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Dimension six corrections to the vector sector of AdS/QCD model

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Abstract

We study the effects of dimension six terms on the predictions of the holographic model for the vector meson form factors and determine the corrections to the electric radius, the magnetic and the quadrupole moments of the ρ-meson. We show that the only dimension six terms which contribute nontrivially to the vector meson form factors are X^2F^2 and F^3. It appears that the effect from the former term is equivalent to the metric deformation and can change only masses, decay constants and charge radii of vector mesons, leaving the magnetic and the quadrupole moments intact. The latter term gives different contributions to the three form factors of the vector meson and changes the values of the magnetic and the quadrupole moments. The results suggest that the addition of the higher dimension terms improves the holographic model.

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1. Introduction

The significant progress of the holographic duals of QCD (based on [1]) in determination of basic hadronic observables (see, e.g., Refs. [2–21]) suggests for further development. In this Letter, we work in the vector sector of the AdS/QCD model with the hard-wall cutoff, proposed in Ref. [2]. We study the effects of dimension six terms on the vector meson form factors and extract the values of observables such as the ρ-meson's electric radius, the mass, the decay constant, the magnetic and the quadrupole moments.

The leading order contribution to the vector meson form factors coming from the F^2 term has already been studied in detail in Refs. [3,4], where it has been shown that the holographic models in Refs. [2,5] reproduce only the trivial structure of vector mesons. In particular, instead of three independent form factors that describe vector meson, these holographic models predict only one.

We show that the inclusion of dimension six terms changes the situation towards a more interesting scenario in which all of the three form factors are corrected in different amounts. We also observe, that the only dimension six terms which give nontrivial contribution to the vector meson form factors are X^2F^2 and F^3. The contribution from the rest of the dimension six terms can be removed by the redefinition of the coupling constant g_5^2.

We find that the addition of a term such as X^2F^2 is equivalent to the AdS metric deformation and, according to Ref. [6], this, in turn, is equivalent to the inclusion of the vacuum condensates. This is in agreement with the point made in Ref. [2] that the higher dimension (HD) operators which appear in the operator product expansion of QCD arise in the holographic model from the higher terms in the 5D Lagrangian such as X^2F^2. We also notice that the term X^2F^2 does not alter the values of the magnetic and the quadrupole moments, however, changes the values of the vector meson electric radius, the mass and the decay constant.

The Letter is organized as follows, in Section 2, we go through the basics of the holographic model given in Refs. [2,7], and in particular, we discuss the leading order action, the equations of motion for the vector bound states and the forms

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of dimension six terms that can enter the action. In Section 3, we demonstrate that the term like X^2F^2 does not change the values of the magnetic and the quadrupole moments and that its effect is equivalent to the AdS metric deformation. We also discuss, how this term, to a first approximation, changes the values of the ρ-meson mass, the decay constant and the electric charge radius. In Section 4, we consider the relevant part of the F^3 Lagrangian and calculate the three-point function which is then used in Section 5 to derive the corrections to the form factors of vector mesons. In Section 6, we calculate the charge radius, the magnetic and the quadrupole moments of the ρ-meson and compare these with the predictions from the other models given in Refs. [23–28]. Finally, we summarize the Letter and also show that the form factor of pion can get corrections only from the term like X^2F^2.

2. Preliminaries

We are working in the background of the sliced AdS metric of the form:

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dz^2\right), \quad 0 < z \leqslant z_0, \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{Diag}(1,-1,-1,-1)$, $z = z_0$ imposes the IR hard wall cutoff, (with $z_0 \sim 1/\Lambda_{\mathrm{QCD}}$) and $z = \epsilon \to 0$ determines the position of UV brane. From the dictionary of the AdS/QCD model, we will correspond to the 4D vector current $J^a_\mu(x) = \bar{q}(x)\gamma_\mu t^a q(x)$ a bulk gauge field $A^a_\mu(x,z)$ whose boundary value is the source for $J^a_\mu(x)$. The 5D gauge action in the AdS_5 space is

$$S_{\mathrm{AdS}} = -\frac{1}{4g_5^2}\,\mathrm{Tr}\int d^4x\,dz\,\sqrt{g}F^{MN}F_{MN}, \tag{2}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$, $A_M = t^a A^a_M$, ($M,N = 0,1,2,3,z$; $\mu,\nu = 0,1,2,3$ and $t^a = \sigma^a/2$, where σ^a are usual Pauli matrices with $a = 1,2,3$). We work in the $A_z = 0$ gauge and require $\partial_\mu A^\mu = 0$.

