Strength of the trilinear Higgs boson coupling in technicolor models

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Received 23 May 2006; received in revised form 17 July 2006; accepted 29 August 2006
Available online 5 September 2006
Editor: T. Yanagida

Abstract

We discuss the strength of the trilinear Higgs boson coupling in technicolor (or composite) models in a model independent way. The coupling is determined as a function of a very general ansatz for the technicolor self-energy, and turns out to be equal or smaller than the one of the Standard Model Higgs boson depending on the dynamics of the theory.

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PACS: 12.60.Nz; 14.80.Cp
Keywords: Technicolor models; Non-Standard Model Higgs bosons

1. Introduction

In the Standard Model of elementary particles the fermion and gauge boson masses are generated due to the interaction of these particles with elementary Higgs scalar bosons. Despite its success there are some points in the model as, for instance, the enormous range of masses between the lightest and heaviest fermions and other peculiarities that could be better explained at a deeper level. The nature of the Higgs boson is one of the most important problems in particle physics, and there are many questions that may be answered in the near future by LHC experiments, such as: is the Higgs boson, if it exists at all, elementary or composite? What are the symmetries behind the Higgs mechanism?

There are many variants for the Higgs mechanism. Our interest in this work will be focused in the models of electroweak symmetry breaking via strongly interacting theories of technicolor type [1]. In these theories the Higgs boson is a composite of the so-called technifermions, and at some extent any model where the Higgs boson is not an elementary field follows more or less the same ideas of the technicolor models. In extensions of the Standard Model the scalar self-couplings can be enhanced, like in the supersymmetric version. If the same happens in models of dynamical symmetry breaking, as far as we know, has not been investigated up to now, and this study is the motivation of our work.

The beautiful characteristics of technicolor (TC) as well as its problems were clearly listed recently by Lane [1,2]. Most of the technicolor problems may be related to the dynamics of the theory as described in Ref.[1]. Although technicolor is a non-Abelian gauge theory it is not necessarily similar to QCD, and if we cannot even say that QCD is fully understood up to now, it is perfectly reasonable to realize the enormous work that is needed to abstract from the fermionic spectrum the underlying technicolor dynamics. The many attempts to build a realistic model of dynamically generated fermion masses are reviewed in Refs. [1,2]. Most of the work in this area try to find the TC dynamics dealing with the particle content of the theory in order to obtain a technifermion self-energy that does not lead to phenomenological problems as in the scheme known as walking technicolor [3].

The idea of this scheme is quite simple. First, remember that the expression for the TC self-energy is proportional to Σ(p^2)_{TC} ∝ ⟨⟨ψψ⟩_{TC}/p^2⟩(p^2/A_{TC}^2)^γ^*, where ⟨ψψ⟩_{TC} is the TC condensate and γ^* its anomalous dimension. Secondly, depending on the behavior of the anomalous dimension we obtain different behaviors for Σ(p^2)_{TC}. A large anomalous dimen-
sion may solve the problems in TC models. In principle we could deal with many different models, varying fermion representations and particle content, finding different expressions for $\Sigma(p^2)_{TC}$ and testing them phenomenologically, i.e., obtaining the fermion mass spectra without any conflict with experiment. Usually the walking behavior is obtained only with a large number of technifermions, although there are recent proposals where the walking behavior is obtained for a very small number of fields with the introduction of technifermions in higher dimensional representations of the technicolor gauge group [4].

As the dynamics in models of dynamical symmetry breaking can be so different from QCD, it is interesting to investigate the behavior of the dynamical Higgs boson self-coupling, verifying if it can be larger or smaller than the one of the Standard Model.

In this work we will consider a very general ansatz for the $\Sigma(p^2)_{TC}$ and testing them phenomenologically, i.e., obtaining the fermion mass spectra without any conflict with experiment. Usually the walking behavior is obtained only with a large number of technifermions, although there are recent proposals where the walking behavior is obtained for a very small number of fields with the introduction of technifermions in higher dimensional representations of the technicolor gauge group [4].

In this work we will consider a very general ansatz for the technifermion self-energy that was introduced in Ref. [5]. This ansatz interpolates between all known forms of technifermionic self-energy. As we vary some parameters in our ansatz for the technifermionic self-energy we go from the standard operator product expansion (OPE) behavior of the self-energy to the one predicted by the extreme limit of a walking technicolor dynamics, i.e., $\gamma^s \to 1$ [3,6,7]. We will discuss the general properties of the trilinear Higgs coupling based on this ansatz.

