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Analysis of free vibration of embedded multi-layered graphene sheets

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Abstract

In this paper, the vibrational properties of four- and five-layered graphene sheets (GSs) embedded in the elastic matrix are analyzed based on a multi-layered continuum model with considering the vdW interaction of any two layers of GSs. The influences of vdW interaction and the surrounding matrix on the vibrational properties four- and five-layered GSs embedded in elastic medium are investigated in detail. The results show that the surrounding matrix has a significant effect on the foundational frequency of MLGSs, while these factors are neglectable for the other higher-order frequencies. These results will be helpful for engineering application of four- and five-layered GSs.

Keywords: multi-layered graphene sheets; vibrational properties; elastic matrix

1. Introduction

Recently, graphene sheet has provoked a great deal of research due to its unique mechanical and physical properties. Especially, these sheets are thought to be used as reinforcements in composite materials to obtain high strength or as nanomechanical resonators in THz frequency range. In order to apply well these materials, it is necessary to acquire their material properties such as vibrational properties via experiments or theoretical methods.

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Bunch et al. [1] studied experimentally the fundamental frequencies and the quality factors of graphene resonators fabricated from single- and multi-layered graphene sheets (MLGSs). Behfar and Naghdabadi [2] investigated vibration of multi-layered graphene sheets embedded in polymer matrices, in which an anisotropic plate model for graphene sheet was used with considering vdW interactions between two adjacent ones. He and his coauthors [3-5] predicted vibration properties of MLGSs using a continuum model including a formula explicitly derived for vdW interaction between any two sheets of (MLGS). They found that the effect of vdW interaction had no influence on the lowest natural frequency of multilayered graphene sheets but played a significant role in all higher natural frequencies. Recently, Shen et al. [6] studied nonlinear vibration behavior of single-layered graphene sheet in thermal environments via nonlocal orthotropic plate model. Pradhan and Kumar [7, 8] studied the small scale effect on the vibration analysis of orthotropic single layered graphene sheets embedded in elastic medium, in which graphene sheets are formulated using nonlocal differential constitutive relations of Eringen. It is found that most of literatures on the vibration of MLGSs are restricted to two- or three-layered sheets graphene sheets with only considered vdW interaction between two adjacent sheets.

In this paper, the vibrational properties of MLGS embedded in elastic medium are investigated using continuum models. In this analysis, an explicit formula [3] used for describing the van der Waals (vdW) interaction between any two layers of graphene sheets is applied. Both Pasternak-type [9] and Winkler type [10] foundations are used to model the interaction between graphene sheets and surrounding elastic medium. Specially, the influences of vdW interaction and the surrounding matrix on the vibrational properties four- and five-layered graphene sheets embedded in elastic medium are analyzed in detail.

2. Formulation

As shown in Fig. 1, a MLGS is embedded in an elastic medium, in which the chemical bonds formed between the GSs and the elastic medium is described by a Pasternak-type foundation model [9] with accounting for both normal pressure and the transverse shear deformation of the surrounding elastic medium. In this analysis, MLGS is modeled as a stack of plates with length of each plate a, width b, thickness h, mass density $\rho$, and Young’s modulus $E$. The interlayer friction between any two adjacent layers is assumed to be negligible. Thus, the governing equations for the vibration of a MLGS ($N$ layers sheets) can be derived as the $N$ coupled equations:

$$D\nabla^4 w_1 + \rho h \frac{\partial^2 w_1}{\partial t^2} = q_1 + K_n w_1 - G h \nabla^2 w_1$$

$$D\nabla^4 w_2 + \rho h \frac{\partial^2 w_2}{\partial t^2} = q_2$$

$$\vdots$$

(1)
\[
DV^4 w_N + \rho h \frac{\partial^2 w_N}{\partial t^2} = q_N + K_w w_N - G_b \frac{\partial^2 w_N}{\partial t^2}
\]

