RIGID STRUCTURE RESPONSE ANALYSIS TO SEISMIC AND BLAST INDUCED GROUND MOTIONS

Hong Hao\textsuperscript{a}\textsuperscript{*} and Yun Zhou\textsuperscript{b}

\textsuperscript{a} School of Civil and Resource Engineering, the University of Western Australia, Perth, WA 6009, Australia
\textsuperscript{b} Earthquake Engineering Research Center, Guangzhou University, Guangzhou, Guangdong 510405, China

Abstract

Comprehensive studies of rigid structure responses to seismic ground excitations have been reported. It was found that the rocking and sliding response of a rigid structure is highly nonlinear. The structure stability depends on the structure slenderness, as well as the ground motion amplitude, frequency and duration. Compared to an earthquake ground motion, ground shock induced by underground or surface explosion has very large amplitude, high frequency and short duration. Moreover, vertical component of a ground shock may be substantially larger than the gravitational acceleration \( g \). This will cause the unanchored rigid structure jump or fly into air. Therefore, the responses and stability regions of a rigid structure to blast induced ground shock will be very different from those under seismic ground motions. No study of rigid structure response to ground shock of amplitude more than 1.0 \( g \) can be found in the literature. As there might be many rigid structures such as computers, document shelfs, and other important equipments in a building or a military command center close to an explosion center, understanding rigid structure response to ground shock is essential for protection of such equipments. In this study, theoretical derivation and numerical prediction of rigid structure response to ground shock are carried out. Numerical results of stability regions of rigid structures to ground shock are derived. Particular attentions are paid to the case when the vertical ground shock is more than 1.0\( g \) and the rigid structure flies into the air. Results are compared to those obtained with earthquake ground motions. Discussions on the rigid structure stability to earthquake motion and ground shock are made.

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1. INTRODUCTION

Overturning damage of rigid structures such as furniture and equipment during strong earthquake excitations has been reported in almost all the previous major earthquakes. This type of damage, although it is nonstructural, might cause enormous economic losses and sometimes casualties. As a result, rocking and sliding responses of rigid structures under earthquake excitations have been widely studied. Most of these studies are theoretical based to define the unstable regions of possible overturning of unanchored structures. Tso and Wong (1989a, b) reported results from a theoretical and an experimental study of the rocking response of rigid blocks subjected to horizontal sinusoidal motion. It was found that the rigid block can respond in several different ways for a given excitation amplitude and frequency. The steady rocking response amplitude is highly sensitive to the size of the block and excitation frequency, but is relatively insensitive to the restitution coefficient of the system. Hogan (1990) further extended the work by Tso and Wong (1989a) and provided explanations of some of the observations made in Tso and Wong (1989a, b). Zhang and Makris (2001) studied the rocking response of rigid blocks subjected to horizontal sinusoidal pulses. Based on the analytical results, they derived the stability regions of the rigid blocks in the acceleration-frequency plane. It is found that the shape of the stable region depends on the coefficient of restitution and was sensitive to the nonlinear nature of the problem. The authors (Makris and Zhang 2001) extended the work to study the rocking responses of rigid blocks with anchors to a horizontal single sine pulse motion. As expected, in most frequency range, the anchored block survived higher acceleration than free-standing blocks. However, there is a finite frequency range where the opposite was observed.

Most previous studies considered only horizontal excitation. In reality, horizontal and vertical excitations exist simultaneously. Yim, et al (1980) derived the governing equations of rocking response of rigid blocks to horizontal and vertical ground motions. Taniguchi (2002) also derived the governing equations of rigid blocks to horizontal and vertical ground motions, and performed nonlinear time history analysis of the rocking and combined rocking-sliding responses of the rigid block to earthquake ground motions. It was concluded that vertical ground motion adds irregularities to the rigid block response. Therefore it should not be ignored in analyzing the rigid block response to earthquake ground motions.

Most of the previous studies assumed ground motions as an idealized sinusoidal impulse. A few studies considered actual earthquake ground motions (Makris and Zhang 1999, Taniguchi 2002). Because all of those studies targeted at analyzing rigid block response to earthquake motions, the frequency range and peak ground acceleration considered are consistent with those in common earthquake motions. On the other hand, intensive research efforts have been spent on modelling underground and surface explosion-induced stress wave propagation and the effect of ground shock on structure responses (Ma, et al. 1998, Ma, et al. 2000, Lu, et al. 2001). It was found that (Hao, et al. 2001), as compared to earthquake ground motion, explosion-induced ground shock has substantially higher frequency contents, larger amplitude but shorter duration. In particular, the vertical component of a ground shock could be larger than the horizontal component and the peak vertical ground acceleration could be larger than the gravitation acceleration, g. Because of this, the unanchored rigid structure may fly into the air under ground shock. This flight mode is very unlikely to occur in an earthquake excitation situation because the vertical ground acceleration is usually less than g. Therefore in previous studies of rocking responses of rigid blocks to earthquake ground motions possible flight mode of the rigid block was not considered.

