Unparticle effects on top quark rare decays

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1. Introduction

After the Large Hadron Collider (LHC) has been launched very recently, next decade will be a stage for a better understanding of the nature of the elementary particles and the interactions among them at TeV scale. On the one hand, LHC is expected to give a perfect understanding of the electroweak symmetry breaking of the Standard Model (SM) which is expressed through the Higgs mechanism. On the other hand, diversity of the new physics scenarios will be sought at the LHC. Having a mass about the electroweak energy scale and being the heaviest particle in the SM the top quark is one of the beacons of the LHC to shed light on the riddles of the energy effects of the scale invariant QCD, and to explore the new physics scenarios is the unparticle physics which is proposed by Refs. [16,17]. According to unparticle physics proposal given by Georgi, if there is a conformal symmetry in nature it must be broken at a very high energy scale which is above the current energy scale of the colliders. Considering the idea of Ref. [18], in the minimal or left–right supersymmetric Standard Model scenario, Refs. [12–14] or in the littlest Higgs model scenario, Ref. [15], or in the left–right supersymmetric model, Ref. [8] those ratios are found about $10^{-3}–10^{-6}$.

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The flavor changing neutral current (FCNC) decays of the top quark are highly suppressed in the SM (namely, the branching ratios for $t \rightarrow qZ$ and $t \rightarrow cg$ are predicted about $\sim 5 \times 10^{-14}$, and $\sim 1 \times 10^{-12}$, respectively. The parameter space of $\lambda$, $\Lambda$, and $d$ is obtained by taking into account the SM predictions and the results of the simulation performed by the ATLAS Collaboration for the branching ratios of $t \rightarrow cy$ and $t \rightarrow cg$ decays.

In this work we study the flavor changing neutral current (FCNC) decays of the top quark, $t \rightarrow cy$ and $t \rightarrow cg$, in the framework of the unparticle physics. The Standard Model predictions for the branching ratios of these decays are about $\sim 5 \times 10^{-14}$, and $\sim 1 \times 10^{-12}$, respectively. The parameter space of $\lambda$, $\Lambda$, and $d$ is obtained by taking into account the SM predictions and the results of the simulation performed by the ATLAS Collaboration for the branching ratios of $t \rightarrow cy$ and $t \rightarrow cg$ decays.

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transmutation. Thus, after the dimensional transmutation Eq. (1) is given as
\[ C_d^U \frac{d^{q+d-d}}{\Lambda_{l_d}^{d-2}} \alpha_{Qd_{l_d}} \frac{C_d^U}{O_{l_d} O_{SM}} \]

where \( d \) is the scaling mass dimension (or anomalous dimension) of the unparticle operator \( O_{l_d} \) (in Ref. [16], \( d = d_{l_d} \)), and the constant \( C_d^U \) is a coefficient function.

Interactions between the unparticles and the SM fields have been listed by [Ref. [19]]. Regarding the Georgi’s original point of view many work on the unparticle physics have been done so far, for example Ref. [20].

In this work, we study flavor changing neutral current decays \( t \rightarrow c'y' \), and \( t \rightarrow cg \) induced by scalar unparticles. Here we would like to make following remark. In principle, these decays can also get contributions via vector and tensor unparticle exchanges, and in present work we neglect them. Discarding these contributions can be explained as follows. Using Mack’s unitary constraint (see Ref. [21]) in Ref. [22] it was obtained that the lower bound of the scaling parameter \( d \) should be larger than three and four for vector and tensor unparticles, respectively. At these values of \( d \) the unparticle contributions to the considered decays are very tiny. For this reason in this work we consider contribution only from the scalar unparticles to the considered decays.

2. \( t \rightarrow c'y', g \) decays through unparticle

The effective interaction between the scalar unparticle and the SM quarks are given as [19]
\[ \frac{1}{\Lambda^d-1} \int \left( \frac{f' f}{\Lambda} + i \gamma_5 \frac{f' f}{\Lambda} \right) f' \]

where \( f \) and \( f' \) denote different flavor of quarks, with the same electric charge.