Working in the Fourier image representation and defining $A^a_\mu(q,z) = A^a_\mu(q)A(q,z)$, we can determine the linearized equation of motion for $A(q,z)$, which is

$$\left[z^2\partial_z^2 - z\partial_z + q^2z^2\right]A(q,z) = 0, \tag{3}$$

with boundary conditions $A(q,0) = 1$ and $\partial_z A(q,z_0) = 0$.

In general, the 5D gauge theories are not renormalizable, since the 5D gauge coupling g_5^2 has negative mass dimension. This means that these theories can only be considered as an effective theories below some scale Λ. In particular, for our case, the cutoff scale Λ should be set by $1/g_5^2$.

Since, the holographic model is an effective theory with physical cutoff scale $\Lambda \sim 1/g_5^2$, we are free to add HD terms into the Lagrangian which respect all the required symmetries. The coefficients in front of the dimension six operators are of the form c/Λ, where c is some dimensionless constant and $\Lambda = v/g_5^2$ (it can be estimated that $v \sim 24\pi^3$). In general, since $g_5^2 = 12\pi^2/N_c$, according to Ref. [2], we have $c/\Lambda = 12\pi^2 c/(vN_c) \sim c/N_c$ and, therefore, for large N_c the HD terms are N_c-suppressed.

There are three groups of dimension six terms one can add into the AdS/QCD Lagrangian, which may contribute to the three-point function,

1. $(\nabla_A F_{MN})^2$, $\quad\left(\nabla_M F^{MN}\right)^2$, $\quad F^3$,
   $F^{MN}\nabla^2 F_{MN}$, $\quad\left(\nabla_K F^{MN}\right)\left(\nabla_N F^K_M\right)$,
2. RF^2, $\quad R^{MN}F_{MK}F^K_N$, $\quad R^{MNKP}F_{MN}F_{KP}$,
3. $X^\dagger XFF$, $\quad X^\dagger FXF$,

where ∇_M is a covariant derivative, R^{MNPK}, R^{MN} are Riemann and Ricci curvature tensors and R is a Ricci scalar. Here, we will ignore the backreaction of the matter on the metric of the AdS space. As a result, the contribution from the terms of the second group becomes formal, since in the AdS space these terms are proportional to F^2 and can be absorbed into the coupling g_5^2.

Using the equation of motion

$$\nabla_M F^{MN} = i\left[A_K, F^{KM}\right] \equiv J^M, \tag{4}$$

it can be shown that the term $(\nabla_M F^{MN})^2$ does not contribute to the two-point and three-point functions. Notice, that the terms $F^{MN}\nabla^2 F_{MN}$ and $(\nabla_A F_{MN})^2$ are equivalent, since they differ by a full covariant derivative which vanishes after the integration because of the boundary conditions on the fields. The terms in the third group contribute to the three-point function in such a way that the magnetic and the quadrupole moments remain unchanged. We will show this on the example with the X^2F^2 term.

The remaining dimension six terms which can contribute to the three-point function are given in the second line of the first group. Using the properties of the covariant derivatives and the equation of motion, it can be shown that

$$F^{MN}\nabla^2 F_{MN} \supset 2F^{MN}\nabla_M J_N \supset 2\nabla_M\left(F^{MN}J_N\right), \tag{5}$$

where we indicated only the parts which are not expressed through the terms in the second group or through the terms which do not contribute to the three-point function. The last term enters into the action as

$$\begin{aligned}&\mathrm{Tr}\int d^5x\,\sqrt{g}\nabla_M\left(F^{MN}J_N\right)\\ &\quad = -i\,\mathrm{Tr}\int d^4x\left(\sqrt{g}F^{z\nu}\left[A^\mu, F_{\mu\nu}\right]\right)_{z=0}.\end{aligned} \tag{6}$$

It can be shown that this term does not contribute to the vector meson form factors. There are different ways to see this. One of the ways is, to notice, that the form factor is obtained as a double residue of the three-point function (see, e.g., Ref. [3]). Then, working in the Fourier image representation, we have $A(q,0) = 1$ and, therefore, the term $[A_\mu, F^{\mu\nu}]_{z=0}$ cannot have any poles. As can be seen from Eq. (25), only the $F^{z\nu} = A^\nu(q)\partial^z A(q, z=0)$ term in (6), that has poles on the UV boundary. Therefore, since, we have only one term which has poles, the double residue will vanish, leading to zero corrections for the vector meson form factors. The similar arguments are applied for the term $(\nabla_K F^{MN})(\nabla_N F^K_M)$. It appears, that only the term F^3 in this group that can give nonzero corrections to the form factors of vector mesons.