With the research advancement on new construction materials and design skills, many techniques have been developed to protect surface and underground structures to against ground shocks. Research on responses of rigid blocks inside a structure to ground shock, however, cannot be found in the open literature. Besides structure protection, protecting important equipments such as computers and electronic maps inside a military command center against bombing attacks is essential for proper functioning of the center. It is also interesting to study the potentials of toppling of furniture and electronic devices in a
terrorist bombing event. In this study, rocking responses to rigid blocks to simultaneous horizontal and vertical ground motions are calculated. Parametric studies of rigid block responses to simultaneous horizontal and vertical sinusoidal pulses with the vertical pulse smaller or larger than gravitational acceleration $g$, simultaneous horizontal and vertical earthquake ground excitation and simultaneous horizontal and vertical explosion-induced ground shock are carried out. Particular attentions are paid on the flight mode of the block to ground shock, which has not previously been investigated in literature. Numerical results confirm that the size and slenderness of the block along with the ground motion frequency, amplitude and duration have significant effects on toppling of rigid blocks. Including vertical ground motion component results in highly nonlinear responses, the vertical ground motion component, especially when it is larger than $g$ that induces the flight mode is too significant to ignore.

2. THEORETICAL DERIVATION

Figure 1 illustrates rocking response of a rigid block subjected to simultaneous horizontal and vertical ground acceleration $g_h$ and $g_v$, and the respective bending moment about the corner point $O$ and $O'$ at different rotational response $\theta$. The width of the block is $b$ and height is $h$, the distance between the center of gravity of the block and the corner is $R$. The rocking response is defined by the angle of rotation $\theta$. Under ground excitation, the block rocks about its corner $O$ and $O'$. In general, the more slender a block, the easier the block to topple. The slenderness of the block $D = b/h$, also known as the critical angle of rotation, is the aspect ratio of the block, i.e., $b/h$. When the angle $\theta$ exceeds $\alpha$, toppling might occur. However, this is not a sufficient condition for toppling. Makris and Zhang (1999) discussed the toppling criteria, and presented cases with $\theta > \alpha$ but the block did not topple.

![Bending Moment Diagram](image)

Figure 1: Rocking of a rigid block subjected to simultaneous horizontal and vertical ground motion

Ground excitation may cause sliding besides rocking, or simultaneous sliding and rocking responses of the block. It was derived by Makris and Zhang (1999) that when $\tan \alpha = \frac{b}{h} \leq \mu$, in which $\mu$ is the coefficient of friction between ground and the block, rocking response initiates without sliding. This condition is assumed in the present study and only rocking response is considered. Assuming that the
coefficient of friction is large enough so that there is no sliding, the equations of rocking motion under simultaneous horizontal and vertical ground accelerations are (Yim, et al. 1980),

\[ I_0 \ddot{\theta}(t) + mg(1 + \frac{\dot{v}_g(t)}{g})R \sin(-\alpha - \theta) = -m\ddot{u}_g(t)R \cos(-\alpha - \theta), \quad \theta < 0 \]  

and

\[ I_0 \ddot{\theta}(t) + mg(1 + \frac{\dot{v}_g(t)}{g})R \sin(\alpha - \theta) = -m\ddot{u}_g(t)R \cos(\alpha - \theta), \quad \theta > 0 \]

where \( I_0 \) is the moment of inertia of the block about \( O \) and \( O' \). For a rectangular block, \( I_0 = \frac{4mR^2}{3} \), in which \( m \) is the mass of the block. Eqs (1) and (2) can be combined together as (Makris and Zhang 1999)

\[ \ddot{\theta}(t) = -p^2 \left\{ \sin[\alpha \text{sgn}[\theta(t)] - \theta(t)](1 + \frac{\dot{v}_g(t)}{g}) + \frac{\ddot{u}_g(t)}{g} \cos[\alpha \text{sgn}[\theta(t)] - \theta(t)] \right\} \]

where \( p = \sqrt{\frac{3g}{4R}} \) is a constant.

