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Anti-screening by quarks and the structure of the inter-quark potential

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Abstract

The inter-quark potential is dominated by anti-screening effects which underly asymptotic freedom. We calculate the order g^6 anti-screening contribution from light fermions and demonstrate that these effects introduce a non-local divergence. These divergences are shown to make it impossible to define a coupling renormalisation scheme that renormalises this minimal, anti-screening potential. Hence the beta function cannot be divided into screening and anti-screening parts beyond lowest order. However, we then demonstrate that renormalisation can be carried out in terms of the anti-screening potential.

1. Introduction

Asymptotic freedom is the paradigm effect of QCD. It has been shown in many approaches [1–12] that the leading order beta function can be divided into screening and anti-screening effects. In QCD with n_f light fermions this decomposition reads

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[4 - \frac{1}{3} - \frac{2n_f}{3}\right], \tag{1}$$

where the dominant term (the 4) corresponds to anti-screening and the smaller $-\frac{1}{3} - \frac{2n_f}{3}$ terms describe screening by glue and by matter. It is well known that the gluonic screening effects are due to physical (gauge invariant) glue. Anti-screening [9] is due to the contribution of glue which is needed to construct a gauge invariant definition of a coloured charge [13].

The static inter-quark potential [14,15] would seem to offer a direct way to study the screening and anti-screening effects in QCD. The potential can be calculated via Wilson loops. At order g^4 the momentum space bare potential, $\tilde{V}^0$, in $D = 4 - 2\epsilon$ dimensions is, up to some finite terms,

$$\tilde{V}^0(\boldsymbol{k}^2) = -4\pi C_F \frac{\alpha_0}{\boldsymbol{k}^2} \times \left\{1 - \frac{\alpha_0}{\pi}\left(\frac{11}{12}C_A - \frac{n_f}{6}\right)\left[\log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right) - \frac{1}{\epsilon}\right]\right\}, \tag{2}$$

where ν is a dimensional scale parameter and, in $SU(N)$, $C_F = (N^2-1)/(2N)$ and $C_A = N$. This may be renormalised by the standard charge renormalisation where the bare coupling $\alpha_0 = \nu^{-2\epsilon} Z_\alpha \alpha$ where

$$Z_\alpha = 1 - \frac{\alpha}{\pi}\left(\frac{11}{12}C_A - \frac{n_f}{6}\right)\frac{1}{\epsilon}. \tag{3}$$

This yields to order α^2

$$\tilde{V}(\boldsymbol{k}^2) = -4\pi C_F \frac{\alpha}{q^2}\left\{1 - \frac{\alpha}{\pi}\left(\frac{11}{12}C_A - \frac{n_f}{6}\right)\log\left(\frac{q^2}{\nu^2}\right)\right\}. \tag{4}$$

The Wilson loop approach does not, however, display the screening/anti-screening decomposition of the potential. We will therefore study here the interaction between two gauge invariant descriptions of the colour charges. This has previously been seen [9,11] to show the decomposition of screening and anti-screening in $3+1$ dimensions as well as in the potential in $2+1$ dimensions [10] (where the beta function vanishes).

The structure of this Letter is as follows. We first review how a gauge invariant description of physical charges directly shows

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how anti-screening effects are produced. Then we calculate, for the first time, how at order g^6 anti-screening effects due to light fermions arise in the quark potential and that, due to non-local divergences, a minimal (or anti-screening) charge renormalisation approach breaks down here. Renormalising the potential directly is shown to consistently handle the non-local structures and the full result for the renormalised potential at this order is given.

It has previously been shown [16,17] that the correct gauge invariant description of static charges in the ground state is given by $h^{-1}\psi$ where h^{-1} is a field dependent dressing that surrounds the matter field ψ. The dressing has a rich structure which in QED is as follows:

$$h^{-1}(x) = \exp\left(ie \int_{-\infty}^{x^0} ds \, \frac{\partial^i F_{i0}}{\nabla^2}(s, \boldsymbol{x})\right) \exp\left(-ie \frac{\partial_i A_i}{\nabla^2}\right). \tag{5}$$

Here the second exponential is the *minimal dressing* which ensures that the minimally dressed matter field, $\exp(-ie\frac{\partial_i A_i}{\nabla^2})\psi$, is gauge invariant. The other factor, which we call the *additional dressing*, is itself gauge invariant.[1]

The structure of the dressing is reflected in calculations of the potential between charges: anti-screening coming from the minimal dressing [9] while the effects of screening were shown to come from the additional gauge invariant dressing [11]. The different factors in the dressing also make their presence separately felt in the infra-red structure of the on-shell Green's functions of dressed fields: soft divergences are cancelled by the effects of the minimal dressing, while phase divergences are removed by the additional dressing.