This Letter is organized as follows: In Section 2 we compute the trilinear self-coupling of a composite Higgs boson assuming the ansatz for the fermionic self-energy shown in Ref. [5]. In Section 3 we review the self-couplings of the Standard Model fundamental Higgs field and compare them with the results shown in the previous section. Finally in Section 4 we draw our conclusions.

2. The trilinear self-coupling for a composite Higgs boson

Using Ward identities we can show the couplings of the scalar boson to fermions to be [6]

$$G^a(p+q,p) = -i \frac{g_w}{2 M_W} \left[ \tau^a \Sigma(p) P_R - \Sigma(p+q) \tau^a P_L \right].$$

where $\tau^a \equiv \frac{1}{2}(1 \pm \gamma_5)$, $\tau^a$ is a $SU(2)$ matrix, and $\Sigma$ is a matrix of fermionic self-energies in weak-isodoublet space. As in Ref. [6] we assume that there is a scalar composite Higgs boson that couples to the fermionic self-energy which is saturated by the top quark [8]. Specifically, we assume that the scalar-to-fermion coupling matrix at large momenta is given by $G(p,p)$, where we do not attempt to distinguish between the two fermion momenta $p$ and $p+q$, since, in all situations with which we will be concerned, $\Sigma(p+q) \approx \Sigma(p)$. Therefore the coupling between a composite Higgs boson with fermions at large momenta is given by

$$\lambda_{Hff}(p) \equiv G(p,p) \approx -\frac{g_w}{2 M_W} \Sigma(p^2),$$

where $\Sigma(p^2)$ is the fermionic self-energy. The trilinear Higgs boson coupling in technicolor models will be dominated by loops of heavy fermions that couple to the scalar Higgs particle as predicted by Eq. (2) [6]. Our purpose in this section is to obtain an expression for the trilinear Higgs boson coupling using the ansatz

$$\Sigma_A(p^2) \sim \Lambda_{TC} \left( \frac{\Lambda_{TC}^2}{p^2} \right)^{\frac{\alpha}{2}} \left[ 1 + a \ln(p^2/\Lambda_{TC}^2) \right]^{-\beta},$$

which was proposed in Ref. [5]. This choice interpolates between the standard OPE result for the technifermion self-energy, which is obtained when $\alpha \to 1$, and the extreme walking technicolor solution obtained when $\alpha \to 0$ [3], i.e., this is the case where the symmetry breaking is dominated by higher order interactions that are relevant at or above the TC scale, leading naturally to a very hard dynamics [6,7]. As we have pointed out in Ref. [8] only such kind of solution is naturally capable of generating a large mass to the third fermionic generation, which has a mass limit almost saturated by the top quark mass. Moreover, as also claimed in the second paper of Ref. [8], there are other possible reasons to have $\alpha \sim 0$, as the existence of an infrared fixed point and a gluon (or technigluon) mass scale [9], which, actually, are related possibilities [10]. It is interesting that many technicolor models make use of the existence of a non-trivial fixed point (or a quasi-conformal theory) to cure their phenomenological problems [3], and exactly for this possibility Brodsky has been claiming that it will be possible to build an skeleton expansion that could allow to capture the non-perturbative effects in a reliable way [11].

In Eq. (3) the scale, $\Lambda_{TC}$ is related to the technicolor condensate by $\langle \bar{\psi} \psi \rangle_{TC} \approx \Lambda_{TC}^3$. We defined $\beta \equiv \gamma_{TC} \cos(\alpha \pi)$, $a \equiv b g_{TC}^2$ with $\gamma_{TC} = 3c/16\pi^2 b$, and $c$ is the quadratic Casimir operator given by

$$c = \frac{1}{2} \left[ C_2(R_1) + C_2(R_1) - C_2(R_3) \right],$$

where $C_2(R_i)$ are the Casimir operators for technifermions in the representations $R_1$ and $R_2$ that condensate in the representation $R_3$, $b$ is the coefficient of the $g^3$ term in the technicolor $\beta(g)$ function.

