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Gravity assisted dark energy dominance and cosmic acceleration

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Abstract

It is proposed that dark energy may become dominant over standard matter due to universe expansion (curvature decrease). Two models: non-linear gravity–matter system and modified gravity may provide an effective phantom or effective quintessence dark energy which complies with the conjecture. The effective quintessence naturally describes current cosmic speed-up.

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1. Introduction

Growing evidence from high redshift surveys of supernovae, WMAP data analysis and other sources indicates that current universe expiriences a cosmic speed-up phase. Moreover, about 70 percent of total universe energy is attributed to puzzling cosmic fluid with large negative pressure (dark energy). The simplest possibility for dark energy is a cosmological constant (for recent review, see [1,2]) with equation of state parameter $w = -1$. Unfortunately, the dark energy due to cosmological constant requires huge fine-tuning. Moreover, it could be that the realistic value for w is slightly less than -1. Such phantom dark energy may be described by scalar field with negative kinetic energy [3]. Unfortunately, phantoms look inconsistent in many respects. In particular, phantom energy becomes infinite in finite time and such universe quickly evolves to Big Rip [4]. (Note, however, that finite time future singularity is possible even if strong-energy condition holds [5].) The number of other proposals for dark energy exist (for a review, see [6]) but most of them have also serious drawbacks. As a rule, the dark energy models do not respond to the question: why dark energy became the dominant contribution to energy density precisely at current epoch?

Recently, a gravitational alternative for dark energy was suggested [7] modifying the Einstein action by

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adding a $1/R^n$ term. Such a theory may produce the current cosmic speed-up, may be naturally generated by M-theory [8] and may give an effective phantom description with w above (or less) than -1. Of course, higher derivative gravity contains a number of instabilities [9] (for a recent discussion of possible deviations from Newton gravity, see [10]). Nevertheless, some modification of such a theory at high curvature (by higher derivatives and ln-terms) [11] leads to viable theory (with some amount of fine-tuning). This picture is supported by quantum effects account [11]. It is interesting that Palatini version of modified gravity may be also consistent [12]. Of course, as deviations from GR occur at low curvatures (for instance, in entropy studies [13]), more checks of such modified gravity should be done. Nevertheless, the important lesson drawn by such modification is possible, very simple explanation of current dark energy dominance. Indeed, gravitational dark energy is increased due to dynamical decrease of the curvature while FRW universe expands. It is extremely interesting to understand if such natural explanation may be applied to another dark energy models where probably gravity itself is less modified.

In the present Letter we make some attempt in the construction of dark energy which grows due to decrease of the curvature. Such gravity assisted dark energy dominance may occur due to direct (non-linear) gravitational coupling with matter-like Lagrangian as we show below.

2. Non-linear matter–gravity coupling as asymptotic dark energy and cosmic speed-up

Let us start from the following action:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{\kappa^2} R + \left(\frac{R}{\mu^2} \right)^\alpha L_d \right\}. \tag{1}$$

Here L_d is matter-like action (dark energy). The choice of parameter μ may keep away the unwanted instabilities which often occur in higher derivative theories. The second term in above action describes the non-linear coupling of matter with gravity (in parallel with $R\phi^2$ term which is usually required by renormalizability condition). Similarly, such term may be induced by quantum effects as non-local effective action. Hence, it is natural to consider that it belongs to matter sector. (Standard matter is not included for simplicity.) It is also interesting to remark that higher derivative kinetic term of above sort was proposed in [14] for the study of cosmological constant problem. However, this model is different from our theory because it also contained R^2 term in gravitational sector while the non-linear coupling of the potential with the curvature was not included.

