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# RCA models with correlated errors

S.S. Appadoo[a], A. Thavaneswaran[a,*], Jagbir Singh[b]

[a] *Department of Statistics, The University of Manitoba, Winnipeg, Manitoba, Canada R3T–2N2*
[b] *Department of Statistics, Fox School of Business and Management, Temple University, Philadelphia, PA, USA*

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**Abstract**

Financial time series data cannot be adequately modelled by a normal distribution and empirical evidence on the non-normality assumption is very well documented in the financial literature; see [R.F. Engle, Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation, Econometrica 50 (1982) 987–1008] and [T. Bollerslev, Generalized autoregressive conditional heteroscedasticity, J. Econometrics 31 (1986) 307–327] for details. The kurtosis of various classes of RCA models has been the subject of a study by Appadoo et al. [S.S. Appadoo, M. Gharahmani, A. Thavaneswaran, Moment properties of some volatility models, Math. Sci. 30 (2005) 50–63] and Thavaneswaran et al. [A. Thavaneswaran, S.S. Appadoo, M. Samanta, Random coefficient GARCH models, Math. Comput. Modelling 41 (2005) 723–733]. In this work we derive the kurtosis of the correlated RCA model as well as the normal GARCH model under the assumption that the errors are correlated.


*Keywords:* Moments properties; Kurtosis; Correlated RCA models; Correlated GARCH models

## 1. Introduction

Rapid developments of time series models and methods addressing nonlinearity in computational finance have recently been reported in the literature. These theories either extend and complement existing time series methodology by introducing more general structures or provide an alternative framework. Volatility modelling has attracted attention in recent years and the quest for heavy-tailed distributions is still an ongoing process. In this regard, we derive the kurtosis, which characterizes the heavy-tail properties, of the correlated RCA model of Nicholls and Quinn [5]. Many financial series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and volatility (varying in time). Kurtosis, measured by the moment ratio $K = \frac{\mu_4}{\mu_2^2}$, gives an estimate of the peakedness of unimodal curves. Estimation for RCA models had been studied in [6] and new predictors have been derived in [7].

## 2. Moment properties

Random coefficient autoregressive time series were introduced by Nicholls and Quinn [5] and some of their properties have been studied recently by Appadoo et al. [4]. RCA models exhibiting long memory properties have

* Corresponding author. Tel.: +1 204 474 9826; fax: +1 204 474 7621.
*E-mail address:* thavane@ccu.umanitoba.ca (A. Thavaneswaran).

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been considered in [8]. A sequence of random variables $\{y_t\}$ is called an RCA(1) time series if it satisfies the equations

$$y_t = (\phi + b_t)y_{t-1} + e_t \qquad t \in Z,$$

where $Z$ denotes the set of integers and

(i) $\begin{pmatrix} b_t \\ e_t \end{pmatrix} \sim \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix} \right)$,
(ii) $\phi^2 + \sigma_b^2 < 1$.

The sequences $\{b_t\}$ and $\{e_t\}$ respectively, are the errors in the model. According to Nicholls and Quinn [5], (ii) is a necessary and sufficient condition for the second-order stationarity of $\{y_t\}$. So, together with (i), it also ensures strict stationarity. Moreover, Feigin and Tweedie [9] showed that $Ey_t^{2k} < \infty$ for some $k \geq 1$ if the moments of the noise sequences satisfy $Ee_t^{2k} < \infty$ and $E(\phi + b_t)^{2k} < 1$, for the same $k$.