The terms of the first group $F^{MN}\nabla^2 F_{MN}$ and $(\nabla_K F^{MN}) \times (\nabla_N F^K_M)$, contribute to the two-point function only through the terms in the second group. Therefore, the effect of these terms on the two point function is trivial and can be absorbed by the coupling g_5^2.

Notice, that the F^3 term is not coming from the expansion of DBI action. In this model, F^3 term is one of the possible terms which should be invariant under the Lorenz and gauge transformations. We also do not allow the violation of the 5D discrete charge conjugation symmetry in the AdS background. As we will show, the corrections associated with this F^3 term are $1/N_c^2$ suppressed.

3. The effects from the X^2F^2 term

Consider the correction to the action (2), of the form

$$S_{X^2F^2} = \kappa g_5^2 \operatorname{Tr} \int d^4x\, dz \sqrt{g} X^\dagger X F^{MN} F_{MN}, \tag{7}$$

where κ is some constant and following Ref. [2], we have $X^2 = \mathbf{1}_{(2\times 2)} v^2(z)/4$. In particular, $v(z) = (m_q z + \sigma z^3)$, where m_q is the quark mass parameter and σ plays the role of the chiral condensate.

We observe that the total action can be written as

$$S_{F^2} + S_{X^2F^2} = -\frac{1}{4g_5^2} \operatorname{Tr} \int d^4x\, dz \frac{p(z)}{z} F^{MN} F_{MN},$$

where the Lorentz indexes are now governed by the flat metric η_{MN}, $p(z) = 1 - \kappa g_5^4 v^2(z)$ and it is clear that, in general, the contribution from all the terms like $X^{2n}F^2$, (n is natural number), will modify $p(z)$ to a function $P(v(z)) \equiv 1 + C_1 g_5^4 v^2(z) + \cdots + C_n g_5^{4n} v^{2n}(z)$, where C_n are some unknown coefficients. Therefore, the inclusion of the X^2F^2 term corresponds effectively to the deformation of the AdS metric, that is instead of the $1/z^2$ factor in the metric (1), we will have $p^2(z)/z^2$. The similar arguments are applied also for the term $X^\dagger F X F$.

This observation allows the direct application of the result from Ref. [3] to the present case, leaving us with the following expression for the elastic form factors:

$$\tilde{F}_{nn}(Q^2) = \int_0^{z_0} dz \frac{p(z)}{z} \mathcal{J}(Q,z) \left|\psi_n(z)\right|^2, \tag{8}$$

where $\psi_n(z)$ are the solutions of the equations of motion,

$$\partial_z \left[\frac{p(z)}{z} \partial_z \psi_n(z)\right] + \frac{p(z)}{z} M_n^2 \psi_n(z) = 0, \tag{9}$$

with b.c. $\psi_n(0) = \psi_n'(z_0) = 0$ and $q^2 = M_n^2$. The function $\mathcal{J}(Q,z)$ is a solution of the same equation of motion but with $q^2 = -Q^2$ instead of M_n^2 and b.c. $\mathcal{J}(Q,0) = 1$, $\partial_z \mathcal{J}(Q,z_0) = 0$. The eigenfunctions of Eq. (9) are normalized as

$$\int_0^{z_0} dz \frac{p(z)}{z} \left|\psi_n(z)\right|^2 = 1. \tag{10}$$

Therefore, $\tilde{F}_{nn}(0) = 1$ and, since, the electric G_C, magnetic G_M and quadrupole G_Q form factors are:

$$G_Q^{(n)}(Q^2) = -\tilde{F}_{nn}(Q^2), \qquad G_M^{(n)}(Q^2) = 2\tilde{F}_{nn}(Q^2),$$
$$G_C^{(n)}(Q^2) = \left(1 - \frac{Q^2}{6M_n^2}\right) \tilde{F}_{nn}(Q^2), \tag{11}$$

one can check that at $Q^2 = 0$, these form factors reproduce the same values for electric charge, magnetic and quadrupole moments, as in the case with $\kappa = 0$, that is in the absence of the X^2F^2 term. This term, however, can change masses and decay constants of vector mesons. Besides, it also changes the electric radius of the ρ-meson.