By the variation over $g_{\mu\nu}$, the equation of motion follows:

$$0 = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = \frac{1}{\kappa^2} \left\{ \frac{1}{2} g^{\mu\nu} R - R^{\mu\nu} \right\} + \tilde{T}^{\mu\nu}. \tag{2}$$

Here the effective energy momentum tensor (EMT) $\tilde{T}_{\mu\nu}$ is defined by

$$\begin{aligned} \tilde{T}^{\mu\nu} \equiv{}& \frac{1}{\mu^{2\alpha}} \big\{ -\alpha R^{\alpha-1} R^{\mu\nu} L_d \\ &+ \alpha \big(\nabla^\mu \nabla^\nu - g^{\mu\nu} \nabla^2 \big) \big(R^{\alpha-1} L_d \big) + R^\alpha T^{\mu\nu} \big\}, \\ T^{\mu\nu} \equiv{}& \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \left(\int d^4x \sqrt{-g} L_d \right). \end{aligned} \tag{3}$$

In accord with our last remark, we consider last term in equation of motion as the effective matter EMT. For scalar matter below one may insist that above theory is just scalar-tensor gravity of unusual form (modified gravity). Nevertheless, other matter (fermion, Yang–Mills field) may be considered too. Then, the interpretation of above theory as modified gravity is doubtful.

Let free massless scalar be a matter

$$L_d = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \tag{4}$$

The metric is chosen to describe the FRW universe with flat 3-space:

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} \big(dx^i\big)^2. \tag{5}$$

If we assume ϕ only depends on t ($\phi = \phi(t)$), the solution of scalar field equation is given by

$$\dot{\phi} = q a^{-3} R^{-\alpha}. \tag{6}$$

Here q is a constant of the integration. Hence $R^\alpha L_d = \frac{q^2}{2a^6 R^\alpha}$, which becomes dominant when R is small (large) compared with the Einstein term $\frac{1}{\kappa^2} R$ if $\alpha > -1$ ($\alpha < -1$). Thus, one arrives at the remarkable possibility that dark energy grows to asymptotic dominance over the usual matter with decrease of the curvature.

Substituting (6) into (2), the $(\mu,\nu)=(t,t)$ component of equation of motion has the following form:

$$0=-\frac{3}{\kappa^2}H^2+\frac{36q^2}{\mu^{2\alpha}a^6(6\dot{H}+12H^2)^{\alpha+2}} \times\left\{\frac{\alpha(\alpha+1)}{4}\ddot{H}H+\frac{\alpha+1}{4}\dot{H}^2 +\left(1+\frac{13}{4}\alpha+\alpha^2\right)\dot{H}H^2+\left(1+\frac{7}{2}\alpha\right)H^4\right\}. \quad (7)$$

Especially when $\alpha=-1$, this equation has only the trivial solution $H=0$ (a is constant).

The accelerating solution of (7) exists

$$a=a_0t^{\frac{\alpha+1}{3}} \qquad \left(H=\frac{\alpha+1}{3t}\right),$$
$$a_0^6\equiv\frac{\kappa^2q^2(2\alpha-1)(\alpha-1)}{\mu^{2\alpha}3(\alpha+1)^{\alpha+1}(\frac{2}{3}(2\alpha-1))^{\alpha+2}}. \quad (8)$$

Eq. (8) tells that the universe accelerates, that is, $\ddot{a}>0$ if $\alpha>2$. If $\alpha<-1$, the solution (8) describes shrinking universe. But if we change the direction of the time as $t\to t_s-t$ (t_s is a constant), we have accelerating and expanding universe. In the solution with $\alpha<-1$ there appears a singularity at $t=t_s$, which is Big Rip singularity. For the matter with the relation $p=w\rho$, where p is the pressure and ρ is the energy density, from the usual FRW equation, one has $a\propto t^{\frac{2}{3(w+1)}}$. For $a\propto t^{h_0}$ it follows $w=-1+\frac{2}{3h_0}$, and the accelerating expansion ($h_0>1$) of the universe occurs if $-1<w<-\frac{1}{3}$. For the case of (8), we find $w=\frac{1-\alpha}{1+\alpha}$. Then if $\alpha<-1$, we have $w<-1$, which is an effective phantom. For the general matter with the relation $p=w\rho$ with constant w, the energy E and the energy density ρ behave as $E\sim a^{-3w}$ and $\rho\sim a^{-3(w+1)}$. Thus, for the standard phantom with $w<-1$, the density becomes large with time and might generate finite time future singularity (Big Rip).

