HISTORIA MATHEMATICA **23** (1996), 437–446
ARTICLE NOS. 0041–0043

REVIEWS

Edited by Catherine Goldstein and Paul R. Wolfson

All books, monographs, journal articles, and other publications (including films and other multisensory materials) relating to the history of mathematics are abstracted in the Abstracts Department. The Reviews Department prints extended reviews of selected publications.

Materials for review, except books, should be sent to the Abstracts Editor, Professor David E. Zitarelli, Department of Mathematics, Temple University, Philadelphia, Pennsylvania 19122, U.S.A. Books in English for review should be sent to Professor Paul R. Wolfson, Department of Mathematics, West Chester University, West Chester, Pennsylvania 19383, U.S.A. Books in other languages for review should be sent to Professor Catherine Goldstein, Bâtiment 425 Mathématiques, Université de Paris-Sud, F-91405 Orsay Cedex, France.

Most reviews are solicited. However, colleagues wishing to review a book are invited to make their wishes known to the appropriate Book Review Editor. (Requests to review books written in the English language should be sent to Prof. Paul R. Wolfson at the above address; requests to review books written in other languages should be sent to Prof. Catherine Goldstein at the above address.) We also welcome retrospective reviews of older books. Colleagues interested in writing such reviews should consult first with the appropriate Book Review Editor (as indicated above, according to the language in which the book is written) to avoid duplication.

Development of Mathematics, 1900–1950. Edited by Jean-Paul Pier. Basel/Boston/Berlin (Birkhäuser-Verlag). 1994. 729 pp. DM 118. £ 43.

Reviewed by Jeremy Gray

Faculty of Mathematics and Computing, Open University, Milton Keynes, MK7 6AA, England

Will we ever know the history of mathematics in the 20th century? The most likely answer is surely that we, at least, will not. Not because too much is too well hidden or too difficult to recover, as is the case with the political history of that violent century. Not because histories that are too Eurocentric are nowadays rightly dismissed as too narrow, thus enlarging the task impossibly. But simply because what is on the surface, written in a few well-known languages and published in accessible journals and books, is too much and too hard for us to master. Learning the history with all its imprecisions is harder, in some ways, than learning the mathematics (albeit easier in others), and we can wonder if we are asking the right questions. Before turning to the essays this book offers, consider its first 34 pages. This is a chronological list of 1000 important papers and books published between 1900 and 1950, compiled by asking some 50 experts. It cannot reasonably be supposed that any one person will have read these works and formed a fresh and accurate opinion of them, let alone of their much larger penumbra. Nor, of course,

0315-0860/96 $18.00

has the editor asked just one person to write the book. Instead we have 12 people who write about different aspects, and still much is left out. It could not be otherwise. Nor will the reader of this review fail to see that this single reviewer has his limitations.

The essays agree in a number of methodological ways. They present the history of mathematics as the growth of numerous branches of mathematics and so as the history of ideas. There are no quantitative methods here, no attempt to describe whole communities, almost no social history, and very little biography. There is very little attempt to show why the chosen subjects matter, or mattered in their own day. Why algebraic topology gets 115 pages of text and 6 of bibliography but partial differential equations only get 21 pages of text and 16 of bibliography is left unexplained. But we know the answer; it simply reflects the expertise available to Pier and his colleagues at that time faced with the impossible task of conveying the history of mathematics, 1900–1950. Even so, the choice of topics, with so much on mathematical logic (137 pages of text, 47 of bibliography), geometry subsumed by topology, and no algebra, makes one want to ask what belongs to mathematics, and what not?

As it happens, the essay on the history of topology is a gem. Jean Dieudonné was able to stand back from the details described in his much longer book [1] and give a remarkable account, with hints of motivation, helpful examples, and indications of personal and intellectual connections; the result is algebraic topology as a living subject, driven by the curiosity and the insights of numerous leading mathematicians (and not particularly constrained by the terminus of 1950 either). The omissions are part of the success of the story; the reader travels light, seeing worthwhile problems and their (sometimes) partial solutions.

Two shorter essays follow which can be read as offering methodological spice, even a hint of dissent. Doob's essay, "The Development of Rigor in Mathematical Probability, (1900–1950)," is a delight. The title already indicates that a sensible way through a vast amount of material has been found. Almost at once Doob writes: "Specific results are mentioned only in so far as they are important in the history of the logical development of mathematical probability." This is a good criterion for selection. Then he offers three famous opinions (by Planck, Poincaré, and Hermite) on progress in their subjects and follows it with three on the law of large numbers (that it is a theorem, a proposition, and a fact) and five on the definition of probability. Doob modestly deduces that this conflict of opinion requires one to separate mathematical probability from its real world applications, but it does much more: it establishes that there was a real debate about a substantial and difficult issue. Then he turns to measure theory, notes some of the work on Brownian motion, work of Borel, and Kolmogorov's memoir of 1933 (which was not immediately accepted). The author concludes by criticising the strange way in which many mathematicians try to hold measure theory and probability theory apart.