Notice, that the eigenvalues of Eq. (9) may be expressed through the eigenfunctions in the following way:

$$M_n^2 = \int_0^{z_0} dz \frac{p(z)}{z} \left|\partial_z \psi_n(z)\right|^2. \tag{12}$$

Up to a first order approximation, using the same eigenfunctions as in case with $\kappa = 0$, that is

$$\psi_n^{(0)}(z) = \frac{\sqrt{2}}{z_0 J_1(\gamma_{0,n})} z J_1\left(M_n^{(0)} z\right), \tag{13}$$

with $M_n^{(0)} = \gamma_{0,n}/z_0$ (where $J_0(\gamma_{0,n}) = 0$) but with metric perturbation $p(z)$, we will have for the ρ-meson mass $M_\rho \equiv M_1$ the following result:

$$M_\rho \simeq M_\rho^{(0)}\left(1 - 0.02\kappa g_5^4\right), \tag{14}$$

where $M_\rho^{(0)}$ is the mass of the ρ-meson in case $\kappa = 0$, and we used the values of parameters: $m_q = 2.3$ MeV, $\sigma = (327\ \text{MeV})^3$, $z_0 = 1/(323\ \text{MeV})$, taken from the Model A of Ref. [2].

The decay constant of the ρ-meson, f_ρ, in terms of the eigenfunctions of the 5D equation of motion has the form

$$f_\rho = \frac{1}{g_5} \left(\frac{p(z)}{z} \partial_z \psi_\rho(z)\right)_{z \to 0}, \tag{15}$$

as was discussed, for example, in Ref. [4]. The solution for $\psi_\rho(z) \equiv \psi_1(z)$ near the $z = 0$ is of the same form as in case $\kappa = 0$ thus,

$$f_\rho = \frac{\sqrt{2} M_\rho}{g_5 z_0 J_1(\gamma_{0,1})}. \tag{16}$$

Therefore, to lowest order in κ, we will have:

$$f_\rho \simeq f_\rho^{(0)}\left(1 - 0.02\kappa g_5^4\right), \tag{17}$$

where $f_\rho^{(0)}$ is the decay constant in case when $\kappa = 0$.

We can also express the electric charge radius of the ρ-meson, $\langle \tilde{r}_\rho^2 \rangle_C$, defined as

$$\langle \tilde{r}_\rho^2 \rangle_C \equiv -6\left(\frac{dG_C^{(1)}(Q^2)}{dQ^2}\right)_{Q^2=0}, \tag{18}$$

in terms of the parameter κ. In this case, using Eqs. (8), (11) and (18), to lowest order in the coefficient κ, the electric charge radius is:

$$\langle \tilde{r}_\rho^2\rangle_C \simeq \left(0.53 - 0.16\kappa g_5^4\right)\ \mathrm{fm}^2, \tag{19}$$

where $0.53\ \mathrm{fm}^2$ is the result for the electric radius obtained in Ref. [3] (again, we used parameters taken from the Model A of Ref. [2]).

The similar analysis can be applied for the case of Model B in Ref. [2], for which we have:

$$\begin{aligned}
&M_\rho \simeq M_\rho^{(0)}\left(1 - 0.01\kappa g_5^4\right),\\
&f_\rho \simeq f_\rho^{(0)}\left(1 - 0.01\kappa g_5^4\right),\\
&\langle \tilde{r}_\rho^2\rangle_C \simeq \left(0.46 - 0.07\kappa g_5^4\right)\ \mathrm{fm}^2.
\end{aligned} \tag{20}$$

Notice, that the coefficients in front of κ, in case of Model B are almost twice as smaller than in the Model A. Also, it is straightforward to see that the contribution from the term $X^\dagger F X F$ can be absorbed by κ.