The description (5) has been extended to QCD order by order in perturbation theory. With the inclusion of colour, the *minimal dressing* up to order g^3 is given by [13]

$$h^{-1}(x) = \exp\big(g\chi(x)\big) + O\big(g^4\big), \tag{6}$$

with $\chi = \chi^a T^a = (\chi_1^a + g\chi_2^a + g^2\chi_3^a)T^a$ and

$$\chi_1^a = \frac{\partial_j A_j^a}{\nabla^2}, \qquad \chi_2^a = f^{abc}\chi^{bc}, \qquad \chi_3^a = f^{acb} f^{cef}\chi^{efb}, \tag{7}$$

where we have defined

$$\chi^{bc} = \frac{\partial_j}{\nabla^2}\left(\chi_1^b A_j^c + \frac{1}{2}\big(\partial_j\chi_1^b\big)\chi_1^c\right), \tag{8}$$

and

$$\chi^{efb} = \frac{\partial_j}{\nabla^2}\bigg(\chi^{ef} A_j^b + \frac{1}{2}A_j^e\chi_1^f\chi_1^b - \frac{1}{2}\chi^{ef}\big(\partial_j\chi_1^b\big) + \frac{1}{2}\big(\partial_j\chi^{ef}\big)\chi_1^b - \frac{1}{6}\big(\partial_j\chi_1^e\big)\chi_1^f\chi_1^b\bigg). \tag{9}$$

To now calculate the potential between such charges, we take a quark and an anti-quark, both dressed according to Eq. (6), average over colours and study the expectation value of the QCD Hamiltonian. The potential is given by the dependence of the energy on the quark separation, $r := |\boldsymbol{y} - \boldsymbol{y}'|$. The lowest order contribution from either charge is of order g and so, to calculate the potential at order g^4 between two charges, we only need to expand the two dressings up to order g^3. The potential is therefore

$$V(r) = \frac{1}{2N^2}\int d^3x \,\big\langle\bar\psi(y)h(y)h^{-1}(y')\psi(y')\big| \times \big(E_i^{a2}(x) + B_i^{a2}(x)\big)\big|\bar\psi(y)h(y)h^{-1}(y')\psi(y')\big\rangle, \tag{10}$$

which implies

$$V(r) = -\frac{1}{N}\,\mathrm{tr}\int d^3x\,\langle 0|\big[E_i^a(x), h^{-1}(y)\big]h(y) \times \big[E_i^a(x), h^{-1}(y')\big]h(y')|0\rangle, \tag{11}$$

where the trace is over colour and we have used the fact that B_i^a commutes with the minimal dressing.

It follows at leading order from (11) that

$$V(r) = -\frac{g^2}{N}\,\mathrm{tr}\int d^3x\,\langle 0|\big[E_i^a(x), \chi_1^d(y)\big] \times T^d T^b\big[E_i^a(x), \chi_1^b(y')\big]|0\rangle. \tag{12}$$

Inserting the fundamental equal time commutator, $[E_i^a(x), A_j^b(y)] = i\delta^{ab}\delta(\boldsymbol{x}-\boldsymbol{y})$, into this last equation gives at leading order

$$V(r) = -\frac{g^2 C_F}{4\pi r}. \tag{13}$$

We recognise the Coulombic inter-quark potential [19,20].

