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Proof of a universal lower bound on the shear viscosity to entropy density ratio

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ABSTRACT

It has been conjectured, on the basis of the gauge-gravity duality, that the ratio of the shear viscosity to the entropy density should be universally bounded from below by $1/4\pi$ in units of the Planck constant divided by the Boltzmann constant. Here, we prove the bound for any ghost-free extension of Einstein gravity and the field-theory dual thereof. Our proof is based on the fact that, for such an extension, any gravitational coupling can only increase from its Einstein value. Therefore, since the shear viscosity is a particular gravitational coupling, it is minimal for Einstein gravity. Meanwhile, we show that the entropy density can always be calibrated to its Einstein value. Our general principles are demonstrated for a pair of specific models, one with ghosts and one without.

Kovtun, Son and Starinets (KSS) have proposed that the ratio of the shear viscosity η to the entropy density s should be universally bounded from below by $1/4\pi$ in units of the Planck constant divided by the Boltzmann constant $\hbar/k_B$ [1]. In kinetic theory, the shear viscosity of a fluid is directly proportional to the mean free path of the quasiparticles, suggesting that η/s is much larger than $\hbar/k_B$ in weakly coupled fluids for which the mean free path of the quasiparticles is always large. Thus, a bound on η/s is mostly relevant to strongly interacting fluids. The uncertainty principle can be used in this context to argue that the ratio η/s in units of $\hbar/k_B$ is bounded from below by a constant of order unity [1]. So far, two classes of quantum fluids are known to have values of η/s that approach $\frac{1}{4\pi}\frac{\hbar}{k_B}$: Strongly correlated ultracold Fermi gases and the quark–gluon plasma. Experiments in both systems have now reached the necessary precision to probe the KSS bound. (See [2] for a recent review and many references.)

The gauge-gravity duality [3,4] provides a hydrodynamic description of strongly coupled field theories in terms of the hydrodynamics of a black brane in an asymptotically anti-de Sitter (AdS) spacetime [5]. It was shown in [6] that, for strongly coupled field theories (from here on, $\hbar$, k_B, $c=1$), $\eta/s = 1/4\pi$ when the bulk gravitational theory is Einstein's. (For references and further discussion, see [7].) However, recent findings have cast doubts over the universal nature of the bound. For instance, when the gravitational Lagrangian includes the square of the 4-index Riemann tensor, the ratio η/s can either be smaller or larger than its Einstein value [8,9]. These modifications can be understood from the observation [10] that η/s is equivalently a ratio of two different gravitational couplings, each associated with a differently polarized graviton [11]. If the gravitational theory is Einstein's or related to Einstein's by a field redefinition, then the couplings will be independent of the polarization and $\eta/s = 1/4\pi$. In general, however, the couplings for differently polarized gravitons are distinct, and there is no longer any reason to expect that $\eta/s = 1/4\pi$.

As will be explained, if we impose the physical requirement that extensions of Einstein gravity must be ghost free, then any gravitational coupling can only *increase* from its Einstein value. We will show, in particular, how this outcome applies to the shear viscosity. It will then be demonstrated that the entropy density for any extension can always be calibrated to its Einstein value. Combining both facts will allow us to establish that, if the ratio η/s does differ from the Einstein result, then it must necessarily be larger than $1/4\pi$.

We will consider generalized theories of gravity in AdS whose action depends on the metric $g_{\mu\nu}$, the Riemann tensor $\mathcal{R}_{\rho\mu\lambda\nu}$, matter fields ϕ and their covariant derivatives: $I = \int d^{d+1}x\sqrt{-g} \times \mathcal{L}(\mathcal{R}_{\rho\mu\lambda\nu}, g_{\mu\nu}, \nabla_\sigma R_{\rho\mu\lambda\nu}, \phi, \nabla\phi, \ldots)$ with $d \geqslant 3$. This action should be viewed as a classical one-particle irreducible (1PI) effective action. The theory extends Einstein gravity $\mathcal{L} = \mathcal{R} + \lambda\delta\mathcal{L}$, with λ parameterizing the strength of the corrections.