Now, since $g_5^2 = 12\pi^2/N_c$, it follows that the corrections to the observables ($\sim \kappa g_5^4$) are $1/N_c^2$ suppressed. The natural constraint on the coefficient κ should come from the requirement that the corrections to the observables are small. This means that, if $N_c = 3$, then for the first two observables in (20), we should have $|\kappa| \ll 0.06$ and for the third one we expect to have $|\kappa| \ll 0.004$. Therefore, we conclude, that it is natural for the coefficient κ to satisfy the condition $|\kappa| \ll 10^{-3}$.

4. Corrections from the F^3 term

The action relevant for finding the corrections to the 3-point function is

$$\begin{aligned}
S_{F^3} &= \frac{1}{2}\alpha g_5^2 \,\mathrm{Tr} \int d^4x\, dz\, \sqrt{g}\left(\left[F_{MN}, F^{NK}\right] F_K{}^M\right)\\
&\supset \frac{i\alpha g_5^2 \epsilon^{abc}}{4} \int d^4x\, dz\, z\left[3\left(\partial_\mu A_\nu^a\right)\left(\partial_z A^{b,\nu}\right)\left(\partial_z A^{c,\mu}\right)\right.\\
&\quad \left. + 2\left(\partial^\mu A^{a,\nu}\right) F_\nu^{b,\alpha} F_{\alpha\mu}^c\right],
\end{aligned} \tag{21}$$

where α is a new dimensionless parameter of the theory and the Lorentz indexes are governed by the Minkowski flat metric $\eta_{\mu\nu}$. Therefore, using the prescription of the holographic model, for the 3-point function we will have:

$$\begin{aligned}
&T_{\mu\alpha\beta}^{abc}(p_1, p_2, q)\\
&\quad \equiv \left\langle J_\alpha^b(p_1) J_\mu^a(q) J_\beta^c(-p_2)\right\rangle\\
&\quad = \epsilon^{abc} T_{\mu\alpha\beta}(p_1, p_2, q)\, i(2\pi)^4 \delta^{(4)}(q - p_2 + p_1),
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
&T_{\mu\alpha\beta}(p_1, p_2, q)\\
&\quad = \frac{3\alpha g_5^2}{4}\left\{\left[q^2 K_2 - K_{11}\right]\eta_{\alpha\beta}(p_1 + p_2)_\mu\right.\\
&\qquad + \left[2M^2 K_2 - K_{12}\right](\eta_{\mu\alpha} q_\beta - \eta_{\mu\beta} q_\alpha)\\
&\qquad \left. - 2K_2 q_\alpha q_\beta (p_1 + p_2)_\mu\right\},
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
K_{11}(p_1, p_2, q) &= \int_0^{z_0} dz\, z\, \partial_z A(q, z) A(p_1, z) \partial_z A(p_2, z),\\
K_{12}(p_1, p_2, q) &= \int_0^{z_0} dz\, z\, \partial_z\left[A(q, z) A(p_1, z)\right] \partial_z A(p_2, z),\\
K_2(p_1, p_2, q) &= \int_0^{z_0} dz\, z\, A(q, z) A(p_1, z) A(p_2, z),
\end{aligned} \tag{24}$$

where we used that the functions $K(p_1, p_2, q)$ are symmetric under the exchange of $p_1 \leftrightarrow p_2$ (to understand this, see Eq. (25)), but not $p_{1,2} \leftrightarrow q$, ($q = p_2 - p_1$) and anticipating the on-shell limit, we applied conditions: $p_1^2 = p_2^2 = M^2$, $(p_1 p_2) = M^2 - q^2/2$ and $(p_2 q) = -(p_1 q) = q^2/2$, for the diagonal transitions (one can easily generalize this to nondiagonal transition). Since we are dealing with the transverse components of the gauge field, to simplify the tensor structure, we applied, as in [4], the transverse projectors $\Pi^{\alpha\alpha'}(p_1) \equiv (\eta^{\alpha\alpha'} - p_1^\alpha p_1^{\alpha'}/p_1^2)$, etc., (that allows us to add or eliminate terms proportional to $p_{1\alpha}$ or $p_{2\beta}$). The solution of the (3) for timelike momentum can be written as an infinite sum:

$$A(p, z) = -g_5 \sum_{m=1}^\infty \frac{f_m \psi_m(z)}{p^2 - M_m^2}, \tag{25}$$

where $\psi_m(z)$ are the solutions of the (3) with b.c. $\psi_m(0) = \psi_m'(z_0) = 0$ and $q^2 = M_m^2$. Then, for a spacelike momentum transfer, $q^2 = -Q^2$, it follows that:

$$T_{\mu\alpha\beta}(p_1, p_2, q) = \frac{3\alpha g_5^4}{4} \sum_{n,k=1}^\infty \frac{f_m f_n R_{\mu\alpha\beta}^{nk}(Q^2)}{(p_1^2 - M_n^2)(p_2^2 - M_k^2)},$$

and for the diagonal $n \leftrightarrow n$ transition:

$$\begin{aligned}
R_{\mu\alpha\beta}^{(n)}(Q^2) &\equiv \lim_{p_1^2 \to M_n^2} \lim_{p_2^2 \to M_n^2} \left(p_1^2 - M_n^2\right)\left(p_2^2 - M_n^2\right) T_{\mu\alpha\beta}\\
&= \frac{3\alpha g_5^4}{4}\left\{-\left[Q^2 W_2^{nn} + W_{11}^{nn}\right]\eta_{\alpha\beta}(p_1 + p_2)_\mu\right.\\
&\quad + \left[2M_n^2 W_2^{nn} - W_{12}^{nn}\right](\eta_{\mu\alpha} q_\beta - \eta_{\mu\beta} q_\alpha)\\
&\quad \left. - 2W_2^{nn} q_\alpha q_\beta (p_1 + p_2)_\mu\right\},
\end{aligned} \tag{26}$$

where we defined new functions as

$$W_{11}^{nn}\left(Q^2\right) = \int_0^{z_0} dz\, z\, \partial_z \mathcal{J}(Q, z) \psi_n(z) \partial_z \psi_n(z), \tag{27}$$

$$W_{12}^{nn}\left(Q^2\right) = \int_0^{z_0} dz\, z\, \partial_z\left[\mathcal{J}(Q, z) \psi_n(z)\right] \partial_z \psi_n(z), \tag{28}$$

$$W_2^{nn}\left(Q^2\right) = \int_0^{z_0} dz\, z\, \mathcal{J}(Q, z) \psi_n(z) \psi_n(z), \tag{29}$$

with

$$\mathcal{J}(Q, z) = Qz\left[K_1(Qz) + I_1(Qz)\frac{K_0(Qz_0)}{I_0(Qz_0)}\right], \tag{30}$$

where $\mathcal{J}(Q,z)=A(Q,z)$ is the solution of Eq. (3).

5. Form factors

Adding the corrections to the form factor coming from the F^3 term to the leading order result from the F^2 term obtained in Ref. [3] gives for the electric $\tilde{G}_C$, magnetic $\tilde{G}_M$ and quadrupole $\tilde{G}_Q$ form factors the following result

$$\begin{aligned}
\tilde{G}_C^{(n)}(Q^2) &= \left[1-\frac{Q^2}{6M_n^2}\right]F_{nn}-\frac{3\alpha g_5^4 Q^2}{4}\left[1+\frac{Q^2}{12M_n^2}\right]W_2^{nn} \\
&\quad -\frac{3\alpha g_5^4}{4}\left[1+\frac{Q^2}{6M_n^2}\right]W_{11}^{nn}+\frac{\alpha g_5^4 Q^2}{8M_n^2}W_{12}^{nn}, \\
\tilde{G}_M^{(n)}(Q^2) &= 2F_{nn}(Q^2)+\frac{3\alpha g_5^4}{4}\left[2M_n^2 W_2^{nn}-W_{12}^{nn}\right], \\
\tilde{G}_Q^{(n)}(Q^2) &= -F_{nn}(Q^2)-\frac{3\alpha g_5^4 Q^2}{8}W_2^{nn} \\
&\quad -\frac{3\alpha g_5^4}{4}\left[W_{11}^{nn}-W_{12}^{nn}\right],
\end{aligned} \tag{31}$$

where

$$F_{nn}(Q^2)=\int_0^{z_0}\frac{dz}{z}\mathcal{J}(Q,z)\left|\psi_n(z)\right|^2, \tag{32}$$

see Ref. [3] for more details. In the AdS/QCD model, with $\alpha=0$ as was shown in [3], these three form factors of vector meson are expressed through the single function $F_{nn}(Q^2)$. Besides, for $Q^2=0$, the AdS/QCD model reproduce the unit electric charge e of the meson, "predict" $\mu\equiv G_M(0)=2$ for the magnetic moment and $D\equiv G_Q(0)/M^2=-1/M^2$ for the quadrupole moment, which are just the canonical values for a vector particle [22]. However, for non zero value of α the situation changes towards a more realistic scenario.

