cases}
\end{align*}
\]
where $A_{xs}$ and $D_{xs}$ are the axial and flexural shear stiffness terms for pipes' material with:

$$
\begin{align*}
K_{xs} &= \sum_{i=1}^{N} G_{xs}^{(i)} h^{(i)} \\
D_{xs} &= \sum_{i=1}^{N} \left[ G_{xs}^{(i)} h^{(i)} \left( \frac{h^2}{4} - (z_i + z_{i-1}) \frac{h}{2} + \frac{1}{3} (z_i^2 + z_i z_{i-1}) \right) \right] \\
A_{xs} &= \sum_{i=1}^{N} G_{xs}^{(i)} h^{(i)} (z_i + z_{i-1} - h)
\end{align*}
$$

(6)

Fig. 2. Resultants forces and moments acting on the cylindrical shell element: (a) moment components; (b) force components.

Fig. 3. Notations used for the layered pipe.
in which, $G_{sx}$ is the shear moduli of the material, and with reference to Fig. 3:

$$z_i = \sum_{m=1}^{i} h^{(i)}$$
$$z_0 = 0$$

(7)

The shear strain related to the stress and moment resultants are given as:

$$\gamma_{sx} = \varphi_{sx} \left( N_{sx} + \frac{3}{2R} M_{sx} \right)$$

(8)

where

$$\varphi_{sx} = \frac{1}{K_{sx} + \frac{3}{R^2} D_{sx} - \frac{3}{R} A_{sx}}$$

(9)

2.3. Force equilibrium

In the joint region, as shown in Figs. 4 and 5, the adhesive resides between inner outer surfaces of the pipes. Based on the adhesive stress and the stress resultants in the pipe and coupling, the condition of the force equilibrium results in the following equation:

$$\frac{d}{dx} \left( N_{sx} + \frac{3}{2R} M_{sx} \right) = - \left( q_s + \frac{m_s}{R} \right)$$

(10)

---

**Fig. 4.** Schematic diagram of a tubular single-lap joint.

**Fig. 5.** Schematic diagram of a tubular socket joint.
where \( q_s \) and \( m_s \) are the resulting shear force and torsional moment on the middle section of interest (see Fig. 6(a) and (b)), which can be expressed by

\[
\begin{align*}
      m_s &= \left[ \left( 1 + \frac{h}{2R} \right) \tau_{xx}^{(h/2)} + \left( 1 - \frac{h}{2R} \right) \tau_{xx}^{(-h/2)} \right] \frac{h}{2} \\
      q_s &= \left( 1 + \frac{h}{2R} \right) \tau_{xx}^{(h/2)} - \left( 1 - \frac{h}{2R} \right) \tau_{xx}^{(-h/2)}
\end{align*}
\]

\( \tau_{xx}^{(h/2)} \) and \( \tau_{xx}^{(-h/2)} \) are defined as the shear stresses applied to the pipe’s outside and inside surfaces, respectively. For example, if a shear stress of \( \tau_a \) is applied to the outside surface of the pipe, one can then express the resulting shear force, \( q_s \), as: \( q_s = \left( 1 + \frac{h_1 + h_0/2}{2R_1} \right) \tau_a \), where \( h_1 \) is the thickness of the pipe, \( h_0 \) is the thickness of the adhesive layer, and \( R_1 \) is the radius of the pipe.

Thus, the force equilibrium for the two joining sections of the pipes is expressed by

\[
\begin{align*}
      \frac{d}{dx} \left( N_{xx}^1 + \frac{3}{2R_1} M_{xx}^1 \right) &= -\left( 1 + \frac{h_1 + h_0/2}{2R_1} \right) \tau_a - \frac{1}{R_1} \left( 1 + \frac{h_1 + h_0/2}{2R_1} \right) \frac{h_1 + h_0/2}{2} \tau_a \\
      \frac{d}{dx} \left( N_{xx}^2 + \frac{3}{2R_1} M_{xx}^2 \right) &= \left( 1 - \frac{h_2 + h_0/2}{2R_2} \right) \tau_a - \frac{1}{R_2} \left( 1 - \frac{h_2 + h_0/2}{2R_2} \right) \frac{h_2 + h_0/2}{2} \tau_a
\end{align*}
\]

where \( \tau_a \) is the shear stress in the adhesive. \( R_1 \) and \( R_2 \) are the radii of pipes 1 and 2, respectively.
3. Adhesive stresses