**Theorem 2.1.** *Let $\{y_t\}$ be an **RCA**(1) time series satisfying conditions* (i) *and* (ii), *and let $\gamma_y$ be its covariance function. Then,*

(a) $Ey_t = 0$, $Ey_t^2 = \frac{\sigma_e^2}{1-\phi^2-\sigma_b^2}$, *the kth lag autocovariance for $y_t$ is given by $\gamma_y(k) = \frac{\phi^k \sigma_e^2}{1-\phi^2-\sigma_b^2}$ and the autocorrelation for $y_t$ is $\rho_k = \phi^k$ for all $k \in Z$. That is, the usual AR(1) process has same autocorrelation as the RCA(1).*
(b) *If $\{b_t\}$ and $\{e_t\}$ are normally distributed random variables and if $e_t$ and $b_t$ are correlated with correlation coefficient $\rho$, then the kurtosis $K^{(y)}$ of the RCA process $\{y_t\}$ is given by*

$$K^{(y)} = \frac{6(\sigma_b^2 + \phi_1^2)[1 - \phi_1^3 - 3\phi_1\sigma_b^2] + 72\phi_1^3\rho^2\sigma_b^2 + 3[1 - (\phi_1^2 + \sigma_b^2)][1 - \phi_1^3 - 3\phi_1\sigma_b^2]}{[1 - \phi_1^3 - 3\phi_1\sigma_b^2][1 - 6\phi_1^2\sigma_b^2 - \phi_1^4 - 3\sigma_b^4]} \times [1 - (\phi_1^2 + \sigma_b^2)] \tag{2.1}$$

*and for an AR(1) process $K^{(y)}$ reduces to 3 and when $\rho = 0$, the kurtosis reduces to the one observed by Appadoo et al.* [4].

**Proof.** The proof of part (a) of Theorem 2.1 is given in Appadoo et al. [4]. Proof of part (b) is as follows:

$$\begin{aligned} y_t &= (\phi_1 + b_t)y_{t-1} + e_t = y_{t-1}\phi_1 + y_{t-1}b_t + e_t \\ y_t^2 &= y_{t-1}^2\phi_1^2 + 2y_{t-1}^2\phi_1 b_t + 2y_{t-1}\phi_1 e_t + y_{t-1}^2 b_t^2 + 2y_{t-1}b_t e_t + e_t^2. \end{aligned}$$

Under the condition of $\phi_1 + \sigma_b^2 < 1$, the second moment of $y_t$ and, hence, the variance is given by $E[y_t^2] = \frac{\sigma_\epsilon^2}{1-(\phi_1^2+\sigma_b^2)}$. The third moment of the process is given by

$$\begin{aligned} y_t^3 &= \phi_1^3 y_{t-1}^3 + 3\phi_1^2 y_{t-1}^3 b_t + 3\phi_1^2 y_{t-1}^2 e_t + 3\phi_1 y_{t-1}^3 b_t^2 + 6\phi_1 y_{t-1}^2 b_t e_t + 3\phi_1 y_{t-1} e_t^2 \\ &\quad + b_t^3 y_{t-1}^3 + 3b_t^2 y_{t-1}^2 e_t + 3b_t y_{t-1} e_t^2 + e_t^3 \\ E(y_t^3) &= E(\phi_1^3 y_{t-1}^3 + 3\phi_1 y_{t-1}^3 b_t^2 + 6\phi_1 y_{t-1}^2 b_t e_t) = \frac{6\phi_1\rho\sigma_e^3\sigma_b}{(1 - \sigma_b^2 - \phi_1^2)[1 - \phi_1^3 - 3\phi_1\sigma_b^2]}. \end{aligned}$$

The fourth moment of the process is given by

$$\begin{aligned} E(y_t^4) &= 6\phi_1^2 E(y_{t-1}^4 b_t^2) + 6E(b_t^2 y_{t-1}^2 e_t^2) + \phi_1^4 E(y_{t-1}^4) + E(b_t^4 y_{t-1}^4) + E(e_t^4) \\ &\quad + 12\phi_1^2 E(y_{t-1}^3 b_t e_t) + 6\phi_1^2 E(y_{t-1}^2 e_t^2) \\ &= 6\phi_1^2\sigma_b^2 E(y_{t-1}^4) + 6\sigma_e^2\sigma_b^2 E(y_{t-1}^2) + \phi_1^4 E(y_{t-1}^4) + 3\sigma_b^4 E(y_{t-1}^4) \\ &\quad + 3\sigma_e^4 + 12\phi_1^2\sigma_e\sigma_b\rho E(y_{t-1}^3) + 6\phi_1^2\sigma_e^2 E(y_{t-1}^2) \end{aligned}$$