We will further assume the existence of stationary $(p+2)$-black brane solutions with a bifurcate Killing horizon that are described by the metric $ds^2 = -g_{tt}(r)\,dt^2 + g_{rr}(r)\,dr^2 + g_{xx}(r)\,dx^i\,dx_i$, $i = 1, \ldots, p$. The black brane horizon is at $r = r_h$, where g_{tt} has a first-order zero, g_{rr} has a first-order pole and all other metric

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components are finite. Since the metric components depend only on r, the metric is Poincare invariant in the (t, x_i) subspace. The AdS boundary is taken to be at $r \to \infty$, where the metric asymptotically approaches its AdS form. The near-horizon geometry of a black brane at $r \to r_h$ is insensitive to both the global properties of the spacetime and the precise structure of the Lagrangian $\mathcal{L}$; rather, it depends only on the value of r_h or equivalently the temperature (and possibly the charges). When we compare η/s for Einstein gravity to its value in the generalized theories, we have to fix the temperature of the brane, leading to the same near-horizon geometry for solutions of all theories.

Let us discuss small perturbations $h_{\mu\nu}$ of the metric that propagate in the z direction. In the radial gauge [12], the highest-helicity polarization of the h_{xy} gravitons decouples from all others. (Obviously, x, y can be replaced by any other orthogonal-to-z transverse dimensions.) A standard procedure [13,7] that involves taking the hydrodynamic limit and using the Kubo formula allows one to extract the shear viscosity from the correlation function of the dissipative energy–momentum tensor $T_{xy} \sim \eta \partial_t h_{xy}$. As explained in [10,14,15], this procedure is valid for extensions of Einstein gravity and amounts to extracting the gravitational coupling for the h_{xy} gravitons and, from it, the shear viscosity in the dual field theory. The very same procedure can also be implemented at any radial distance from the brane [16,17].

We wish to determine the h_{xy} coupling in the vicinity of the horizon and in the hydrodynamic limit. We will do so by calculating the propagator in the one-particle exchange approximation, which is valid because $h_{xy} \ll 1$. We can use the $p+1$-dimensional flat-space propagator for a $p+2$-dimensional AdS brane theory. This is because the Klein–Gordon equation for brane-propagating modes reduces to the radial analogue of an infinitely damped harmonic oscillator. For example, for 5D AdS, $\Box_5 h^x{}_y = 0$ simplifies in the hydrodynamic and near-horizon limits to $\partial_r[(r/L)^3 g_{tt}]\partial_r h^x{}_y = 0$ (when $g_{tt}g_{rr} = -1$). Since $r_h/L \gg 1$, the radial mode will then be trapped on the brane, effectively confined to a conformally flat slice of the bulk.

For Einstein gravity, only massless spin-2 gravitons are exchanged, but gravitons can, for a general theory, be either massless or massive and of either spin-0 or spin-2. Particles of any other spin, in particular vectors, cannot couple linearly to a conserved source and so can safely be neglected when evaluating the propagator. Accordingly, the 1PI graviton propagator $[\mathcal{D}(q^2)]_\mu{}^\nu{}_\alpha{}^\beta \equiv \langle h_\mu{}^\nu(q) h_\alpha{}^\beta(-q)\rangle$ must be of the following irreducibly decomposed form [18–21]:

$$\begin{aligned}\left[\mathcal{D}(q^2)\right]_\mu{}^\nu{}_\alpha{}^\beta &= \left(\rho_E(q^2) + \rho_{NE}(q^2)\right)\left[\delta_\mu{}^\beta\delta_\alpha{}^\nu - \frac{1}{2}\delta_\mu{}^\nu\delta_\alpha{}^\beta\right]\frac{G_E}{q^2}\\ &\quad + \sum_i \rho^i_{NE}(q^2)\left(\delta_\mu{}^\beta\delta_\alpha{}^\nu - \frac{1}{3}\delta_\mu{}^\nu\delta_\alpha{}^\beta\right)\frac{G_E}{q^2+m_i^2}\\ &\quad + \sum_j \tilde{\rho}^j_{NE}(q^2)\delta_\mu{}^\nu\delta_\alpha{}^\beta\frac{G_E}{q^2+\tilde{m}_j^2}. \qquad (1)\end{aligned}$$