The coupling between the sandwich pipe adherends is established through the constitutive relations for the interface/"resin rich" layer, which is assumed to be homogeneous, isotropic and linear elastic. The constitutive equations are suggested as follows:

\[
\tau_a = \frac{G_a}{h_a} (u_{a2} - u_{a1}) - \frac{G_a}{2} \left( \frac{R_a}{R_2} \frac{3}{4R_2} u_{a2} + \frac{R_a}{R_1} \frac{3}{4R_1} u_{a1} \right) + \frac{G_a}{2} \frac{u_{a2} + u_{a1}}{R_a} = x_2 u_{a2} - x_1 u_{a1} \quad (13)
\]

where

\[
x_2 = G_a \left( \frac{1}{h_a} - \frac{3R_a}{8R_2^2} + \frac{1}{2R_a} \right)
\]

\[
x_1 = G_a \left( \frac{1}{h_a} + \frac{3R_a}{8R_1^2} - \frac{1}{2R_a} \right) \quad (14)
\]

where \( u_{a1} \) and \( u_{a2} \) are defined in Fig. 1.

4. Governing equations

Differentiating the interface constitutive equation (13), yields:

\[
\frac{d\tau_a}{dx} = x_2 \frac{du_{a2}}{dx} - x_1 \frac{du_{a1}}{dx} = x_2 \psi_{a2} \left( N_{a2}^2 + \frac{3}{2R_2} M_{a2}^2 \right) - x_1 \psi_{a1} \left( N_{a1}^1 + \frac{3}{2R_1} M_{a1}^1 \right) \quad (15)
\]

\[
\frac{d^2\tau_a}{dx^2} = \frac{d}{dx} \left( \frac{d\tau_a}{dx} \right) = x_2 \psi_{a2}^3 \left[ 1 - \frac{h_0/2 + h_2}{2R_2} - \frac{1}{R_2} \left( 1 - \frac{(h_0/2 + h_2)^2}{2} \right) \left( \frac{h_0/2 + h_2}{2} \right) \right] \tau_a
\]

\[
+ x_1 \psi_{a1}^1 \left[ 1 + \frac{h_0/2 + h_1}{2R_1} + \frac{1}{R_1} \left( 1 + \frac{(h_0/2 + h_2)^2}{2} \right) \left( \frac{h_0/2 + h_2}{2} \right) \right] \tau_a \quad (16)
\]

Letting

\[
\beta = x_2 \psi_{a2}^3 \left[ 1 - \frac{h_0/2 + h_2}{2R_2} - \frac{1}{R_2} \left( 1 - \frac{(h_0/2 + h_2)^2}{2} \right) \left( \frac{h_0/2 + h_2}{2} \right) \right] \tau_a
\]

\[
+ x_1 \psi_{a1}^1 \left[ 1 + \frac{h_0/2 + h_1}{2R_1} + \frac{1}{R_1} \left( 1 + \frac{(h_0/2 + h_2)^2}{2} \right) \left( \frac{h_0/2 + h_2}{2} \right) \right] \tau_a \quad (17)
\]

Eq. (16), therefore, can be expressed by simply:

\[
\frac{d^2\tau_a}{dx^2} - \beta \tau_a = 0 \quad (18)
\]

The general solution of Eq. (18) can be given by

\[
\tau_a(x) = \Psi_1 \cosh(\xi x) + \Psi_2 \sinh(\xi x) \quad (19)
\]

in which

\[
\xi^2 = \beta
\]

5. Boundary conditions

It can be shown that expressing the boundary conditions in term of \( \tau_a \) along the boundary \( x = \pm c \) (as shown in Figs. 4 and 5), one can obtain the following expressions:
(a) For tubular lap pipe joint (with reference to Fig. 4)

\[
\left[ 1 + \frac{h_1 + h_0}{2R_1} + \frac{1}{R_1} \left( 1 + \frac{h_1 + h_0}{2R_1} \right) \frac{h_1 + h_0}{2} \right] \int_{-c}^{c} \tau_a(x) \, dx
\]

\[
= \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} \right] M_{xx}^1 \right|_{x=-c}^c \right) - \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} \right] M_{xx}^1 \right|_{x=c}^c \right)
\]

\[
\left. \frac{d\tau_a(x)}{dx} \right|_{x=c} = \alpha_2 \phi_{ss}^2 \left( \left. \left[ \frac{N_{xx}^2 + 3}{2R_1} M_{xx}^2 \right] \right|_{x=c}^c \right) - \alpha_1 \phi_{ss}^1 \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} M_{xx}^1 \right] \right|_{x=c}^c \right)
\]

(20)

(21)