The scalar unparticle propagator is given as
\[ \Delta_f (p^2) = \frac{A_d}{2} \left( -p^2 - i \epsilon \right) d^{-2} \]

where
\[ A_d = \frac{16 \pi \sin^2 \theta}{(2\pi)^2} \Gamma(d + 1/2) \Gamma(d - 1/2)(2d). \]

The Feynman diagrams for the \( t \rightarrow cV \) decays through scalar unparticle is depicted in Fig. 1. The matrix element for the \( t \rightarrow cV \) \((V = \gamma, g)\) decay in general form can be written as follows
\[ M = \epsilon_{\mu \nu}^{(a)} \epsilon_{\mu \nu}^{(b)} (A_5 + A_6 \gamma_5) + \gamma_{\mu} (C + D \gamma_5) \]
\[ + \alpha_{\mu} (E + F \gamma_5) \]

where \( \epsilon_{\mu \nu}^{(a)} \) and \( \epsilon_{\mu \nu}^{(b)} \) are the polarization, and the momentum vector of the photon (gluon), respectively, and \( A_5, A_6, C, D, E \) and \( F \) are invariant amplitudes. From the gauge invariance we have \( C = D = 0 \). Since the photon (gluon) is on shell, i.e. \( q^2 = 0 \), and the transversality condition \( q_{\mu} \epsilon_{\mu}^{(a)} = 0 \), leads to the last term in Eq. (6) can safely be omitted. Other words, the \( t \rightarrow cV \) decay is described by magnetic moment type transition
\[ M = \epsilon_{\mu \nu}^{(a)} \epsilon_{\mu \nu}^{(b)} (A_5 + A_6 \gamma_5) \]

Fig. 1. Feynman diagrams for FCNC decays of the top quark through scalar unparticle.

Obviously the contribution of Figs. 1(a) and 1(b) are proportional to \( \epsilon_{\mu \nu}^{(a)} \epsilon_{\mu \nu}^{(b)} \) and \( \epsilon_{\mu \nu}^{(a)} \epsilon_{\mu \nu}^{(b)} \), therefore, can be omitted since they do not contribute to the structure \( \sigma_{\mu \nu} \). So, only diagram (c) presented in Fig. 1 should be considered. After some calculation for the invariant amplitudes \( A_5 \) and \( A_6 \) we get

\[ A_5 = \frac{A_{d6}^2}{2} \frac{N}{(4\pi)^2} \int_0^1 \frac{dx}{y(1 - x - y)^{d-1}} \]
\[ \times \left[ m_y (1 - x - y) (\lambda^c \lambda^c + \lambda^g \lambda^g) \right] \]
\[ - m_y (x + y) (\lambda^c \lambda^g + \lambda^g \lambda^c) \]
\[ - m_y (x + y) (\lambda^c \lambda^g + \lambda^g \lambda^c) \]
\[ \times \left[ - (p^2 + m_y^2)^2 + p^2 (x + y) - m_y^2 (x + y)^d - 2 \right]. \]

\[ A_6 = \frac{A_{d6}^2}{2} \frac{N}{(4\pi)^2} \int_0^1 \frac{dx}{y(1 - x - y)^{d-1}} \]
\[ \times \left[ m_y (1 - x - y) (\lambda^c \lambda^c + \lambda^g \lambda^g) \right] \]
\[ - m_y (x + y) (\lambda^c \lambda^g + \lambda^g \lambda^c) \]
\[ \times \left[ - (p^2 + m_y^2)^2 + p^2 (x + y) - m_y^2 (x + y)^d - 2 \right]. \]

where \( q = \{u, c, t\} \), when \( t \) or \( c \) quark running at loop only one of vertices contain flavor changing and another vertex is flavor diagonal. But when \( u \) quark runs at loop both vertices are flavor changing and therefore its contribution to the considered process compared to the \( t \) and \( c \) quark contributions should be very small. For this reason we will neglect \( u \) quark contributions in all next discussions. \( \lambda^c \) \((\lambda^g)\) and \( \Lambda \) are the scalar (pseudo-scalar) couplings and energy scale of unparticles, respectively. The couplings for the vector bosons are defined as \( g^V = Q_{\gamma, g} \), \( g^V = g_{\gamma, g} \).

Taking the square and the average of the amplitude gives
\[ |M| = \frac{N}{(4\pi)^2} \left[ A_5^2 + A_6^2 \right] \]
\[ \frac{m_t^2}{32\pi^4} \frac{N}{(4\pi)^2} \left[ p \cdot q \right] \]

where \( N \) is color factor given by \( \frac{4}{3} \) for the \( t \rightarrow cg \) and 1 for the \( t \rightarrow cV \) decay. Therefore, the FCNC decay width can be written as
\[ \Gamma = \frac{N}{(4\pi)^2} \frac{m_t^2}{32\pi^4} \left[ p \cdot q \right] \]