For the solution (8), the first and second terms in (1) are of the same order. This is true for both of the early time (small t) and late time (large t) epochs. Let us take the case that the second term is dominant and the first term may be neglected. Assuming $H=\frac{h_0}{t}$, we find

$$h_0=\frac{1+\frac{13}{4}\alpha+\alpha^2\pm\sqrt{\alpha^4-\frac{\alpha^3}{2}}}{2(1+\frac{7}{2}\alpha)},$$

which is real if $\alpha\leqslant 0$ or $\alpha>\frac{1}{2}$. Since $R^\alpha L_d=\frac{q^2}{2a^6R^\alpha}$, the second term in (1) behaves as

$$R^\alpha L_d\sim t^{2\alpha-6h_0}=t^{\frac{-3-\frac{31}{4}\alpha+4\alpha^2\mp\sqrt{\alpha^4-\frac{\alpha}{2}}}{1+\frac{7}{2}\alpha}}.$$

Especially if α is large, $R^\alpha L_d\sim t^{2\alpha-6h_0}=t^{\frac{2}{7}\alpha,2\alpha}$. As the scalar curvature behaves as $R\sim t^{-2}$, if α is positive and large, the second term in (1) becomes surely dominant compared with the first Einstein term when R is small, that is, t is large. In the early universe (small t), the second term might be suppressed. Thus, if w is negative but bigger than -1 in the current universe, the model under discussion (effective quintessence [15]) describes the current cosmic speed-up. On the same time, such dark energy dominance is again explained by the universe expansion. One can also show that corrections to Newton law in this scenario are small.

In [16], the stability of the solution (8) has been investigated by replacing $a=at^{\frac{\alpha+1}{3}}$ with $a=at^{\frac{\alpha+1}{3}}(1+\delta)$ ($|\delta|\ll 1$). There δ has been assumed to only depend on the time variable t. Then the equation with account of perturbations is

$$0=\frac{1}{t}\frac{d^3\delta}{dt^3}+\frac{A_1}{t^2}\frac{d^2\delta}{dt^2}+\frac{A_2}{t^3}\frac{d\delta}{dt}+\frac{A_3}{t^4}\delta, \quad (9)$$

with constants A_1, A_2, A_3, and A_4. One may consider the case that δ depends on the spatial coordinates. Since this dependence appears through the d'Alembertian, after replacing the Laplacian with $-k^2$ (k is the magnitude of the momentum), Eq. (9) should be modified as

$$0=\frac{1}{t}\frac{d^3\delta}{dt^3}+\frac{A_1}{t^2}\frac{d^2\delta}{dt^2}+\frac{A_2}{t^3}\frac{d\delta}{dt}+\frac{A_3}{t^4}\delta +B_1\frac{k^2}{t^2}\frac{d^2\delta}{dt^2}+\frac{B_2k^2}{t}\frac{d\delta}{dt} \quad (10)$$

with constants B_1 and B_2. The newly added terms in (10) may be dominant when t is large but less dominant when t is small. When $w<-1$, after replacing t by t_0-t, small t corresponds to the case $t\sim t_0$. Then the inhomogeneity of the universe does not grow up when $w<-1$.

One can rewrite the action (1) by using the auxiliary field(s). First we introduce two scalar field ζ and η and rewrite (1) as

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{\kappa^2}\zeta+\zeta^\alpha L_d+\eta(R-\zeta)\right\}. \quad (11)$$

Using the equation $\zeta = R$ given by the variation over η, the action (11) is reduced into the original one (1). Varying over ζ, we obtain $\eta = \frac{1}{\kappa^2} + \alpha\zeta^{\alpha-1}L_d$. For $\alpha \neq 1$, one can delete ζ in (11) as

$$S = \int d^4x\sqrt{-g}\left\{\eta R + \left(\frac{1}{\alpha} - 1\right) \times \left(\eta - \frac{1}{\kappa^2}\right)^{\frac{1}{1-\alpha}} (\alpha L_d)^{\frac{1}{1-\alpha}}\right\}. \quad (12)$$