$$[1 - 6\phi_1^2\sigma_b^2 - \phi_1^4 - 3\sigma_b^4]E(y_t^4) = 6\sigma_e^2\sigma_b^2 E(y_{t-1}^2) + 3\sigma_e^4 + 12\phi_1^2\sigma_e\sigma_b\rho E(y_{t-1}^3) + 6\phi_1^2\sigma_e^2 E(y_{t-1}^2)$$

$$[1 - 6\phi_1^2\sigma_b^2 - \phi_1^4 - 3\sigma_b^4]E(y_t^4) = 6\sigma_e^2 E(y_{t-1}^2)[\sigma_b^2 + \phi_1^2] + 3\sigma_e^4 + 12\phi_1^2\sigma_e\sigma_b\rho E(y_{t-1}^3)$$

$$[1 - 6\phi_1^2\sigma_b^2 - \phi_1^4 - 3\sigma_b^4]E(y_{t-1}^4) = \frac{6\sigma_e^4(\sigma_b^2 + \phi_1^2)}{1 - (\phi_1^2 + \sigma_b^2)} + 3\sigma_e^4 + \frac{72\phi_1^3\rho^2\sigma_b^2\sigma_e^4}{1 - (\phi_1^2 + \sigma_b^2)[1 - \phi_1^3 - 3\phi_1\sigma_b^2]}.$$

By substitution,

$$E(y_{t-1}^4) = \frac{6\sigma_e^4(\sigma_b^2+\phi_1^2)[1-\phi_1^3-3\phi_1\sigma_b^2] + 72\phi_1^3\rho^2\sigma_b^2\sigma_e^4 + 3\sigma_e^4[1-(\phi_1^2+\sigma_b^2)][1-\phi_1^3-3\phi_1\sigma_b^2]}{[1-(\phi_1^2+\sigma_b^2)][1-\phi_1^3-3\phi_1\sigma_b^2][1-6\phi_1^2\sigma_b^2-\phi_1^4-3\sigma_b^4]}$$

when $\rho = 0$,

$$E(y_{t-1}^4) = \frac{3\sigma_e^4[\sigma_b^2+\phi_1^2+1]}{[1-(\phi_1^2+\sigma_b^2)][1-6\phi_1^2\sigma_b^2-\phi_1^4-3\sigma_b^4]}.$$

$$\begin{aligned} K^{(y)} &= \frac{6\sigma_e^4(\sigma_b^2+\phi_1^2)[1-\phi_1^3-3\phi_1\sigma_b^2] + 72\phi_1^3\rho^2\sigma_b^2\sigma_e^4 + 3\sigma_e^4[1-(\phi_1^2+\sigma_b^2)][1-\phi_1^3-3\phi_1\sigma_b^2]}{[1-(\phi_1^2+\sigma_b^2)][1-\phi_1^3-3\phi_1\sigma_b^2][1-6\phi_1^2\sigma_b^2-\phi_1^4-3\sigma_b^4]} \\ &\quad \times \left[\frac{[1-(\phi_1^2+\sigma_b^2)]^2}{\sigma_e^4}\right] \\ &= \frac{6(\sigma_b^2+\phi_1^2)[1-\phi_1^3-3\phi_1\sigma_b^2] + 72\phi_1^3\rho^2\sigma_b^2 + 3[1-(\phi_1^2+\sigma_b^2)][1-\phi_1^3-3\phi_1\sigma_b^2]}{[1-\phi_1^3-3\phi_1\sigma_b^2][1-6\phi_1^2\sigma_b^2-\phi_1^4-3\sigma_b^4]} \\ &\quad \times [1-(\phi_1^2+\sigma_b^2)]. \end{aligned}$$

When $\rho = 0$, our results converge to the one reported in the literature:

$$K^{(y)} = \frac{3[1+(\sigma_b^2+\phi_1^2)][1-(\phi_1^2+\sigma_b^2)]}{[1-6\phi_1^2\sigma_b^2-\phi_1^4-3\sigma_b^4]} = \frac{3[1-(\phi_1^2+\sigma_b^2)^2]}{[1-(\phi_1^4+6\phi_1^2\sigma_b^2+3\sigma_b^4)]}.$$

The kurtosis of the RCA model is a special case of Theorem 2.1. The correlated RCA model has a higher kurtosis than its uncorrelated counterpart and easy computation leads to the following inequality for the kurtosis for the different classes of RCA models: $K_{\text{AR}}^{(y)} \le K_{\text{RCA}}^{(y)} \le K_{\text{CRCA}}^{(y)}$.