3. NUMERICAL SOLUTIONS

Eqs. (1) to (3) are nonlinear differential equations. Closed form solutions of these equations are not straightforward especially when the ground acceleration \( \ddot{u}_g \) and \( \dot{v}_g \) are not defined by any mathematical functions such as the cases of earthquake and ground shock time histories. In this study, a computer code based on step-by-step integration with central difference method is developed to solve the above equations. Letting

\[ \dot{\theta}_i = \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t} \]  

\[ \ddot{\theta}_i = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{\Delta t^2} \]

and substituting Eqs. (4) and (5) into Eqs. (1) and (2), and rearranging the equations, it has

\[ \theta_{i+1} = \frac{\Delta t^2}{I_0} \left\{ -m\ddot{u}_g R \cos(-\alpha - \theta_i) - mg(1 + \frac{\dot{v}_g(t)}{g})R \sin(-\alpha - \theta_i) \right\} + 2\theta_i - \theta_{i-1}, \quad \theta_i < 0 \]

\[ \theta_{i+1} = \frac{\Delta t^2}{I_0} \left\{ -m\ddot{u}_g R \cos(\alpha - \theta_i) - mg(1 + \frac{\dot{v}_g(t)}{g})R \sin(\alpha - \theta_i) \right\} + 2\theta_i - \theta_{i-1}, \quad \theta_i > 0 \]

With these two equations, the response \( \theta \) at every time step \( i \) can be obtained. In this study, the initial conditions \( \theta_0 \) and \( \theta_{-1} \) are set to zero. It should be noted that when the response \( \theta \) changes sign, i.e.,
rocking about $O$ is changed to rocking about $O'$, the block impacts on the ground, and this impact dissipates certain energy. Assume the rotation continues smoothly from $O$ to $O'$, it has $\theta_i>0$ and $\theta_i<0$, or vice-versa, from the conservation of momentum about point $O'$, it has

$$I_0\dot{\theta}_i - m\dot{\theta}_i b R\sin(\alpha) = I_0\dot{\theta}_{i+1}$$

(8)

in which step $i$ is the rocking just before the impact on $O'$ and step $i+1$ is right after the impact. The ratio of the kinetic energy after and before the impact is the coefficient of restitution $r$

$$r = \frac{\dot{\theta}_{i+1}^2}{\dot{\theta}_i^2}$$

(9)

then it has $\dot{\theta}_{i+1} = \sqrt{r}\dot{\theta}_i$, indicating the rotational velocity after the impact is only $\sqrt{r}$ times of the velocity before the impact because of energy loss owing to impact. Therefore, in step-by-step solutions, whenever the rocking response $\theta$ changes sign, the rotation velocity is reduced by an amount of $\sqrt{r}$. Using forward different for $\dot{\theta}_{i+1}$, and central difference defined in Eq.(4) for $\dot{\theta}_i$, it can be derived that

$$\theta_{i+1} = \frac{2}{2 - \sqrt{r}} (\theta_i - \frac{\sqrt{r}}{2} \theta_{i-1})$$

(10)

This formula is used in the present study to calculate rotational response when rotational angle changes sign. The coefficient of restitution depends on many parameters. In this study, various values of $r$ are assumed to study its influence on rocking responses.

In the study by Makris and Zhang (1999), rocking response of a rigid block to a horizontal sinusoidal sine pulse is calculated. The horizontal ground acceleration was defined as

$$\ddot{u}_g(t) = \begin{cases} 
  a_p \sin(\omega_p t + \psi), & -\psi/\omega_p \leq t \leq (2\pi - \psi)/\omega_p \\
  0, & \text{otherwise}
\end{cases}$$

(11)

where $\psi = \sin^{-1}(\alpha g / a_p)$ is the phase angle when rocking initiates, $a_p$ is the peak ground acceleration and $\omega_p$ is the circular frequency of the sinusoidal impulse. Assuming $\alpha$ is small, Eqs. (1) and (2) can be approximately linearized, and closed form solution was derived (Makris and Zhang 1999). Figure 2 shows the peak acceleration $a_p$ with respect to different frequencies obtained with closed-form (analytical) and numerical solutions to cause overturning of a rigid block of $h=3.11$ m, $b=0.79$ m and $r=0.9$ in Makris and Zhang (1999). The unstable region of this block is also determined using the computer code described above. The results are also shown in Figure 2. As shown, the present simulations give exactly the same results as the nonlinear numerical simulation results in Makris and Zhang (1999). The validity of the present code is therefore verified.
4. FLIGHT MODE