The next short essay, by Fichera, focuses on the evaluation of Volterra's work on functional analysis. He draws out its presuppositions and shows accordingly what it could do and where it was misleading, thus explaining its impact—ultimately negative—in Italy, because of Volterra's prestige. This vindicates the well-placed criticism of Dieudonné's account of the same material with which Fichera begins his essay: the distinction between doing history and writing an historical review. The latter exercise admits modern insights denied to the protagonists and invites historical misrepresentation. Reader—beware.

The other long essay, by Guillaume on mathematical logic, raises an awkward historical problem. No one disputes the profundity of the topic, but the field is deep and narrow, its relation to the rest of mathematics is difficult to elucidate and since the 1930s has become tenuous. Guillaume begins by invoking a number of the 19th-century sources of mathematical logic and then surveys a wide variety of issues in set theory, logic, and the foundations of mathematics. Despite the clarity of each paragraph, the overall effect is confusing. One is left wondering what the import of all this material was (in its day) and is (for the historian). In 137 pages of text and 47 bibliography, there has been some attempt at completeness, even though the author rightly admits at the very start that a completely faithful history would require several hundred pages. What, indeed, was the historical question at issue here? Most likely an attempt to say a bit about almost everything that seems to have lasted. Now it is a commonplace that Gödel's theorems put an end to Hilbert's programme to rigorise mathematics and with it the serious commitment of mathematicians to mathematical logic and the foundations of mathematics. The famous indifference of Bourbaki to these issues belongs here. But a commonplace need not be true. One might reasonably ask an historian to confront the question, and one might well ask Guillaume because his essay takes a sharp turn with Gödel's work. After the mid-1930s it becomes an account of specialists doing difficult technical work, innocently reinforcing our sense (if we are not logicians) of its remoteness. But although Gödel's theorems are said to have been an earthquake—and nine specific issues are listed as flowing from them as well as quite an amount of literature (good and not so good)—one misses any sense of what their historical significance was. If indeed the 19th-century programmes to make sense of mathematics all perished at this moment, then how and why? (I owe to a conversation with Ray Monk the realisation that the matter really is not simple.) Is the subsequent work as a result as dry, even irrelevant, as it seems? Better use of the growing historical literature might have helped to shape this essay.

The reader now enters the second half of the book: eight essays in just under 250 pages. There is a skill needed here by all of us, for the 30-page essay is what books and journals like. It helps if you have a topic of about the right size. Houzel's account of the prehistory of the Weil conjectures is one such topic. A small number of mathematicians progressively elucidated an area until one could formulate a series of rich questions which even as they were asked issued a profound challenge to the existing techniques (a rich mixture of algebraic number theory and algebraic geometry). Kahane's essay on Taylor series and Brownian motion addresses the issue of what mathematicians meant when they said something held in general; the question is a good one and the answer instructive. Mahwin's account of how questions in nonlinear ordinary differential equations led to the work of, amongst others, Leray and Schauder is rapid and dense, but it shares with Dieudonné's the feel that questions and problems led, in a natural way, to new methods and new problems. One picks up a sense of excitement.

The editor of the book, Jean-Paul Pier, contributed an essay on integration and measure (but not probability). It shares with Mahwin's the ability to connect things, to show this mathematician responding to that one. The emphasis is on the power of the techniques being developed, and it would have been interesting to see them do more; they were not, after all, put forward just for their own sake. The connection with functional analysis is left until the appearance of Schwartz, and that is also a pity, but this is the inevitable downside of a correct decision to pursue one aspect and make it intelligible.

The essays by Lichnerowicz, Nirenberg, Hayman, and Schwarz are less successful. The first of these is perhaps too short, with the result that the mathematics is too far away to be seen clearly, and one gets generalisations where precision was needed. Nirenberg's essay on partial differential equations is little more than a list of results (who first proved what, when). A whole book needs to be written on the topic, but in a limited space it would surely have been better to tell much less, with more spirit. Much the same can be said of Hayman's report on topics in complex analysis, which also says little new, but with less excuse. The paradox here is that while in its day Nevanlinna's theory was highly praised (by Hilbert and Weyl, no less), the topic of single variable complex function theory has ever since dwindled in esteem. This is not the type of historical development this genre of book can deal with easily. Schwarz's essay on the prime number theorem is equally factual, chronological, and unexciting.

The presence of Jean Dieudonné dominates the book. His is the first photograph to appear, and the first essay, but influence is felt in less visible ways: not perhaps in the choice of topics, but in the approach to history. This book is written in the dominant mode of history of mathematics, which emphasises the mathematical results. I have no criticisms of that mode, provided that one admits others elsewhere. What turns out to be curiously intangible is how the same approach can produce excitement in one essay and boredom in another. It may be partly what the reader (or reviewer) brings to the topic. It may be a literary skill. But sometimes the narrative mode is gripping: you want to read on, to know what happened next and why. Sometimes the result is facts, and one longs to bring them to life with questions. The story-telling skill of Dieudonné, the astute criticism of Fichera, and the dexterity of Doob are good examples for the historian to ponder, not least because they do not point in the same direction.

REFERENCES

1. J. Dieudonné, *A History of Algebraic and Differential Topology, 1900–1960,* Boston/Basel: Birkhäuser, 1989.