Here, $q^2 = -q^\mu q_\mu$ is the spacelike momentum (a positive number), the propagator is evaluated in the vacuum state, G_E is Newton's constant and we have denoted the Einstein and "Non-Einstein" parts of the gravitational couplings ρ by the subscripts E and NE. We have separated the contribution of the massless spin-2 particles, massive spin-2 particles with mass m_i and scalar particles with mass $\tilde{m}_j$. Some of the masses m_i, $\tilde{m}_j$ may vanish or be parametrically small in certain cases. The couplings or ρ's are dimensionless quantities and can depend on the momentum scale q; in particular, $\rho_E(0) = 1$, fixing the Newtonian force at large distances $\sim G_E/|x|^{p-1}$.

For a ghost-free theory, all of the couplings must remain positive at all energy scales [20]; meaning that the propagator can only *increase* relative to its Einstein value. To understand this, recall the Kallen–Lehmann representation of the 1PI propagator as discussed in, for instance, Ch. 12 of [22]. The ρ's of Eq. (1) are spectral densities that can be computed in the microscopic theory by inserting a complete set of single- and multi-particle states and can be expressed schematically as $\rho = \sum_n \langle 0|h|n\rangle\langle n|h|0\rangle$. If all such states have a positive norm, then each additional state can only make a positive contribution to a given ρ. By the same logic, a generalized theory that extends Einstein's can only make a positive contribution relative to any Einstein spectral density. For example, for 4D flat space in configuration space, a typical modification of ρ results in a deviation from the Newtonian force law $G_E/r^2 \to G_E/r^2 + c/r^2 e^{-mr}$ with $c > 0$. One might still be concerned that the Einstein contribution for a generalized theory might be reduced due to the geometry changing from the Einstein background. However, as explained previously, since the temperature is fixed, so is the near-horizon geometry.

The shear viscosity for *any* theory of gravity can be determined directly from the propagator $\langle h_{xy}(q)h_{xy}(-q)\rangle$ when taken to the hydrodynamic limit. In this limit, the temperature T is the largest relevant scale, so that both the energy ω and momentum $\vec{q}$ have to satisfy ω/T, $|\vec{q}|/T \ll 1$ (with ω and $|\vec{q}|$ not necessarily of the same magnitude). For Einstein's theory, the above procedure yields the well-known answer $\eta_E = 1/(16\pi G_E)$. For a general theory, the corrections can be read off the propagator in Eq. (1). As we only have to consider mode contributions such that $m/T \to 0$, the shear viscosity η_X for a generic theory X can be expressed as follows:

$$\frac{\eta_X}{\eta_E} = \left[\frac{\langle h_x{}^y h_y{}^x\rangle_X}{\langle h_x{}^y h_y{}^x\rangle_E}\right] = 1 + \frac{1}{\rho_E(q^2 \to 0)}\sum_i \rho^i_{NE}(q^2 \to 0). \qquad (2)$$

The sum represents the non-Einstein contribution of spin-2 particles to the xy-polarization channel in Eq. (1). The scalars and the trace parts of the massless and massive gravitons do not contribute to the sum in Eq. (2).

Irrespective of the precise nature of the corrections, we know that $\rho_E(0) = 1$ and that, for any ghost-free theory of gravity, the ρ's must be positive. It follows that their effect can only be to increase η relative to its Einstein value:

$$\frac{\eta_X}{\eta_E} \geqslant 1. \qquad (3)$$

This lower bound must be true for any coordinate system or choice of field definitions, as the absence of ghosts is an invariant statement that is insensitive to these choices. That is, the smallest η for a field theory must be for the theory dual to Einstein gravity.

Let us next consider the entropy density. The gauge-gravity duality tells us that the entropy density for a given field theory is the same as that of its black brane dual. Also, the temperature T of the field theory can be identified with the Hawking temperature of the black brane. The latter is fixed by the horizon radius r_h (along with any charges) in units of the AdS curvature scale and depends explicitly on the geometry but *not* on the underlying Lagrangian. The dimensionless black brane entropy S_X for a generalized gravity theory can be different from the Einstein value S_E. However, the entropy density depends on the transverse volume. So, it is always possible to find units (by a trivial field redefinition as we show below) that make the *entropy density* s_X numerically equal to the Einstein entropy density s_E.