The FCNC top quark decay width \( \Gamma(t \rightarrow Vc) \) is calculated in terms of the unparticle coupling to the quarks \( \lambda \), the unparticle scale \( \Lambda \) and the scaling dimension \( d \). In numerical analysis, without loss of generality, for simplicity, we take \( \lambda = \lambda^c = \lambda^g \), and \( m_t = 175 \) GeV, \( m_c = 1.2 \) GeV, and \( \alpha = 1/128 \). We consider the total width of the top quark decay as \( \Gamma_{tot} = 1.5 \) GeV, which is mainly determined by the decay width of \( t \rightarrow bW^+ \).
In Figs. 2 and 3, we present the branching ratios for \( t \rightarrow c\gamma \) and \( t \rightarrow cg \) decays with respect to the scaling dimension \( d \) for various values of the coupling \( \lambda \) at \( \Lambda = 1 \) TeV. In these and the following figures the line (EXP) means the result of the simulations performed by the ATLAS Collaboration where the upper limits of the considered decays are obtained about \( 10^{-5} \) at 95% C.L., Ref. [10]. The SM prediction is represented by the solid horizontal line. From those figures we see that the branching ratio of \( t \rightarrow c\gamma \) and \( t \rightarrow cg \) decays decreases strongly with increasing \( d \), except \( d = 2 \). It is well known that for scalar unparticles at \( d = 2 \) there is infrared singularity. From the figures it also follows that the branching ratio of \( t \rightarrow c\gamma \) \( (t \rightarrow cg) \) decay becomes smaller than the SM prediction when \( d \leq 1.4 \) at \( \lambda = 10^{-2} \). If the coupling constant is larger than \( 10^{-2} \) then practically at all values of \( d \) in the considered region \( 1 < d < 2 \) branching ratio of \( t \rightarrow c\gamma \) \( (t \rightarrow cg) \) decay in the unparticle theory exceeds the SM one.

In Figs. 4 and 5 we present the dependence of the branching ratios on the parameter \( d \) for various values of the energy scale \( \Lambda \) at \( \lambda = 10^{-2} \). From these figures it follows that for \( \Lambda = 1–10 \) TeV up to \( d = 1.4 \) the branching ratio of \( t \rightarrow c\gamma \) \( (t \rightarrow cg) \) decay exceeds the SM one.

In Figs. 6 and 7 we present the dependence of the branching ratios to the coupling parameter \( \lambda \) for given values of the parameter \( d \). From these figures, one can observe that the branching ratio exceeds the SM prediction if \( \lambda > 10^{-2} \).

In Tables 1 and 2 we present numerical values of the branching ratios for \( t \rightarrow cg \), and \( t \rightarrow c\gamma \), respectively. One can explicitly see that experimental sensitivity is appropriate for only \( d < 1.3 \) for \( \lambda > 1 \times 10^{-1} \), however if the experimental sensitivity can be increased then the unparticle effects can be detected even if the coupling is about \( 10^{-2} \).

3. Conclusions

In present work, we study the FCNC rare decays of the top quark \( t \rightarrow c\gamma \) and \( t \rightarrow cg \) through scalar unparticle. Regarding the latest simulation performed by the ATLAS Collaboration, Ref. [10], the sensitivity to these rare decays of the top quark at 95% C.L. are \( \text{Br}(t \rightarrow c\gamma) = 2.8 \times 10^{-5} \), and \( \text{Br}(t \rightarrow cg) = 1.6 \times 10^{-5} \). If there is such a rare decay it will give a window to see the beyond SM physics effects. Using the low energy effective field description of the unparticle physics we show that FCNC decay of the top quark is very good channel to explore for and to put limits on the unparticle effects. We use the limits given by the ATLAS Collaboration to constrain the unparticle parameters. According to our results, one could expect to see unparticle effects for \( \Lambda = 1–10 \) TeV if the coupling is about \( \lambda > 10^{-2} \) for \( d < 1.3 \). This is consistent with the
existing results in the literature (see, for example Refs. [20,23–25], and references therein).

We want to remark that $t \rightarrow c \gamma$ or $t \rightarrow cg$ are loop level processes both in the SM and in the unparticle physics. However, $t \rightarrow cg$ can take place at tree level in the unparticle physics. The unparticle effects in the rare $t \rightarrow cg$ decays has been studied in Ref. [25]. In Table 3, we present a comparison our branching ratios $Br(t \rightarrow c \gamma)$, and $Br(t \rightarrow cg)$ with the branching ratios found in Ref. [25] for various values of the scaling parameter $d$. One could understand this behavior with the observation that the $t \rightarrow c \gamma$ or $t \rightarrow cg$ decays are proportional with $\alpha_{em}$ or $\alpha_t$ but the $t \rightarrow cgy$ or $t \rightarrow cgg$ decays depend on the unparticle coupling $\lambda$ which we take $10^{-2}$, is smaller than $\alpha_t$ but bigger than $\alpha_{em}$. Therefore, the behaviors of the branching ratios of $t \rightarrow cgy(g)$, and $t \rightarrow cgy(g)$ in the SM, and the unparticle physics are different.

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**References**