### *2.1. Random coefficient ARCH(1) model*

In analogy with the RCA models, we introduce a class of RCA versions of GARCH models. Consider the class of ARCH(1) model for the time series $y_t$, where

$$y_t = \sqrt{h_t} Z_t \qquad h_t = \omega + (\alpha_1 + b_{t-1}) y_{t-1}^2 \tag{2.2}$$

and $Z_t$ is a sequence of independent, identically distributed random variables with zero mean and unit variance. Let $u_t = y_t^2 - h_t$ be the martingale difference and let $\sigma_u^2$ be the variance of $u_t$. On writing the model as

$$y_t^2 = \omega + (\alpha_1 + b_{t-1}) y_{t-1}^2 + u_t, \tag{2.3}$$

the minimum mean square error forecast is optimal for $y_t^2$; however, for the random coefficient ARCH(1) model introduced in [1] given by (2.2), the minimum mean square error forecast of $y_t$ is not optimal (see [7] for more details).

**Lemma 2.1.** *For the GARCH model considered in* [2] $y_t = \sqrt{h_t} Z_t$, $h_t = \omega_0 + (\alpha_1 + b_{t-1}) y_{t-1}^2$, *where* $Z_t \sim N(0, \sigma_Z^2)$ *and* $b_t \sim N(0, \sigma_a^2)$, *the kurtosis is given by* $K^{(y)} = \frac{3[1-\alpha_1^2\sigma_Z^4]}{[1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)]}$.

**Proof.** $y_t^2 = h_t Z_t^2$ and $E[y_t^2] = E[h_t]\sigma_Z^2$.
We will make use of the above relationship to find the expected value of $E(h_t)$:

$$E[h_t] = \omega_0 + \alpha_1 E[y_{t-1}^2] = \omega_0 + \alpha_1 E[h_{t-1}]\sigma_Z^2 = \frac{\omega_0}{[1-\alpha_1\sigma_Z^2]}$$

$$\begin{aligned} E[h_t^2] &= \omega_0^2 + 2\omega_0\alpha_1\sigma_Z^2 E[h_{t-1}] + 3\alpha_1^2\sigma_Z^4 E[h_{t-1}^2] + 3\sigma_Z^4 E[h_{t-1}^2]\sigma_b^2 \\ &= \frac{\omega_0^2 + 2\omega_0\alpha_1\sigma_Z^2 E[h_{t-1}]}{1 - 3\alpha_1^2\sigma_Z^2 - 3\sigma_Z^4\sigma_b^2} = \frac{(1-\alpha_1\sigma_Z^2)\omega_0^2 + 2\omega_0^2\alpha_1\sigma_Z^2}{(1 - 3\sigma_Z^4(\alpha_1^2 + \sigma_b^2))} \\ &= \frac{\omega_0^2(1+\alpha_1\sigma_Z^2)}{1 - 3\sigma_Z^4(\alpha_1^2 + \sigma_b^2)} \end{aligned}$$

$E[y_t^2] = E[h_t]\sigma_Z^2 = [\frac{\sigma_Z^2\omega_0}{1-\alpha_1\sigma_Z^2}]$,

$$E[y_t^4] = E[Z_t^4 h_t^2] = E[Z_t^4]E[h_t^2] = 3\sigma_Z^4 E[h_t^2] = 3\sigma_Z^4\left[\frac{\omega_0^2(1+\alpha_1\sigma_Z^2)}{1 - 3\sigma_Z^2(\alpha_1^2 + \sigma_b^2)}\right].$$

Hence, $K^{(y)} = 3[\frac{(1-\alpha_1^2\sigma_Z^4)}{1-3\sigma_Z^2(\alpha_1^2+\sigma_b^2)}]$.