We can determine one expression for the trilinear coupling for any theory where the Higgs boson is composite by considering the diagram shown in Fig. 1. The contribution of Fig. 1 is certainly the dominant one [6]. Assuming the coupling of the scalar boson to the fermions to be given by Eq. (1), and with the fermion propagator written as

$$S_F(p) = \frac{\langle \phi + \Sigma(p^2) \rangle}{(p^2 - \Sigma^2(p^2))}.$$
we find that
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \left( \frac{3 n_F}{M_W^3} \right) \int_0^\infty \frac{p^4}{(p^2 + \Sigma^2(p)^2)^3} \, dp^2.
\] (5)

where \( n_F \) is the number of technifermions included in the model. Considering the ansatz given by Eq. (3), and introducing it into Eq. (5), we obtain
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \frac{3 n_F}{M_W^3} \Lambda_{TC}^4 (\Lambda_{TC})^{4a} I(p^2).
\] (6)

with
\[
I(p^2) = \frac{1}{\Gamma(4\beta)} \int_0^\infty dz z^{4\beta-2} e^{-z} (\Lambda_{TC})^{a z} \int_0^\infty dp^2 \frac{(p^2)^{2-4a-az}}{(p^2 + \Lambda_{TC}^2)^3}.
\]

To compute this last expression we have used the following Mellin transform
\[
[1 + A \ln B]^{-\eta} = \frac{1}{\Gamma(\eta)} \int_0^\infty dz z^{\eta-1} e^{-z} (B)^{-Az}.
\] (7)

After performing the \( p^2 \) integration in Eq. (6), we can write this equation as
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \frac{3 n_F}{M_W^3} \Lambda_{TC}^4 (\Lambda_{TC})^{4a} I(p^2).
\] (8)

We will present our analysis of \( \lambda_{HHH}^T \) for two different regions of the parameter \( \alpha \). We will start with the case \( \alpha \approx 0 \). Therefore we can make the following expansion in Eq. (8)
\[
\frac{1}{4a + az} \approx \frac{1}{az} \left[ 1 - \frac{4a}{az} + O(a^2) \ldots \right].
\] (9)

Then Eq. (8) can be cast in the form
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \frac{3 n_F}{M_W^3} \Lambda_{TC}^4 (\Lambda_{TC})^{4a} \int_0^\infty dz z^{4\beta-2} e^{-z}
\]
\[
- \frac{4a}{a} \int_0^\infty dz z^{4\beta-3} e^{-z} + O(a^2) \ldots
\]

Retaining only the first two terms in the \( \alpha \) expansion and performing the \( z \) integration, we finally can write
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \frac{3 n_F}{M_W^3} \Lambda_{TC}^4 \left( 1 - \frac{4a}{a(4\beta - 1)} \right).
\] (10)

When \( \alpha \approx 1 \), we can consider a similar expansion, and following the same steps we obtain
\[
\lambda_{HHH}^T \approx \frac{3 g_W^3}{64 \pi^2} \frac{3 n_F}{M_W^3} \Lambda_{TC}^4 \left( 1 - \frac{4a}{a(4\beta - 2)} \right).
\] (11)

The above expressions for the trilinear Higgs coupling are quite dependent on the scale \( \Lambda_{TC} \). This is not the best formula to compute this coupling, since \( \Lambda_{TC} \), which in principle is related to the value of the dynamical technifermion mass at the origin, is not directly fixed by the symmetry breaking of the Standard Model. A more appropriate quantity that can be used to describe this coupling is the technipion decay constant, which is fixed by the \( W \) and \( Z \) gauge boson masses.

Considering our comments in the previous paragraph we will express the trilinear Higgs coupling as a function of the technipion decay constant (\( F_\Pi \)) instead of the scale \( \Lambda_{TC} \). \( F_\Pi \) can be computed through the known Pagels and Stokar relation [12]
\[
F_\Pi^2 = \frac{N_{TC}}{4 \pi} \int_0^\infty \frac{dp^2}{(p^2 + \Sigma^2(p)^2)} \times \left[ \Sigma^2(p)^2 - \frac{p^2 d \Sigma(p)^2}{2 dp^2} \Sigma(p)^2 \right].
\]

where \( N_{TC} \) is the technicolor number.

We compute the technipion decay constant using the ansatz Eq. (3). After some calculation we obtain the following expression for \( F_\Pi \)
\[
F_\Pi^2 = \frac{N_{TC}}{4 \pi} \Lambda_{TC}^2 f(k),
\] (12)

where
\[
f(k) = \frac{(1 + k/2)}{(1 + 2k)} \csc[\pi/(1 + 2k)]
\] (13)

with
\[
k = \alpha + 3 \cos(\alpha \pi)/4 \pi.
\]

To obtain this expression we have assumed the scaling law \( c_{\alpha TC} \sim 1 \) [13]. To be consistent with Eqs. (10) and (11), we also need to expand Eq. (12) for \( \alpha \approx 0 \) and \( \alpha \approx 1 \). In this case, we obtain
\[
F_\Pi^2 = \frac{N_{TC}}{8 \pi} \Lambda_{TC}^2 [1 - S(\alpha)].
\] (14)

with
\[
S(\alpha) = \begin{cases} 5\alpha & \text{for } \alpha \approx 0, \\ \alpha/2 & \text{for } \alpha \approx 1. \end{cases}
\]