where \( t \) is time, \( w_j \) \((i=1,2, \ldots , N)\) is the deflection of the \( i \)-th sheet, which is assumed to be positive in the upward direction, \( D \) is the bending stiffness of the individual sheet, \( K_w \) is the Winkler foundation modulus [10], \( G_b \) is the stiffness of the shearing layer [9]. \( q_i \) is the pressure that is exerted on \( i \)-th sheet due to vdW interaction from any other layers. The equilibrium distance between GSs is about 0.34 nm, and thus the initial pressure between layers can be ignored. As only infinitesimal vibration is considered, the net pressure due to the vdW interaction [3-5] is assumed to be linearly proportional to the deflection between any two layers, i.e.,

\[
q_i = \sum_{j=1}^{N} c_{ij} (w_i - w_j) = w_i \sum_{j=1}^{N} c_{ij} - \sum_{j=1}^{N} c_{ij} w_j,
\]

where the vdW interaction coefficients \( c_{ij} \) [3-5] can be obtained via the Lennard-Jones pair potential [11].

\[
c_{ij} = \left( \frac{4 \sqrt{3}}{9a} \right)^2 \frac{24e}{\sigma^2} \left( \frac{\sigma}{a} \right)^8 \left[ \frac{3003\pi}{256} \sum_{k=0}^{\infty} \frac{(-1)^k (5)}{2k+1} k \left( \frac{\sigma}{a} \right)^6 \frac{1}{(z_i - z_j)^2} \right] - \frac{35\pi}{8} \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)}{2k+1} k \left( \frac{1}{(z_i - z_j)^6} \right), \quad i \neq j
\]

where \( a = 0.142 \text{nm} \) is the C–C bond length, \( z_j = z_i/a \) (where \( z_i \) is the coordinate of the \( i \)-th layer in the thickness direction of the MLGS). It is emphasized that the pressure model here includes the vdW interaction between any two of the layers, rather than two adjacent layers only.

With the assumption of all the edges simply supported, then the deflection of all of the layers can be described by a periodic solution of the form:

\[
w_k(x,y,t) = A_k \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{i\omega t},
\]

where \( i = \sqrt{-1} \), \( \omega \) is the frequency of natural vibration, \( A_k \) \((k=1,2, \ldots , N)\) are \( N \) unknown coefficients, and \( m \) and \( n \) are the half wavenumbers in the direction of \( x \) and \( y \), respectively.

Substituting Eqs. (2-4) into Eq. (1) one can obtain an eigenvalue equation. Solving the resultant eigenvalue equation will give the natural frequencies and associated vibration modes that are relative to the wavenumbers of \( m \) and \( n \), as well as the influence of the surrounding matrix on the vibration behavior. Due to the limitation of space, the derived formulation will not be given here.

3. Results and Discussion

In this analysis, four- and five-layered MLGSs embedded within the elastic matrix are investigated respectively. The dimensions \( a \) and \( b \) are both set as 10 nm for all MLGSs, and the thickness of each GS is assumed to be 0.34 nm. The Young’s modulus of each GS is taken as \( E =1.06 \text{TPa} \), the Poisson ratio \( \nu =0.25 \), and the mass density \( \rho =2250 \text{kg/m}^3 \). It is worth noting that the following parameters \( \overline{K}_w = K_w a^4/D \), \( \overline{G}_b = G_b a^2/D \) and \( \overline{\omega} = \omega \sqrt{\rho a^4/D} \) are introduced in the present analysis.
Fig. 2 shows the influence of the Wrinkler modulus parameter on the natural frequency parameters \( (m = n = 1) \) of four- and five-layered square GSs with the elastic matrix modeled as a Wrinkler foundation (i.e., \( \bar{G}_b = 0 \)). It is noted that \( \sigma_1 \) is the same for the four- and five-layered GSs when \( \bar{K}_w = 0 \) and \( \bar{G}_b = 0 \), i.e., there is no the elastic matrix considered, because \( \sigma_1 \) is not dependent on the vdW interaction for MLGSs with any number of layers. It can be found from Fig.2 that as the Wrinkler modulus parameter increases the natural frequency parameter \( \sigma_1 \) decreases gradually, and finally descends sharply to zero. In contrast to the natural frequency parameter \( \sigma_1 \), the other nature frequencies parameters \( \sigma_i \) (\( i = 2, \ldots, 5 \)) are almost unchanged for four- and five-layered GSs as the Wrinkler modulus increases. Thus, it can be concluded that the influence of the Wrinkler modulus parameter on the nature frequency parameters \( \sigma_i \) (\( i = 2, \ldots, 5 \)) can be ignored. Besides, it can be also observed that the natural frequency parameters \( \sigma_i \) (\( i \geq 2 \)) are much higher than the natural frequency parameter \( \sigma_1 \), and thus it can be concluded that the vdW interaction plays a dominant role in the foundational natural frequency.