For a $(p+2)$-black brane with $p \geqslant 2$ transverse dimensions, the entropy for Einstein's theory is $S_E = V_\perp r_h^p/(4G_E)$, where $V_\perp$ is the transverse volume of the brane which includes all other numerical factors. For an extended gravity theory, by Wald's formula [23,24],

$S_X = S_E + \lambda\delta S + \mathcal{O}[\lambda^2]$. (Recall that $\mathcal{L} = \mathcal{R} + \lambda\delta\mathcal{L}$.) Moreover, since $T_X = T_E$, one finds that the higher-order corrections vanish [25]. Since δS depends strictly on the horizon geometry and r_h (or T) has already been fixed, this correction can be treated as a constant quantity. It is natural to regard δS as a modification to the gravitational coupling [11] or $G_X^{-1} = G_E^{-1}[1 + \lambda\delta S]$, however, it is always possible to perform a simple conformal transformation, along with a redefinition of the coupling, that transfers this correction to the transverse volume. That is, if $\widetilde{X}$ is the transformed theory, then $G_{\widetilde{X}} = G_E$ and $\widetilde{V}_\perp = V_\perp[1+\lambda\delta S]$. Note that the position of the horizon is determined by the largest root of $|g_{tt}/g_{rr}|$ and so remains unaffected under such a transformation. Indeed, $S_{\widetilde{X}}/S_E = \widetilde{V}_\perp/V_\perp$, and so

$$s_{\widetilde{X}} = s_E. \tag{4}$$

This last result and the gauge-gravity duality implies that the entropy densities of the dual field theories are also equal, while Eq. (3) tells us that their shear viscosities satisfy $\eta_{\widetilde{X}} \geqslant \eta_E$. Hence,

$$\frac{\eta_{\widetilde{X}}}{s_{\widetilde{X}}} = \frac{\eta_X}{s_X} \geqslant \frac{\eta_E}{s_E} = \frac{1}{4\pi}, \tag{5}$$

where the first equality follows from the ratio η/s being trivially invariant under a *constant* rescaling of the metric and G. This is because η, like s, is a density (so, independent of $V_\perp$) and both scale numerically as G^{-1}. We have thus proved the KSS bound for any consistent extension of Einstein gravity and its field-theory dual.

Let us now discuss some examples. Obviously, in any theory which is equivalent to Einstein gravity with simple enough matter interactions, $\eta/s = 1/4\pi$. This can occur for theories that contain only topological corrections such as Gauss-Bonnet gravity in $4D$ and Lovelock gravity in higher even-numbered dimensions, or for theories that can be brought into Einstein's by a field redefinition such as $f(\mathcal{R})$ gravity.

To obtain a gravity theory without ghosts that extends Einstein gravity in a non-trivial way, one can start with a ghost-free theory and then consistently integrate out some of the matter or gravity degrees of freedom. A simple model in this class is Einstein's theory in $4+n$ dimensions, with the n extra dimensions compactified on a torus of radius R. From a $4D$ point of view, the correct description is the higher-derivative theory that results from integrating out the Kaluza–Klein (KK) modes. Each extra dimension $i = 1, 2, \ldots, n$ induces an infinite tower of massive KK modes with uniformly spaced masses $m_{k_i} \sim k_i/R$ ($k_i = 1, 2, \ldots, \infty$) for particles of spin-0, 1 and 2. Only the spin-2 particles will be relevant to the shear (xy) channel of the two-graviton propagator (1), which now goes as $[\mathcal{D}_{KK}]_x{}^y{}_y{}^x \sim R^2 G_E \sum_{i=1}^n \sum_{k_i=1}^\infty \frac{\rho_{k_i}(q^2)}{R^2q^2+k_i^2}$, with non-negative ρ_{k_i} at all energy scales. One finds that $\mathcal{D}_{KK}$ is vanishingly small for $q \ll 1/R$ and goes as $\mathcal{D}_{KK} \sim R/q$ for $q \gtrsim 1/R$. The latter dominates over the standard $1/q^2$ contribution when the contributing masses satisfy the hydrodynamic condition $m \sim 1/R \ll T$ in the UV ($R \gg T^{-1}$) regime.