### *2.2. RCA–GARCH models*

Consider the following random coefficient model:

$$y_t = \sqrt{h_t}Z_t, h_t = \omega_0 + (\alpha_1 + b_{t-1})y_{t-1}^2 + \beta_1 h_{t-1} \; Z_t \sim N(0, \sigma_Z^2) \text{ and } b_t \sim N(0, \sigma_b^2).$$

**Lemma 2.2.** *Under suitable stationary conditions, the kurtosis of $y_t$ is given by*

$$K^{(y)} = \frac{3[1 - (\alpha_1^2\sigma_Z^2 + \beta_1)^2]}{[1 - 2\alpha_1\beta_1\sigma_Z^2 - 3\alpha_1^2\sigma_Z^4 - 3\sigma_b^2\sigma_Z^4 - \beta_1^2]}.$$

**Proof.**

$$y_t = \sqrt{h_t}Z_t \tag{2.4}$$

$$h_t = \omega_0 + (\alpha_1 + b_{t-1})y_{t-1}^2 + \beta_1 h_{t-1}. \tag{2.5}$$

Thus, we have

$$y_t^2 = h_t Z_t^2 \tag{2.6}$$

$$E[y_t^2] = E[h_t Z_t^2] = E[h_t]\sigma_z^2. \tag{2.7}$$

The mean of $h_t$ is given by

$$\begin{aligned} E[h_t] &= E[\omega_0 + (\alpha_1 + b_{t-1})y_{t-1}^2 + \beta_1 h_{t-1}] \\ &= E(\omega_0) + E(\alpha_1 y_{t-1}^2) + E(b_{t-1}y_{t-1}^2) + E(\beta_1 h_{t-1}) \\ &= \omega_0 + \alpha_1\sigma_Z^2 E(h_{t-1}) + \beta_1 E(h_{t-1}) \end{aligned}$$

$$E[h_t] - \alpha_1\sigma_Z^2 E(h_{t-1}) - \beta_1 E(h_{t-1}) = \omega_0$$

$$E[h_t](1 - \alpha_1\sigma_Z^2 - \beta_1) = \omega_0$$

$$E[h_t] = \left[\frac{\omega_0}{1 - \alpha_1\sigma_Z^2 - \beta_1}\right]. \tag{2.8}$$

Computation of $E(h_t^2)$:

$$h_t^2 = (\omega_0 + (\alpha_1 + b_{t-1})y_{t-1}^2 + \beta_1 h_{t-1})^2$$

$$\begin{aligned} h_t^2 &= (\omega_0 + (\alpha_1 + b_{t-1})y_{t-1}^2 + \beta_1 h_{t-1})^2 \\ &= \omega_0^2 + 2\omega_0 y_{t-1}^2\alpha_1 + 2\omega_0 y_{t-1}^2 b_{t-1} + 2\omega_0\beta_1 h_{t-1} + y_{t-1}^4\alpha_1^2 \\ &\quad + 2y_{t-1}^4\alpha_1 b_{t-1} + 2y_{t-1}^2\alpha_1\beta_1 h_{t-1} + y_{t-1}^4 b_{t-1}^2 + 2y_{t-1}^2 b_{t-1}\beta_1 h_{t-1} + \beta_1^2 h_{t-1}^2 \end{aligned}$$

$$\begin{aligned} E[h_t^2] &= \omega_0^2 + 2\omega_0\alpha_1\sigma_Z^2 E[h_{t-1}] + 2\omega_0\beta_1 E[h_{t-1}] + 3\sigma_Z^4\alpha_1^2 E[h_{t-1}^2] \\ &\quad + 2\sigma_Z^2\alpha_1\beta_1 E[h_{t-1}^2] + 3\sigma_Z^4\sigma_b^2 E[h_{t-1}^2] + \beta_1^2 E[h_{t-1}^2]. \end{aligned} \tag{2.9}$$