(22)

where \( T \) is the torsional moment. The explicit analytical solution for the coefficients \( \Psi_1, \Psi_2 \) can be represented by

\[
\Psi_1 = \frac{T}{4\pi R_1^2} \frac{\xi}{\xi \sinh(\xi c)}
\]

\[
\Psi_2 = \frac{T}{4\pi R_1^2} \frac{2\alpha_1 \phi_{ss}^1 - \xi^2}{\xi \cosh(\xi c)}
\]

(23)

in which

\[
\xi = \left[ 1 + \frac{h_1 + h_0}{2R_1} + \frac{1}{R_1} \left( 1 + \frac{h_1 + h_0}{2R_1} \right) \frac{h_1 + h_0}{2} \right]
\]

(24)

(b) For adhesive bonded socket pipe joint (with reference to Fig. 5)

\[
\left[ 1 + \frac{h_1 + h_0}{2R_1} + \frac{1}{R_1} \left( 1 + \frac{h_1 + h_0}{2R_1} \right) \frac{h_1 + h_0}{2} \right] \int_{-c/2}^{c/2} \tau_a(x) \, dx
\]

\[
= \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} \right] M_{xx}^1 \right|_{x=-c/2}^{c/2} \right) - \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} \right] M_{xx}^1 \right|_{x=c/2}^{c/2} \right)
\]

\[
\left. \frac{d\tau_a(x)}{dx} \right|_{x=c/2} = \alpha_2 \phi_{ss}^2 \left( \left. \left[ \frac{N_{xx}^2 + 3}{2R_1} M_{xx}^2 \right] \right|_{x=c/2}^{c/2} \right) - \alpha_1 \phi_{ss}^1 \left( \left. \left[ \frac{N_{xx}^1 + 3}{2R_1} M_{xx}^1 \right] \right|_{x=c/2}^{c/2} \right)
\]

(25)

(26)

Table 2
Material properties of the adherends

<table>
<thead>
<tr>
<th></th>
<th>Aluminum adherend</th>
<th>Steel adherend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (GPa)</td>
<td>68.9</td>
<td>206</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.30</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 3
Mechanical and physical properties of the sandwich composite

<table>
<thead>
<tr>
<th>Sandwich composite</th>
<th>Young's modulus (GPa)</th>
<th>Poisson's ratio</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top face sheet</td>
<td>( E_1 = 15, E_2 = 7.2 )</td>
<td>( v_{12} = 0.24, v_{21} = 0.12 )</td>
<td>0.5</td>
</tr>
<tr>
<td>Core</td>
<td>( E = 8.5 )</td>
<td>( v = 0.14 )</td>
<td>0.5</td>
</tr>
<tr>
<td>Bottom face sheet</td>
<td>( E_1 = 15, E_2 = 7.2 )</td>
<td>( v_{12} = 0.24, v_{21} = 0.12 )</td>
<td>0.5</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
N_{xx|x=c/2}^2 &= M_{xx|x=c/2}^2 = 0 \\
N_{xx|x=-c/2}^1 &= \frac{T}{2\pi R_1^2}, \quad M_{xx|x=-c/2}^1 = 0 \\
N_{xx|x=c/2}^1 &= \frac{T}{2\pi R_1^2}, \quad M_{xx|x=c/2}^1 = 0
\end{align*}
\]

Similarly, with the outlined tubular pipe joint procedure, the explicit solution of the coefficients $\Psi_1$, $\Psi_2$ for the socket pipe joint in torsion can be stated as:

![Graph](image)

Fig. 7. Comparison of the normalized adhesive shear stress results of the proposed solution to those of Adams and Peppiatt’s analytical and Hipol’s FEM models.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Mechanical properties of the adhesive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adhesive (epoxy resin)</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>3.33</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.34</td>
</tr>
</tbody>
</table>

![Graph](image)

Fig. 8. FEM mesh for the aluminum–aluminum tubular lap joint.
\[
\Psi_1 = -\frac{T}{4\pi R_1^2} \frac{\zeta}{\lambda \sinh \left(\frac{\zeta}{2}\right)} \\
\Psi_2 = -\frac{T}{4\pi R_1^2} \frac{2\zeta \psi_1 - \zeta^2}{\lambda \cosh \left(\frac{\zeta}{2}\right)}
\]

(28)

6. Applications and numerical results

In our numerical calculations, unless otherwise specified, we consider sandwich composite tubes that are bonded to a carbon steel tube. The material properties for the carbon steel and epoxy adhesive are given in Table 2. The sandwich composite material properties and dimensions are given in Table 3.