Fig. 3 shows the influence of the Wrinkler modulus parameter on the natural frequencies \( (m = n = 1) \) of a square MLGS with the surrounding matrix modeled as a Pasternak foundation (\( \bar{G}_b = 20 \)): (a) four-layered GS, (b) five-layered GS.
Fig. 3 shows that the influence of the Wrinkler modulus parameter on the natural frequency parameters \((n = m = 1)\) of four- and five-layered GSs with the elastic matrix modeled as a Pasternak foundation \((\bar{G}_b = 20)\). Firstly, it can be found from Fig. 3 that the foundational frequency parameters \(\sigma_1\) are decreased gradually till to zero both for four- and five-layered GSs, which is similar in tendency to the case in Fig. 2. For the higher frequency parameters \(\sigma_i (i \geq 2)\) in Fig. 3(a) and (b), they are almost unchanged with increasing the Wrinkler modulus parameter. Then, comparing Fig. 3 with Fig. 2, it can be found that all the frequency parameters \(\sigma_i (i \geq 1)\) for \(\bar{G}_b = 20\) are larger than the counterpart for \(\bar{G}_b = 0\) with the same \(\bar{K}_w\) both for four- and five-layered GSs.

![Fig. 4. Effect of the shear modulus parameter on the natural frequencies \((m = n = 1)\) of a square MLGS with the surrounding matrix modeled as a Pasternak foundation \((\bar{K}_w = 300)\): (a) four-layered GS, (b) five-layered GS.](image)

Fig. 4 shows that the influence of the shear modulus parameter on the natural frequency parameters \((n = m = 1)\) of four- and five-layered GSs with the elastic matrix modeled as a Pasternak foundation \((\bar{K}_w = 300)\). It can be observed from Fig. 4 that the foundational frequency parameter \(\sigma_1\) of four-layered GS is less than one of five-layered GS for a fixed group of \(\bar{K}_w\) and \(\bar{G}_b\). With the increase of the shear modulus parameter, the foundational frequency parameter \(\sigma_1\) of four- and five-layered GSs are both reduced gradually, while the other higher frequency parameters \(\sigma_i (i \geq 2)\) are not sensitive to the shear modulus parameters.

4. Conclusion

Based on a multi-layered continuum model, the vibrational properties of four- and five-layered GSs embedded in elastic medium are investigated with considering the vdW interaction of any two layers GSs. The results show that the Wrinkler modulus and the shear modulus parameters are very significant for the foundational frequency parameter \(\sigma_1\) of MLGSs, while they have negligible influence on the higher frequency parameters \(\sigma_i (i \geq 2)\). For a given elastic matrix (i.e., a fixed group of parameters \(\bar{K}_w\) and \(\bar{G}_b\)), the foundational frequency parameter \(\sigma_1\) of five-layered GS is larger than the counterpart of four-layered GS. These results will be help for engineering application of MLGSs.

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