Writing η as $\eta = e^{-\sigma}$ and rescaling the metric as $g_{\mu\nu} \to e^{\sigma} g_{\mu\nu}$, the Einstein frame action follows:

$$S = \int d^4x\sqrt{-g}\left\{R - \frac{3}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \left(\frac{1}{\alpha} - 1\right) \times \left(e^{-\sigma} - \frac{1}{\kappa^2}\right)^{\frac{1}{1-\alpha}} \left(\alpha L_d\left(e^{\sigma} g_{\mu\nu}, \phi\right)\right)^{\frac{1}{1-\alpha}}\right\}. \quad (13)$$

Such the non-linear action includes Brans–Dicke type scalar σ and the scalar ϕ corresponding to the dark energy, that is, two scalars appear. However, as is expected the (non-standard) kinetic term for ϕ describes the coupling with σ on the same time. Some remark about the equivalence principle may be in order. First, note that there is a trivial solution with $q = 0$ in (5), where $H = R = 0$ and $\phi = 0$. Since the curvature in the present universe is small, one may assume H, R, $\phi \sim 0$. What about the perturbation around the solution $R = H = \phi = 0$ in the action (1)? If $\alpha > 0$, there does not appear ϕ in the perturbed action. Hence, if the usual matter action does not couple with ϕ directly, the equivalence principle is not violated.

Immediate generalization of (4) is to include the potential:

$$L_d = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \quad (14)$$

The solution may be found for a special choice (as an illustrative example) of $V(\phi) = V_0\phi^{2-\frac{2}{\alpha}}$ with a constant V_0, if we assume the FRW metric (5) and if $\alpha \neq 1$ and $\alpha \neq -1 + 3h_0$:

$$\phi = \phi_0 t^{\alpha}, \qquad H = \frac{h_0}{t} \qquad \left(a = a_0 t^{h_0}\right). \quad (15)$$

Here the constants ϕ_0 and h_0 are expressed in terms of the theory parameters. If $\alpha = -1 + 3h_0 < 0$, there is only trivial solution with $\phi_0 = h_0 = 0$. On the other hand, if $\alpha = -1 + 3h_0 > 0$ and $V_0 \neq 0$, there is no solution.

For the solution (15) with dominant second term in (1) and putting $\frac{1}{\kappa^2} \to 0$ one gets $h_0 = \frac{\alpha-3}{3(\alpha-2)}$. Hence, if $\frac{3}{2} < \alpha < 2$, $h_0 > 0$, what may correspond to the cosmic speed-up with $w = \frac{\alpha-1}{\alpha-3}$. This is an effective quintessence dark energy. If $1 < \alpha < 2$, we obtain an effective phantom with $w < -1$. In both cases, the current dark energy dominance is explained by the universe expansion. Similarly, one can analyze the potentials of other form.

As the generalization of other type we consider the model:

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{\kappa^2}R + f(R)L_d\right\}. \quad (16)$$

Assuming that $f(R)$ behaves as $f(R) \sim R^{\alpha}$ when R is small and $f(R) \sim R^{\beta}$ when R is large, as an example, one can take $f(R) = aR^{\alpha} + bR^{\beta}$, with $\alpha < \beta$. L_d is chosen in the form (4). When R is small, we obtain (8) again. On the other hand, when R is large:

$$a \propto t^{\frac{\beta+1}{3}}, \qquad w \sim \frac{1-\beta}{1+\beta}. \quad (17)$$

If $\beta > -1$, at the early time, the universe expands as (17). With the growth of time, $R \sim t^{-2}$ and when it becomes sufficiently small, the universe accelerates. Thus, the possibility to unify the early time inflation with current (asymptotic) dark energy dominance appears.