On the other hand, the entropy density can always be computed in either $4+n$ dimensions or just 4, with the same result. The compactified dimensions make the same R^n contribution to both the area density in the numerator and the gravitational coupling in the denominator. Consequently, from a $4D$ point of view, η/s saturates the bound (5) in the IR where the theory is Einstein's and then increases towards the UV due to the increase in η.

Our assertion is that the bound (5) has to hold for a ghost-free theory. However, the converse is not true. It is possible, as demonstrated below, that the bound holds for some theories with ghosts but not for others; apparently, some ghosts are "friendlier" than others. For concreteness, let us discuss $5D$ Riemann-squared gravity. The Lagrangian density of this theory is $\mathcal{L} = \mathcal{R} + 12 + \lambda\mathcal{R}_{abcd}\mathcal{R}^{abcd}$, where we have set the AdS curvature radius equal to unity and λ is a constant. It is well known that this theory has ghosts for *any* value of λ, due to fourth-order time derivatives in the gravitational field equations.

The graviton propagator is calculated as follows: We expand the metric $g_{\mu\nu} \to g_{\mu\nu}^{(0)} + h_{\mu\nu}$ (with a superscript of (0) always denoting the $\lambda = 0$ solution) and then calculate the graviton kinetic terms which contain exactly two h's and two derivatives. The $\lambda = 0$ solution is the well-known AdS 3-brane, for which $-g_{tt} = g^{rr} = r^2[1 - \frac{r_h^4}{r^4}]$ (with $r_h = T/\pi$) and $g_{xx} = g_{yy} = g_{zz} = r^2$. It is also useful to note that $(\mathcal{R})^{(0)} = -20$ and $(\mathcal{R}_a{}^b)^{(0)} = -4\delta_a{}^b$ at any value of r. The values of $R^{ab}{}_{cd}$ on the horizon are $\mathcal{R}^{xy}{}_{xy} = 0$ and $\mathcal{R}^{rt}{}_{rt} = -\mathcal{R}^{rx}{}_{rx} = -\mathcal{R}^{xt}{}_{xt} = 2$, with all other possibilities either being redundant or trivially zero. Then

$$\mathcal{L}_{kin} = h_{ab}\Box h^{ab} - h\Box h + 8\lambda\left(\sum_{a,b\neq a}^{\{r,t\}} -2\sum_{a}^{\{r,t\}}\sum_{b}^{\{x,y,z\}}\right)\left(h_{ab}\Box h^{ab} - h^a{}_a\Box h^b{}_b\right), \tag{6}$$

where some boundary terms have been dropped, as well as a bulk term that is not of the kinetic form, and the usual summation conventions should be ignored where there is an explicit sum. The leading-order term is just the standard graviton propagator prior to any gauge fixing, and so we need to compare the sign of the higher-order terms with this one. The above form makes it clear that there are ghost terms for any non-vanishing value of λ.

Reading directly off of Eq. (6), we see that η maintains its Einstein value. Meanwhile, Wald's formalism [23–25] can be used to show that $s_X = (1 + 8\lambda)s_E$. One can deduce the exact same result from Eq. (6) by reading off the corrections to the propagator for the h_{rt} gravitons [11]. The net result is that $\eta/s = \frac{1}{4\pi}(1 - 8\lambda + \mathcal{O}[\lambda^2])$. So, although any $\lambda \neq 0$ induces a negative or ghost contribution to the propagator, the KSS bound is only violated for $\lambda > 0$.

Some readers might incorrectly view the bound-violating models that are obtained from a low-energy expansion of compactified string models (*e.g.*, [9,26]) as counter-examples to our proof. Such models are, of course, inherently ghost free and have a consistent low-energy expansion [26]. However, the truncated higher-curvature theories do have Planck/string-scale ghosts in their spectrum. It is usually argued that such apparent "fake" ghosts are immaterial, being at an energy scale that is above the validity of the low-energy effective theory [27]. Since the shear viscosity and the propagator (at vanishingly small momentum) are essentially the same entity, any bound violations in the former must be as physically meaningful (or meaningless) as the ghosts in the latter. That is, consistency dictates that one must dismiss these (apparent) bound violations along with the apparent ghosts.

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