Finally, assuming this last equation, we can write Eqs. (10) and (11) in the form
\[
\lambda_{HHH}^{Ta} = \frac{3 n_F}{N_{TC}^2} F_\Pi f(\alpha),
\] (15)

where for convenience we defined
\[
f(\alpha) = \begin{cases} \frac{3}{a(4\beta - 1)} \left( 1 - 4a/(a(4\beta - 2)) \right) & \text{when } \alpha \approx 0, \\ \frac{3}{4} \frac{1 - 4(\alpha - 1)/a}{(1 - a^2)^2} & \text{when } \alpha \approx 1 \end{cases}
\]

and will assume \( F_\Pi = 125 \) GeV.\footnote{In TC models containing \( N_D \) doublets of technifermions \( F_\Pi = 250 \) GeV/\( \sqrt{N_D} \), and in this work we will be assuming \( N_D = 4 \).}
3. Trilinear coupling: fundamental × composite Higgs boson

In this section we review the expression for the trilinear coupling in the case of the Standard Model fundamental Higgs boson, and compare it to the ones found in the previous section. We start writing the expression for the Higgs boson potential in the Standard Model

\[ V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda \varphi^\dagger \varphi^2. \]  

(16)

The self-couplings are uniquely determined in the Standard Model by the mass of the Higgs boson, which is related to the quadrilinear coupling \( \lambda \) by the following expression

\[ M_H^2 = 2\lambda v^2. \]

After introducing the physical Higgs field \( H \) in the neutral component of the doublet \( \varphi = (v + H)/\sqrt{2} \) we can write the potential as

\[ V(H) = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4. \]  

(17)

The multiple Higgs couplings can be derived from the potential \( V(H) \), and the trilinear and quadrilinear couplings of the Higgs field \( H \) are given by

\[ \lambda_{3H} = 3 \frac{M_H^2}{M_Z^2} \lambda_0, \]

\[ \lambda_{4H} = 3 \frac{M_H^2}{M_Z^2} \lambda_0. \]  

(18)

To obtain these expressions we assumed the normalization \( \lambda_0 = M_Z^2/v \).

In the case of a composite Higgs boson it is possible to show that its mass can be expected to be of the following order [14]:

\[ M_H \sim 2\Lambda_{TC}. \]

This result is independent of the dynamics and is originated from the similarity between the Schwinger–Dyson equation for the technifermion self-energy and the Bethe–Salpeter equation for the scalar channel [14]. Of course, as discussed in the previous section, we write \( M_H \) as a function of \( F_\pi \) instead of \( \Lambda_{TC} \).

To compare the results of the previous section with the couplings shown above we can write the couplings for the composite Higgs boson as a function of its mass. Assuming the mass relation given above, considering Eq. (15) and rewriting it in terms of the parameter \( \lambda_0 \), we obtain in the case of \( \alpha = 0 \)

\[ \lambda_{3H}^T = \left( \frac{1}{14} \right) n_F M_H N_{TC} \sqrt{2/\pi} N_{TC} a(4\beta - 1), \]  

(19)

and in the case when \( \alpha = 1 \) we obtain

\[ \lambda_{3H}^T = \left( \frac{1}{28} \right) \frac{n_F M_H}{N_{TC} \sqrt{2/\pi} N_{TC}} \hat{\lambda}_0. \]  

(20)

where \( \hat{\lambda}_0 \equiv \lambda_0/(1 \text{ GeV}) \).

This coupling could also be computed by means of naive dimensional analysis (nda), which would give \( \lambda_{3H}^{nda} \sim M_H^2/v \). Our Eqs. (19) and (20) for the different trilinear couplings, corresponding to the extreme walking (\( \alpha = 0 \)) and standard OPE (\( \alpha = 1 \)) solutions of the technifermion self-energy, which have the same dimensional factors, can also be put in the form of Eq. (18). However it has to be noticed that the self-energies functional form are quite different in the extreme cases (\( \alpha = 0 \) and \( \alpha = 1 \)), and after the integration of Eq. (5) it is natural to expect a different numerical result, which appears in Eqs. (19) and (20) in the form of different factors (\( a, 4\beta - 1 \) and \( \sqrt{2} \)). This is the origin of the different values for the trilinear couplings. This numerical and group structure would appear in the nda result due to the fact that \( M_H \) and \( v \) vary with the different self-energies. The only parameter that must be kept is the dimensional constant \( F_\pi \) in order to give the right masses to the weak bosons.