3. Modified gravity with time-dependent coefficients

Another class of models may be considered:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{\kappa^2} + \frac{\alpha(t)}{R} + \beta(t)\right). \quad (18)$$

Here $\alpha(t)$ and $\beta(t)$ are time-dependent phenomenological parameters. The above non-covariant action may be considered as an effective theory in parallel with the effective (non-covariant) theory of time-dependent cosmological constant. From another side, it may origin from more fundamental covariant gravity–matter system of the sort described in previous section.[2] In the FRW metric (5), the simple solution of

[2] The action (18) may origin from more complicated, non-linear action like $S = \int d^4x\sqrt{-g}(\frac{R}{\kappa^2} + \frac{L_\alpha}{R} + L_\beta)$. In fact one may choose

equation of motion is $H = \frac{h_0}{t}$, $\alpha = \frac{\alpha_0}{t^4}$, and $\beta = \frac{\beta_0}{t^2}$. Here the constants h_0, α_0, and β_0 are related by

$$0 = -\frac{3h_0^2}{\kappa^2} - \frac{\beta_0}{2} - \frac{(3h_0 - 2)\alpha_0}{12(2h_0 - 1)^2 h_0}. \tag{19}$$

Eq. (19) tells that α_0 and/or β_0 should be negative when $h_0 > \frac{2}{3}$. Since $a \propto t^{h_0}$, the cosmic acceleration ($\dot{a} > 0, \ddot{a} > 0$) occurs if $h_0 > 1$. $w \sim -1$ corresponds to the limit of $|h_0| \to \infty$. In case $|h_0| \gg 1$, in order that h_0 is real, one gets $\alpha_0 < 0$. Since $w = -1 + \frac{2}{3h_0}$, w can be found as

$$w = -1 \pm \frac{2}{3\sqrt{-\alpha_0\kappa^2}} \sqrt{1 + \sqrt{1 - \frac{3\alpha_0}{\beta_0^2\kappa^2}}}. \tag{20}$$

Then the plus sign in (20) corresponds to quintessence and the minus one to phantom. Again the interpretation of dark energy dominance due to the curvature decrease is immediate.

The action (18) may be rewritten in the form of the scalar-tensor theory. First by using the auxiliary field σ, the action (18) is

$$S = \int d^4x \sqrt{-g} \left\{ \left(1 - \frac{\kappa^2\sigma^2}{4\alpha(t)}\right) \frac{R}{\kappa^2} + \sigma + \beta(t) \right\}. \tag{21}$$

By defining new field φ as $e^{-\varphi} \equiv 1 - \frac{\kappa^2\sigma^2}{4\alpha(t)}$, we rescale the metric tensor as $g_{\mu\nu} \to e^{\varphi} g_{\mu\nu}$. Then the action (21) describes scalar-tensor theory with time-dependent potential:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{\kappa^2} \left(R - \frac{3}{2} \partial_\mu\varphi \partial^\mu \varphi \right) \pm \frac{2}{\kappa} e^{2\varphi} \sqrt{\alpha(t)\left(1 - e^{-\varphi}\right)} + e^{2\varphi}\beta(t) \right\}. \tag{22}$$

This indicates that such models of dark energy dominance are in different class (more close to modified gravity [7,11]) with non-linear gravity–matter theory of previous section. They may contain the instabilities [9].

L_α as in (14), $L_\alpha = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{0\alpha} \phi^4$, which corresponds to $\alpha = -1$ and L_β as $L_\beta = -\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - V_{0\beta} e^{-2\frac{\varphi}{\varphi_0}}$. Then after solving FRW equations, we find $L_\alpha \propto t^{-4}$ and $L_\beta \propto t^{-2}$. It is interesting that such model may be rewritten in the Einstein frame (similar to Eq. (20)) as two scalars-tensor gravity.

In summary, gravity assisted dark energy (the possible origin of cosmic speed-up) may become dominant over standard matter just because of the universe expansion (curvature decrease). It is a challenge to understand if such conjecture is true and if one of simple models proposed in this work may lead to dark energy theory which complies with observational data.

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