![Adhesive shear stress distributions](image)

Fig. 9. Comparison of the adhesive shear stress distributions obtained by the proposed analytical solution and the FEM solution for the tabular joints: (a) steel-steel; (b) aluminum-aluminum.
6.1. Isotropic adherends tubular pipe joint

The validity of the present formulation for use in assessing the integrity of an adhesive layer was examined by considering the analytical work of Adams and Peppiatt (1977) and FEM modeling of Hipol (1984). These studies investigated behaviour of tubular lap joints connecting similar isotropic adherends. The lap joint geometry considered had length of $2L = 40$ mm, the coupling length, $2c = 10$ mm, and the adhesive thickness, $h_0 = 0.2$ mm. The inner diameter of the joint, $R_m$, is $18.9$ mm. Each adherend was $1$ mm thick. The adherends material properties were: $E = 70,000$ MPa, $v = 0.33$. The adhesive material properties were: $E = 3500$ MPa, $v = 0.30$. The adhesive layer shear stress was normalized with respect to the mean adhesive shear stress obtained by $\tau_m = T/2\pi r_a c$, in which $T$ is the torque; $r_a$ and $c$ are the adhesive’s mean radii and the half of overlap length, respectively. The normalized results are compared with the results of Adams and Peppiatt.

![Fig. 10. Comparison of the adhesive shear stresses for the tubular pipe joints made of different adherend materials.](image1)

![Fig. 11. Comparison of the adhesive shear stress distributions obtained by the proposed analytical solution and the two FEM solutions for the aluminum-to-composite tubular pipe joint ($h_0 = 0.1$ mm).](image2)
(1977) and Hipol (1984) in Fig. 7, in which \(x\) is defined from the left overlap free end to the middle overlap section. Note that the results of the current study are slightly higher than those predicted by Hipol (1984). This slight discrepancy is believed to be due to the stiffer nature of FEM results, i.e., the stresses calculated by the FEM are always little lower than the analytical results.

6.2. Symmetric adherend material tubular joint

A tubular lap pipe joints with same symmetric adherends is considered in this example. The dimension of the analyzed geometry are: a total length of \(2L = 178\) mm, a coupling length of \(2c = 25\) mm, and the adhesive thickness of \(h_0 = 0.1\) mm. The outer diameter of the adherend, \(R_{out}\) is 16 mm and the inner diameter is variable, while the coupling thickness \(h_2\) is kept constant at 1.5 mm. The applied torsional load is
Three different adherends are used, namely: steel, aluminum and sandwich composite with material properties tabulated in Tables 2 and 3. The adhesive’s material properties are given in Table 4. The commercial software NISA (2004) was used to perform the analysis. The FEM model was constructed using NISA’s 3-D solid element (NKTP = 4), with a typical mesh illustrated in Fig. 8. Two mesh densities were used to conduct the analysis. A coarse mesh with the mesh density of 32 rows of elements circumferentially × 2 rows (radially) × 30 rows (axially), was used to model the pipe region, while the mesh density was doubled to model the joint region. Moreover, the mesh of the joint region was graded along the axial direction of the pipe, finer toward the free edges of the joint (as shown in Fig. 8), such that the length of elements near the free edge was 1/20 of the elements’ length sitting in the mid-span of the joint region. The boundary conditions for the problem were set as follows: \( u_r = 0 \), at \( x = 0 \) and \( x = 2L \); \( u_x = 0 \), \( x = 0 \); and \( u_r = 0 \), \( x = 0 \). Four force couples were applied at the free end of the composite pipe. The shear stress distribution on the adhesive along the axial direction of the system for the steel–steel and aluminum–aluminum tubular lap joints compared with the FEM results are shown in Fig. 9 (a) and (b), respectively. The figures indicate that the results obtained from the proposed analytical solutions are in good comparison with the FEM results. Moreover, the FEM results converge and become more closer to the analytical solution at the free edges of the joint as the mesh is refined. The comparison of the adhesive shear stress for the tubular lap joints formed of steel–steel, aluminum–aluminum and composite–composite adherends are given in Fig. 10. From Fig. 10, we can observe that the maximum shear stress is developed in the composite–composite tubular joint, and the minimum shear stress distribution is exhibited in the steel–steel tubular joint.

### 6.3. Aluminum-to-composite tubular lap joint

An aluminum-sandwich composite tubular lap pipe joint subject to a torsional load of \( T = 1,355,800 \text{ N mm} \) is considered in this example. The configuration is such that an aluminum pipe on the left is connected to a sandwich FRP pipe on the right. The dimensions of the components of the present example are the same as those outlined in the symmetric adherend material tubular joint considered earlier. The material properties are given in Tables 2–4.