In Fig. 2 the behavior of the trilinear Higgs couplings is plotted as a function of the Higgs boson mass. The solid line represents the contribution of the fundamental Higgs boson, i.e., the Standard Model Higgs boson.

To compare the trilinear Higgs coupling for fundamental and composite scalar bosons we will consider technicolor models with technifermions in the fundamental representation and will choose appropriately the number \( n_F \) of technifermions in order to obtain the desired walking behavior. For example, if the technicolor group is \( SU(2)^{\text{TC}} \), the walking limit is going to be obtained with \( n_F = 8 \). The 8 technifermions can be recognized as a colored weak doublet \( Q = (U^a, D^a)^\dagger \) and a color-singlet weak doublet \( L = (E_N) \). If the technicolor theory is described by the \( SU(4)^{\text{TC}} \) non-Abelian group, the extreme walking behavior is obtained when \( n_F \sim 14 \), which can be built with the addition of two colored weak singlets \( R^a, S^a \). It is clear that we are not discussing about phenomenologically viable models, but the cases that we are presenting are plausible examples to make the comparison between the “composite” and the elementary coupling.

In Fig. 2 the continuous curve shows the behavior of the trilinear Higgs boson self-coupling given by Eq. (18). In the same figure we indicate by (□, ■) the values of the trilinear composite Higgs couplings obtained respectively with the help of Eqs. (19) and (20) (\( \alpha \to 0, \alpha \to 1 \)) in the case of the \( SU(2)^{\text{TC}} \) technicolor group. We also indicate by (△, ▲) in Fig. 2 the values of the trilinear coupling obtained for the \( SU(4)^{\text{TC}} \) when the parameter \( \alpha \) has respectively the following behavior (\( \alpha \to 0, \alpha \to 1 \)).

It is possible to verify in Fig. 2 that the trilinear Higgs couplings generated by the dynamics in the limit \( \alpha \to 0 \), which corresponds to the extreme walking technicolor limit, are quite...
close to the values obtained in the case of the fundamental Standard Model Higgs boson. However, in the limit $\alpha \to 1$ the behavior predicted for the trilinear Higgs coupling is very different; it decreases the more the technicolor dynamics approaches the standard result predicted by simple OPE analysis. The arrow in Fig. 2 shows roughly the expected change in the trilinear coupling as we go from $\alpha \to 0$ to $\alpha \to 1$. Our result complements the findings of Ref. [6] where it was shown that in the extreme case ($\alpha = 0$) the strongly interacting composite sector is similar to the Standard Model.

4. Conclusions

In this work we have presented a discussion about the general properties of the trilinear self-coupling of a composite Higgs boson based on a general ansatz for the technifermion self-energy. If the Higgs boson is composite we can expect it to be, at least in the most usual models, a very massive part.

As can be seen in Fig. 2, in the limit that $\alpha \to 0$, the trilinear coupling of a composite Higgs boson practically will not differ from the one of a fundamental Higgs boson. In the limit that $\alpha \to 1$, the trilinear coupling of the composite Higgs boson is much smaller than the one predicted by the Standard Model. Since the trilinear coupling of a fundamental Higgs boson, in principle, could barely be measured at the LHC, observing the subprocess $gg \to HH$ [15], we can imagine that it would not be easier to measure such coupling if the Higgs boson is a composite one. If this coupling could be measured at the LHC, it would be with one leg off-shell and the others probably on-shell, what is different from the coupling that we have calculated in this work, resulting from the simplification performed when passing from Eq. (1) to Eq. (2). Apart from the fact that this would be an extremely difficult measurement, we can say that we shall expect the same group factors to play a role in the full calculation of this coupling, mostly because the integral of Eq. (5) is dominated by the ultraviolet behavior of the self-energies, meaning that the $q$ momentum dependence of Eq. (1) would not introduce large differences in the calculation of Eq. (5). Of course, if this coupling is measured we certainly would need a more sophisticated calculation than the one presented here, and would not be able even to present simple analytic expressions as the ones shown in Eqs. (19) and (20).

According to what is known for a long time, the technifermion dynamics with $\alpha \to 1$ is exactly the one that leads to the many phenomenological problems in technicolor models. The solution of these problems ask for a dynamics where $\alpha \to 0$, and, fortunately, this very same extreme dynamics is the one that would still allow for a study of the trilinear composite Higgs boson coupling, at the same level as it would be possible to observe in the case of a fundamental boson.

Acknowledgements

This research was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (A.A.N.) and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (A.D.).

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