In general, and especially at higher orders, it is simpler to work in momentum space. Integral representations based upon the identity

$$\frac{1}{(\boldsymbol{x}^2)^a} = \frac{4^{\frac{d}{2}-a}\pi^{\frac{d}{2}}\Gamma(\frac{d}{2}-a)}{\Gamma(a)}\int \frac{d^dq}{(2\pi)^d}\frac{1}{(\boldsymbol{q}^2)^{\frac{d}{2}-a}}e^{i\boldsymbol{q}\cdot\boldsymbol{x}}, \tag{14}$$

make the calculations much easier. In this way the contribution to the quark potential from the minimal dressing at $\mathcal{O}(g^4)$ has previously [9] been shown to be

$$-3g^4 C_F C_A \frac{k_l k_m}{\boldsymbol{k}^4}\int \frac{d^dp}{(2\pi)^d}\frac{i\tilde D_{lm}^{TT}(\boldsymbol{p})}{(\boldsymbol{k}-\boldsymbol{p})^2}, \tag{15}$$

where d is the number of spatial dimensions ($d = 3 - 2\epsilon$) and the tree level equal time gluon propagator in momentum space is given by

$$i\tilde D_{lm}(\boldsymbol{p}) = \int d^dx\, iD_{lm}(0, \boldsymbol{x})e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}. \tag{16}$$

The superscript T in (15) signifies projection upon the transverse components, $k_i A_i^T = 0$. This shows the gauge invariance of (15) and it is straightforward, if tedious, to show that the longitudinal, gauge dependent A_i^L fields cancel in this result. At order g^4 this corresponds to inserting the free transverse projected, equal time propagator

$$\big\langle A_j^T(w)A_k^T(z)\big\rangle = \frac{1}{2\pi^2}\frac{(z-w)_j(z-w)_k}{|\boldsymbol{z}-\boldsymbol{w}|^4}, \tag{17}$$

[1] The dressing may be obtained from the requirement of gauge invariance plus an additional dressing equation which may be derived [16] from the heavy charge effective theory or from a study [18] of the asymptotic dynamics of charged particles.

which yields the dominant part of the bare potential corresponding to anti-screening:

$$\tilde{V}^0_{\min}(\boldsymbol{k}^2) = -4\pi C_F \frac{\alpha_0}{\boldsymbol{k}^2} \times \left\{1 - \frac{\alpha_0}{\pi} C_A \left[\log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right) + 2\log(2) - \frac{7}{3} - \frac{1}{\epsilon}\right]\right\}. \tag{18}$$

This should be contrasted with the bare potential (2). The difference between the divergences in these two results is due to screening. The equations clearly show that gluons screen as well as anti-screen. The screening effect is due to transverse, gauge invariant glue from the additional dressing. The relative weighting of gluonic anti-screening to screening by glue is 12 to 1. (There is no anti-screening contribution from the matter fields at this order.)

These effects have also been calculated [10] in $2+1$ dimensions where it was seen that the relative weighting of anti-screening and screening in the potential is the same within 1%.

2. The leading in n_f potential at order g^6

At the next order in the coupling there are contributions from gluons and from light quarks. Here we will calculate the quark contribution, i.e., the n_f dependent terms, to the minimal dressing. As is well known, quarks produce at next to leading order a screening of (electric and) colour charges and we have seen above that, at order g^4, there are no contributions from quarks to the minimal anti-screening potential. However, we will now show that at next to next to leading order quarks also produce an anti-screening effect. This contribution is needed to ensure gauge invariance at higher orders. It occurs through the one loop, fermionic correction to the gluon propagator in (15).

In addition to (15) there are other contributions to the minimal potential at order g^6. They arise by higher order expansions of the dressings and will involve Green's functions such as $g^5\langle 0|AAA|0\rangle$ and $g^6\langle 0|AAAA|0\rangle$. These Green's functions will only depend on the n_f light fermions through loops and it is easy to see that they will first introduce quark contributions beyond order g^6 in the coupling. We conclude that the first quark contribution to the anti-screening potential comes from (15) alone.

It should also be noted that although the QCD two point function $\langle 0|A^T A^T|0\rangle$ in (15) is not generally gauge invariant at higher orders (see Appendix A of [13]), at one loop its n_f dependent part is indeed gauge invariant. At order g^2 we have the well-known n_f dependent term from the one loop contribution to the gluon polarisation

$$\varPi(p) = \frac{g^2 n_f}{(4\pi)^{\frac{D}{2}}} \frac{D-2}{D-1} \left(-p^2\right)^{\frac{D}{2}-2} \frac{\Gamma(2-\frac{D}{2})\Gamma^2(\frac{D}{2}-1)}{\Gamma(D-2)}, \tag{19}$$

where we skip the obvious transverse projection tensor. This enters the one loop propagator via the contribution, $iDi\varPi iD$, which implies

$$\int d^D x\, \langle 0|\mathrm{T}A_i(x)A_j(0)|0\rangle \mathrm{e}^{-ip\cdot x} = -\frac{i}{(p_0^2-\boldsymbol{p}^2)^2}\left[p_i p_j + \delta_{ij}\left(p_0^2-\boldsymbol{p}^2\right)\right]\varPi\left(p_0^2-\boldsymbol{p}^2\right). \tag{20}$$