The distribution of the shear stress along the axial direction of the pipe obtained by the proposed solution is compared with those obtained through the finite element analyses using different meshes, as illustrated in Fig. 11. From the figure we can see that the rate of change of shear stress is much steeper near the composite side and the predicted results agree well with the 3D FEM results. Moreover, results of the FEM analysis with the coarse and fine meshes compare very well, indicating the mesh convergency.
The influence of the adhesive thickness on stress distribution is shown in Fig. 12. It is clear from the figure that the rate of change of the stress is reduced when the thickness of adhesive is increased, and the maximum shear stress occurs in the thinner adhesive.

The influence of the lap length on the shear stress is illustrated in Fig. 13. The analysis examined joints with three different lap length of $2c = 12.5$ mm, $2c = 25$ mm and $2c = 50$ mm. One can see from the results that although the shear stress at mid-plane of the adhesive layer remains the same for most part of the lap length, the rate of change of the stress however varies as the adhesive length changes, and the maximum stress increases as the length of the adhesive decreases.

6.4. Symmetric adhesive bonded socket joint

Various socket pipe joints with same adherend (steel–steel, aluminum–aluminum and composite–composite) subjected to a torsion force with $T = 1,355,800$ N mm are considered in this section. The material prop-

![Fig. 15. Comparison of the adhesive shear stress distributions obtained by the proposed analytical solution and the FEM solutions for the symmetric socket pipe joints with different adherends: (a) steel–steel; (b) aluminum–aluminum.](image-url)
erties are those outlined for the symmetric tubular joint considered earlier. The adhesive thickness is \( h_0 = 0.1 \) mm. The dimensions of the analyzed geometry are: total length of \( 2L = 178 \) mm and a coupling length of \( 2c = 25 \) mm. The outer diameter of the adherend \( R_{out} \) is 16 mm and the inner diameter is variable. The coupling thickness \( h_2 \) is 1.5 mm. The joint was also analyzed by NISA finite element software. Nisa’s 3-D element (NTYPE = 4) was used to model the system, and similar coarse and fine meshes with the mesh refinement scheme described earlier were used to conduct the analysis. The boundary conditions and the meshes of this analysis are illustrated in Fig. 14. The adhesive shear stress distributions along the axial for the steel-to-steel and aluminum-to-aluminum socket joints are shown in Fig. 15(a) and (b), respectively. The results agree fairly closely with both coarse mesh and refined mesh FEM results. In order to investigate the effect of adherends on the torsional shear stress, a comparison of adhesive shear stress for the socket joints with steel–steel, aluminum–aluminum and composite–composite is shown in Fig. 16. It is seen from the figure that the rate of change of the shear stress is the highest for the composite–composite socket joint, and is the least for the steel-to-steel socket pipe joint.

6.5. Aluminum-to-composite socket joint

The aluminum-sandwich composite socket pipe joint under consideration is subjected to a torsional load of \( T = 1,355,800 \) N mm. The configuration of the joint is such that the two composite pipes are adhesively bonded by an aluminum pipe (i.e., the composite pipes are inside the aluminum pipe at the joint region). The dimensions of the components are the same as those outlined in the symmetric adherend material socket joint considered earlier. The materials’ properties are also tabulated in Tables 2–4. The shear stress distribution along the axial direction obtained by the proposed solution with different adherends’ materials is illustrated in Fig. 16. From the figure we can see that the rate of change of shear stress is much steeper near the left and right free ends for the composite–composite adherends’ joint.

The influence of variation of the adherends’ thickness on the adhesive shear stress for the socket joint system was examined by varying the thickness of the aluminum adherend of the aluminum-to-composite socket pipe joint. The results are plotted in Fig. 17. The figure indicates that after a certain thickness (in this case 1 mm), the increase in the thickness of the aluminum does not influence the shear stress distribution.

![Adhesive shear stress distributions versus normalized adherend overlap position obtained through the proposed model for the symmetric socket pipe joints with different materials.](image-url)
7. Conclusions

A theoretical formulation for evaluating the distribution of shear stress in adhesively bonded joint with various configurations was developed and presented. A three component joint system, consisted of a coupling section, adhesive, and pipes, was used for the development of the solution. Tubular and socket pipe joints with metallic and/or sandwich composite adherends subjected to torsional loading were considered to demonstrate the integrity of the proposed solution.

Even though the proposed model cannot predict the strength of a joint (because it does not employ any failure criteria for thin-film adhesive or resin matrix), the proposed solution can however be effectively used to reduce the peak interfacial shear stress of the adhesive by selecting appropriate geometrical entities. For example, through parametric analyses, one can investigate the influence of the thickness and material properties of adherends, as well as the thickness of adhesives, thereby optimizing the joint. With this analytical tool therefore, a composite pipe designer can gain a greater confidence when designing joints in piping applications.

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References