Simple computation leads us to

$$E[h_t^2]\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]=\omega_0^2+2\omega_0(\alpha_1\sigma_Z^2+\beta_1)E[h_{t-1}]$$

$$E[h_t^2]\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]=\omega_0^2+\left[\frac{2\omega_0^2(\alpha_1\sigma_Z^2+\beta_1)}{1-\alpha_1\sigma_Z^2-\beta_1}\right].$$

Thus,

$$\begin{aligned}E[h_t^2]&=\frac{\omega_0^2+\left[\frac{2\omega_0^2(\alpha_1\sigma_Z^2+\beta_1)}{1-\alpha_1\sigma_Z^2-\beta_1}\right]}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]}\\&=\left[\frac{\omega_0^2\left(1-\alpha_1\sigma_Z^2-\beta_1\right)+2\omega_0^2(\alpha_1\sigma_Z^2+\beta_1)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]\left[1-\alpha_1\sigma_Z^2-\beta_1\right]}\right]\\&=\left[\frac{\omega_0^2(1+\alpha_1\sigma_Z^2+\beta_1)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]\left[1-\alpha_1\sigma_Z^2-\beta_1\right]}\right].\end{aligned}\tag{2.10}$$

The kurtosis of the process is given by

$$\begin{aligned}K^{(y)}&=\frac{E[y_t^4]}{E\left[y_t^2\right]^2}=\frac{E[h_t^2]E[Z_t^4]}{E\left[h_tZ_t^2\right]^2}=\frac{3\sigma_Z^4E[h_t^2]}{\left(E[h_t]\sigma_z^2\right)^2}=\frac{3\sigma_Z^4E[h_t^2]}{\sigma_z^4\left(E[h_t]\right)^2}=\frac{3E[h_t^2]}{(E[h_t])^2}\\&=\frac{3\left[\frac{\omega_0^2(1+\alpha_1\sigma_Z^2+\beta_1)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]\left[1-\alpha_1\sigma_Z^2-\beta_1\right]}\right]}{\left[\frac{\omega_0}{1-\alpha_1\sigma_Z^2-\beta_1}\right]^2}\\&=\frac{\left[\frac{3\omega_0^2(1+\alpha_1\sigma_Z^2+\beta_1)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]\left[1-\alpha_1\sigma_Z^2-\beta_1\right]}\right]}{\left[\frac{\omega_0^2}{\left(1-\alpha_1\sigma_Z^2-\beta_1\right)^2}\right]}\\&=\left[\frac{3\omega_0^2(1+\alpha_1\sigma_Z^2+\beta_1)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]\left[1-\alpha_1\sigma_Z^2-\beta_1\right]}\right]\left[\frac{\left(1-\alpha_1\sigma_Z^2-\beta_1\right)^2}{\omega_0^2}\right]\\&=\left[\frac{3(1+\alpha_1\sigma_Z^2+\beta_1)\left(1-\alpha_1\sigma_Z^2-\beta_1\right)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]}\right]=\left[\frac{3(1-(\alpha_1\sigma_Z^2+\beta_1)^2)}{\left[1-2\sigma_Z^2\alpha_1\beta_1-3\sigma_Z^4(\alpha_1^2+\sigma_b^2)-\beta_1^2\right]}\right].\end{aligned}\tag{2.11}$$

For a GARCH(1,1) process, we have

$$K^{(y)}=\left[\frac{3(1-(\alpha_1+\beta_1)^2)}{1-2\alpha_1\beta_1-3\alpha_1^2-\beta_1^2}\right].\tag{2.12}$$

## 3. Conclusions

In this work, the kurtosis of the correlated RCA model and that of the random coefficient ARCH(1), GARCH(1,1), are derived. The correlated random coefficient GARCH model may be viewed as a special case of a state space model for $y_t^2$ and the parameter process $\theta_t$, and inferences for these processes may be studied as in [6]. The kurtosis for these processes, important in finance, has applications including in comparative simulations. The results for random coefficient GARCH given in [3] are also extended to RCGARCH with correlated errors.

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