The one loop equal time propagator in momentum space, $i\tilde{D}_{ij}$ is now defined to be

$$i\tilde{D}_{ij}(\boldsymbol{p}) = -\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \frac{i}{(p_0^2-\boldsymbol{p}^2)^2}\left[p_i p_j + \delta_{ij}\left(p_0^2-\boldsymbol{p}^2\right)\right] \times \varPi\left(p_0^2-\boldsymbol{p}^2\right). \tag{21}$$

Projecting onto the transverse components (which are gauge invariant at this order in g) via $\delta_{il} - p_i p_l/\boldsymbol{p}^2$ and $\delta_{jm} - p_j p_m/\boldsymbol{p}^2$ gives

$$i\tilde{D}^{TT}_{lm}(\boldsymbol{p}) = -\left(\delta_{lm} - \frac{p_l p_m}{\boldsymbol{p}^2}\right)\int_{-\infty}^{\infty} \frac{dp_0}{2\pi}\frac{i}{(p_0^2-\boldsymbol{p}^2)}\varPi\left(p_0^2-\boldsymbol{p}^2\right). \tag{22}$$

Inserting (19) yields

$$i\tilde{D}^{TT}_{lm}(\boldsymbol{p}) = -g^2 n_f \left(\delta_{lm} - \frac{p_l p_m}{\boldsymbol{p}^2}\right) \times \frac{1}{(\boldsymbol{p}^2)^{2-\frac{d}{2}}} \frac{1}{2^{2+d}\pi^{\frac{d}{2}}} \frac{\Gamma(3-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{5-d}{2})\Gamma(\frac{2+d}{2})}. \tag{23}$$

To calculate the n_f dependent part of the potential, we now insert this into (15). It is helpful, though, to rewrite the resulting expression via

$$1 - \frac{(\boldsymbol{k}\cdot\boldsymbol{p})^2}{\boldsymbol{k}^2\boldsymbol{p}^2} = \frac{1}{4}\left(2 - \frac{\boldsymbol{k}^2}{\boldsymbol{p}^2} - \frac{\boldsymbol{p}^2}{\boldsymbol{k}^2}\right) + \cdots, \tag{24}$$

where we have *dropped* terms that only contribute massless tadpoles in the subsequent integral and will hence vanish in dimensional regularisation. This leads to the order g^6 contribution to the potential (18)

$$\frac{g^6 n_f C_F C_A}{\boldsymbol{k}^2} \frac{3}{2^{4+d}\pi^{\frac{d}{2}}} \frac{\Gamma(3-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{5-d}{2})\Gamma(\frac{2+d}{2})} \times \int \frac{d^d p}{(2\pi)^d} \frac{1}{(\boldsymbol{p}^2)^{2-\frac{d}{2}}(\boldsymbol{k}-\boldsymbol{p})^2}\left(2 - \frac{\boldsymbol{k}^2}{\boldsymbol{p}^2} - \frac{\boldsymbol{p}^2}{\boldsymbol{k}^2}\right). \tag{25}$$

The divergent part of this contribution, in terms of the bare coupling α_0, is

$$\frac{\alpha_0^3 n_f C_F C_A}{3\boldsymbol{k}^2\pi} \times \left\{\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left[\frac{14}{3} - 2\gamma_E - 2\log\left(\frac{\boldsymbol{k}^2}{4\pi\nu^2}\right) - 4\log(2)\right]\right\}. \tag{26}$$

Note that these divergences are ultra-violet singularities as can be seen from power counting in (15) and (23). The leading singularity here is local, but the sub-leading divergences include the $\frac{1}{\epsilon}\log(\boldsymbol{k}^2)$ term which is a non-local divergence. The immediate question is can renormalisation deal with this infinity?