6. Results

One can verify that at $Q^2=0$, we have $W_{11}^{nn}(0)=0$, because $\partial_z\mathcal{J}(0,z)=0$, since

$$\partial_z\mathcal{J}(Q,z)=-zQ^2\left[K_0(Qz)-I_0(Qz)\frac{K_0(Qz_0)}{I_0(Qz_0)}\right]. \tag{33}$$

Besides

$$W_{12}^{11}(0)=\int_0^{z_0}dz\,z\left(\partial_z\psi_1(z)\right)^2=\frac{2M^2z_0^2}{J_1^2(\gamma_{0,1})}\int_0^1 d\zeta\,\zeta^3 J_0^2(\gamma_{0,1}\zeta), \tag{34}$$

$$W_2^{11}(0)=\int_0^{z_0}dz\,z\psi_1^2(z)=\frac{2z_0^2}{J_1^2(\gamma_{0,1})}\int_0^1 d\zeta\,\zeta^3 J_1^2(\gamma_{0,1}\zeta), \tag{35}$$

where $J_0(\gamma_{0,1})=0$, $M=\gamma_{0,1}/z_0$ is the mass of the ρ-meson and we took into account that

$$\psi_1(z)=\frac{\sqrt{2}}{z_0J_1(\gamma_{0,1})}zJ_1(Mz). \tag{36}$$

After partial integrations and using the properties of Bessel functions we will have

$$W_{12}^{11}(0)=M^2W_2^{11}(0)-2. \tag{37}$$

Now, defining $w\equiv W_{12}^{11}(0)\simeq 1.261$, we find ($e=1$),

$$\begin{aligned}
\mu &\equiv \tilde{G}_M^{(1)}(0)=2+\frac{3\alpha g_5^4}{4}(w+4), \\
DM^2 &\equiv \tilde{G}_Q^{(1)}(0)=-1+\frac{3\alpha g_5^4 w}{4}.
\end{aligned} \tag{38}$$

The electric radius of the ρ-meson is

$$\begin{aligned}
\langle\tilde{r}_\rho^2\rangle_C &\equiv -6\left(\frac{d\tilde{G}_C^{(1)}(Q^2)}{dQ^2}\right)_{Q^2=0} \\
&= \langle r_\rho^2\rangle_C+\alpha g_5^4\left[\frac{3}{4M^2}(5w+12)\right. \\
&\quad \left.+\frac{9}{2}\left(\frac{dW_{11}^{11}(Q^2)}{dQ^2}\right)_{Q^2=0}\right],
\end{aligned} \tag{39}$$

where the first term is $\langle r_\rho^2\rangle_C=0.53\ \mathrm{fm}^2$, found in Ref. [3], and the second term in the square brackets is the correction to the ρ-meson's radius. Using Eqs. (27), (33) and (36) one can find that

$$\begin{aligned}
&\frac{9}{2}\left(\frac{dW_{11}^{11}(Q^2)}{dQ^2}\right)_{Q^2=0} \\
&\quad =\frac{9\gamma_{0,1}z_0^2}{J_1^2(\gamma_{0,1})}\int_0^1 d\zeta\,\zeta^4\ln\zeta\,J_0(\gamma_{0,1}\zeta)J_1(\gamma_{0,1}\zeta),
\end{aligned} \tag{40}$$

which is $\simeq -0.255\ \mathrm{fm}^2$. Therefore,

$$\sigma\equiv\left(\langle\tilde{r}_\rho^2\rangle_C-\langle r_\rho^2\rangle_C\right)/\mathrm{fm}^2\simeq 0.647\alpha g_5^4\simeq 252\alpha. \tag{41}$$

Now, in terms of σ, the magnetic and quadrupole moments of the ρ-meson are: $\mu\simeq 2+6.1\sigma$ and $DM^2=1.46\sigma-1$. Table 1 shows possible values for electric radius, magnetic and quadrupole moments in terms of reasonable range of values for σ, where $r^2\equiv\langle\tilde{r}_\rho^2\rangle_C/\mathrm{fm}^2$. These results depend explicitly on α (or σ) and implicitly on z_0 which is fixed by the mass of the ρ-meson. Notice, that $g_5^4|\alpha|<0.23$, therefore, we are not outside of the perturbative domain and our calculations are consistent. For comparison with other models, see Table 2.