As discussed above, previous studies did not consider the possible flight mode when the vertical ground acceleration is larger than g that causes separations of the free standing block from the ground and results in the block flying in the air. Because explosion induced ground shock usually has peak ground accelerations higher than g, it is important to investigate the responses of rigid block under such responses. To investigate the influences of the flight mode on block responses, responses of the same rigid block defined above subjected to simultaneous one-cycle sinusoidal ground motion are calculated. The calculated acceleration, velocity and displacement responses, together with the input ground motions are shown in Figure 3. As shown, when the vertical ground acceleration is upward, the block moves together with the ground, no separation occurs. When ground acceleration is downward and is less than g, the block still moves together with the ground. When the downward ground acceleration is larger than g, separation of block from the ground occurs and the block experiences a free-fall in air with a constant acceleration of g. The block will remain in the air for a certain time until it is in touch with the ground again. Then the block will again move together with the ground in the vertical direction, but some relative displacement between block and ground in the horizontal direction occurs. These results demonstrate that separation of the block from the ground substantially affect its responses. It should be noted that rocking response is simulated continuously during the time when the block flies in the air.
5. NUMERICAL SIMULATIONS

In this study, responses of blocks to actual recorded ground motions from earthquakes and underground explosions are calculated. Figure 4 shows a typical recorded ground motions during the 1987 Cadoux earthquake. The magnitude of the earthquake is ML=4.5, epicentral distance is 6 km, and focal depth is 5 km. The ground motion is recorded on weathered bedrock. As shown, a strong horizontal impulse is recorded with the peak horizontal acceleration about 2g. The corresponding vertical impulse is slightly more than 1g. Figure 5 shows the recorded ground shock in a field blast testing. The explosion weight was 25 kg TNT equivalent, detonated in an underground chamber of dimension 250 m³ in granite mass. The explosion center was 8.5 m below the ground surface and ground shock was recorded on ground surface 25 m from the center of explosion chamber. As shown, both horizontal and vertical peak ground accelerations are larger than g. These recorded ground motions are used in numerical analysis as input to calculate rocking responses of rigid blocks of different dimensions.
Simulation results indicated that under earthquake ground motions separation of the block from the ground does not occur although vertical ground motion is slightly larger than g. The vertical response of the block follows exactly the ground motion. Under ground shock, however, the flight mode of the block is induced. Figure 6 shows the horizontal and vertical response time histories of the block. As shown, block separates from the ground at about 0.002 s, and lands on the ground at about 0.014 s only for an instant, where it loses its free-fall velocity, but re-enters into the flight mode immediately as the ground acceleration is less than g. It finally lands on the ground at about 0.016 sec and after that the vertical response follows the ground movements. As shown, when the block separates from the ground, it continues to move upwards owing to momentum from the initial strong impulse. Although the separation distance between the block and the ground is very small, it leads to a large portion of the ground shock excitation being skipped. Therefore, entering into a flight mode could be beneficial to the block stability. As shown in the figure, although peak vertical ground shock is about 9g, and that of the horizontal ground shock is about 3g, overturning of the block does not occur. This is because the block response depends not only on the ground acceleration amplitude, but also on the ground acceleration frequency and duration. Ground shock frequency is very high and duration very short, therefore a large amplitude, as compared to earthquake ground motions which have smaller frequency and longer duration, is needed to overturn the block.

**Figure 6**: Response time histories of the block to simultaneous horizontal and vertical ground shock

**Figure 7**: Peak earthquake and ground shock acceleration required to topple the block of different sizes and $a=0.25$ and $r=0.9$
Figure 7 shows the multipliers of the earthquake ground motion and ground shock shown in Figures 4 and 5 required to overturn blocks of different sizes, but same slenderness and restitution coefficient. As shown, the larger the block, the larger ground motion amplitude required to overturn the block. The required ground shock amplitude to topple the block is substantially larger than earthquake ground motion. This is expected because earthquake ground motion has lower frequencies and longer duration than ground shock. It was also observed in Makris and Zhang (1999) that overturning depends not only on peak ground acceleration, but also on ground velocity. Owing to its very high frequency, ground shock velocity is small although it has large acceleration, as compared to the earthquake motion. Therefore, it is more difficult for a ground shock to overturn a rigid block than an earthquake ground motion.

6. CONCLUSIONS

This study presents numerical simulation results of responses of free-standing rigid blocks to simultaneous horizontal and vertical earthquake motion and underground explosion induced ground shock. It is found that when vertical ground motion amplitude is larger than g, separation of the rigid block from ground may occur and the block enters into a flight mode. Although the separation distance between block and ground is small, it leads to the block free from the ground excitation. This may reduce the toppling potential of rigid block to ground excitations. It is also found that because of its higher frequency contents and short duration, the required ground shock amplitude to topple a rigid block is substantially larger than that of an earthquake ground motion.

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REFERENCES