3. Renormalising the minimal potential

The minimal part of the inter-quark potential has been previously calculated at order g^4 in both four and three dimensions. The result in four dimensions (18) may be renormalised by using *minimal* charge renormalisation where we define the minimally renormalised coupling, α', through $\alpha_0 = Z_{\alpha'}^{\min}\alpha'$ where

$$Z_{\alpha'}^{\min} = 1 - \frac{\alpha'}{\pi} C_A \frac{1}{\epsilon}. \tag{27}$$

This anti-screening renormalisation is defined so that

$$\tilde{V}_{\min}(\boldsymbol{k}^2) = -4\pi C_F \frac{\alpha'}{\boldsymbol{k}^2}\left[1 - \frac{\alpha'}{\pi} C_A \frac{1}{\epsilon}\right] \times \left\{1 - \frac{\alpha'}{\pi} C_A\left[\log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right) - \frac{1}{\epsilon}\right]\right\},$$

is finite at this order in the minimal coupling:

$$\tilde{V}_{\min}(\boldsymbol{k}^2) = -4\pi C_F \frac{\alpha'}{\boldsymbol{k}^2}\left\{1 - \frac{\alpha'}{\pi} C_A \log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right)\right\} + \mathcal{O}(\alpha'^3). \tag{28}$$

This is a very direct way to extract the minimal, anti-screening beta function which has also been observed in very different ways [4]. This minimal coupling clarifies the nature and importance of anti-screening in non-Abelian gauge theories.

It is, however, very simple to show that the 'anti-screening coupling' cannot be used at next order. We have seen that there is a non-local, n_f dependent divergence in the minimal potential at order g^6 and this is the *only* n_f dependence in the minimal potential (26) up to this order. There is a logarithm at order α_0^2 which might help produce non-local divergences at α_0^2 but it is not n_f dependent and in the leading anti-screening charge renormalisation (27) there is no n_f dependence either. Any n_f dependence in $Z_{\alpha'}^{\min}$ at order α'^3 would, of course, be local and not introduce any logs into the potential at order α'^3. Thus *nothing* can cancel the non-local divergence at order α'^3 in this approach. We are forced to conclude that the anti-screening or minimal charge renormalisation of the minimal potential breaks down beyond leading order. It is, in other words, impossible to define a coupling renormalisation scheme that renormalises the minimal, anti-screening potential. We cannot, beyond lowest order, speak of screening and anti-screening structures in the beta function.

It is, however, not necessary to use the anti-screening coupling in the minimal potential. Instead one can use full coupling renormalisation (3) plus an additional multiplicative renormalisation of the minimal potential in (18):

$$\tilde{V}_{\min} = \nu^{-2\epsilon} Z_V \tilde{V}_{\min}^0(\boldsymbol{k}^2), \tag{29}$$

where we write

$$Z_V = 1 + \delta_V^1 \frac{\alpha}{\pi} + \delta_V^2\left(\frac{\alpha}{\pi}\right)^2 + \cdots. \tag{30}$$

The minimal potential is easily seen to be finite at this order if

$$\delta_V^1 = -\left(\frac{1}{12} C_A + \frac{n_f}{6}\right)\frac{1}{\epsilon}. \tag{31}$$

This corresponds to

$$\tilde{V}_{\min} = -4\pi C_F \frac{\alpha}{\boldsymbol{k}^2}\left\{1 - \frac{\alpha}{\pi} C_A \log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right)\right\} + \mathcal{O}(\alpha^3). \tag{32}$$

Our interpretation of this additional factor, Z_V, is that it is a renormalisation of the additional potential energy between excited, minimally dressed charges compared to the true ground state of the fully dressed system, i.e., with screening effects included.

We will now show that this second approach may still be used at the next order of perturbation theory, i.e., the minimal potential is indeed renormalised by the full coupling (3) and the potential renormalisation of (29) and (30). At order α^3 scheme dependence appears and we use the $\overline{\text{MS}}$ scheme. We require the standard two loop coupling renormalisation

$$Z_\alpha = 1 + z_\alpha^1 \frac{\alpha}{\pi} + z_\alpha^2\left(\frac{\alpha}{\pi}\right)^2, \tag{33}$$

where

$$z_\alpha^2 = \frac{1}{\epsilon^2}\left(\frac{11 C_A}{12} - \frac{n_f}{6}\right)^2 - \frac{1}{\epsilon}\left(\frac{17 C_A^2}{48} - \frac{C_F n_f}{16} - \frac{5 C_A n_f}{48}\right). \tag{34}$$

We now define

$$\delta_V^2 = \delta_V^{2a}\frac{1}{\epsilon^2} + \delta_V^{2b}\frac{1}{\epsilon}. \tag{35}$$

At order $n_f \alpha^3$ in the potential, we first consider the $1/\epsilon^2$ terms. Inserting all the above renormalisation constants and demanding the cancellation of $1/\epsilon^2$ terms leads to

$$\delta_V^{2a} = C_A\left(\frac{2}{3} C_A + \frac{n_f}{12}\right). \tag{36}$$

(Note that the n_f independent term must be corrected by gluonic anti-screening effects which we neglect.)