It is interesting, that the only HD term in the 5D effective theory that can alter the canonical values of the magnetic and the quadrupole moments is the term F^3. Therefore, the more precise knowledge of either one of these observables (μ, D or r^2) can put more stringent constraints on the coefficient α. Here, we showed that the corrections are proportional to αg_5^4 and thus,

Table 1
The observables for different values of σ

σ	−0.15	−0.1	−0.05	−0.01	0.01	0.05	0.1	0.15
r^2	0.38	0.43	0.48	0.52	0.54	0.58	0.63	0.68
μ	1.09	1.39	1.7	1.94	2.06	2.31	2.61	2.92
$-DM^2$	1.22	1.15	1.07	1.01	0.99	0.93	0.85	0.78

Table 2
The observables in different models

Models	[23]	[24]	[25]	[26]	[27]	[28]
r^2	0.27	0.37	0.37	0.39	0.54	0.55
μ	1.92	2.69	2.14	2.48	2.01	2.25
$-DM^2$	0.43	0.84	0.79	0.89	0.41	0.11

are $1/N_c^2$ suppressed as expected. Finally, our estimates suggest that $|\alpha| < 10^{-4}$.

7. Summary

In this Letter, as one of the possible ways to test and improve the AdS/QCD model proposed in Ref. [2], we considered the addition of dimension six terms into the vector sector of the AdS/QCD Lagrangian and study their effect on the vector meson form factors.

We discussed that ignoring the backreaction of the matter to the metric, the effect from the terms of the second group involving the AdS curvature tensors and Ricci scalar, is equivalent to the redefinition of the coupling g_5^2. We showed that the term, like X^2F^2, does not change the electric charge, the magnetic and the quadrupole moments, but affects the charge radius, the masses and the decay constants of the vector mesons. The effect of this term is equivalent to the AdS metric deformation and, in agreement with [2] and [6], it is also equivalent to the addition of the vacuum condensates. However, one should keep in mind that the metric deformations are also coming from the matter fields, which we ignore compared to the explicit or effective metric deformations from the X^2F^2 term.

By calculating the form factors, we found a relation between electric charge radius, mass and decay constant of the ρ-meson on the coefficient κ (to lowest order) with which the term X^2F^2 enters the action. Also, we expressed electric radius, magnetic and quadrupole moments of the ρ-meson in terms of the dimensionless parameter α, with which the term F^3 enters the action. These results can be straightforwardly generalized to the case of the soft wall model [4,5].

It is also interesting to study the contribution of the dimension six terms to the form factor of pion. As it was discussed in Ref. [20], in the full AdS/QCD model the pion form factor is derived from the variation of the action with respect to the two longitudinal axial-vector fields and one transverse vector field. As a result, only the term like X^2F^2 can contribute to the form factor of pion. To demonstrate this, first, consider the term $F_A^2F_V$, where F_A is related to the axial-vector field. This term may contribute to the three-point function in such a way that only the linear pieces of the field strength tensors can enter. However, since these linear pieces vanish for the longitudinal axial-vector field, there cannot be any contribution from the term like F^3 to the form factor of pion (this question was also discussed in Ref. [21]).

The other relevant dimension six terms $(\nabla_A F_{MN})^2$ and $(\nabla_K F^{MN})(\nabla_N F^K{}_M)$ also cannot contribute to the form factor of pion. We demonstrate this on the example with the term $(\nabla_A F_{MN})^2$ which, as shown above contributes to the action in the form given in Eq. (6). However, this term contains two field strength tensors, and at least one should vanish for the longitudinal components. Similar arguments can be also applied for the second term.

Finally, we think that the results obtained here are in the range of the values from the other models. This is encouraging and suggests that the further addition of the HD terms can improve the holographic dual model of QCD.

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