Inserting this into the potential and demanding the vanishing of the local $1/\epsilon$ terms yields

$$\delta_V^{2b} = -n_f\left(\frac{5}{48} C_A + \frac{C_F}{16}\right), \tag{37}$$

plus various n_f independent terms from the purely gluonic contributions to the anti-screening potential.

Having now fixed the renormalisation constant, it is very satisfying to see that the non-local divergences in (26) are cancelled in this scheme. At order α^3 there are three such non-local terms: they are generated by n_f dependent local divergences in the renormalisation constants multiplying the logarithm in the one loop potential (18). One is from the n_f part of Z_V:

$$-\frac{4 C_F}{\boldsymbol{k}^2}\frac{\alpha^3}{\pi}\frac{n_f}{6}\log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right)\frac{1}{\epsilon}, \tag{38}$$

while there are two further n_f dependent contributions from the coupling constant renormalisation (3) since α_0 occurs twice in (18). Each of these yields

$$-\frac{2 C_F}{\boldsymbol{k}^2}\frac{\alpha^3}{\pi}\frac{n_f}{6}\log\left(\frac{\boldsymbol{k}^2}{\nu^2}\right)\frac{1}{\epsilon}, \tag{39}$$

and adding all three of these terms together we see that the n_f dependent, non-local divergences in $V_{\min}$ at order α^3 indeed cancel. We stress that this cancellation is a stringent test of the method since there was no freedom in the calculation. We conclude that this renormalisation programme can be carried through.

Our final result for the renormalised, anti-screening potential is

$$\tilde{V}_{\min}(\boldsymbol{k}^2) = -\frac{4\pi\alpha C_F}{\boldsymbol{k}^2} + \alpha^2 \tilde{V}^2_{\min}(\boldsymbol{k}^2) + \alpha^3 \tilde{V}^3_{\min}(\boldsymbol{k}^2) + \cdots, \tag{40}$$

where

$$\tilde{V}^2_{\min}(\boldsymbol{k}^2) = \frac{4C_A C_F}{3\boldsymbol{k}^2}\left(-7 + 6\log(2) + 3\log\left(\frac{\boldsymbol{k}^2}{\boldsymbol{\nu}^2}\right)\right), \tag{41}$$

and the n_f dependent terms

$$\tilde{V}^3_{\min}(\boldsymbol{k}^2) = \frac{C_A C_F n_f}{27\pi\boldsymbol{k}^2}\left(125 - 3\pi^2 + 12\log(2)\left[-7 + 3\log(2)\right] + 3\log\left(\frac{\boldsymbol{k}^2}{\boldsymbol{\nu}^2}\right)\left[-14 + 12\log(2) + 3\log\left(\frac{\boldsymbol{k}^2}{\boldsymbol{\nu}^2}\right)\right]\right), \tag{42}$$

where $\boldsymbol{\nu}^2 = 4\pi\nu^2 \mathrm{e}^{-\gamma_E}$.

4. Conclusions

We have seen that the decomposition of the beta function into screening and anti-screening structures breaks down beyond one loop. This we saw by calculating the light quark contributions to the anti-screening potential: non-local divergences arose in fermion loops in the minimal potential at order g^6 which are not cancelled by an anti-screening beta function. This is due to anti-screening effects from light fermions which are necessary consequences of a gauge invariant construction of charges.

However, we have seen that it is possible to renormalise this potential via full charge renormalisation plus a multiplicative renormalisation of the potential. This renormalisation provided a stringent test of the method. It is to be understood as a renormalisation of the additional energy due to the neglect of screening interactions in a minimally dressed construction of charges.

The results presented here suggest that a decomposition of the potential into a minimal, anti-screening part plus an additional screening structure is indeed possible. Further studies of this decomposition may help to clarify the structure of the forces between heavy quarks.

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