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# Multi-criterion multi-attribute decision-making for an EOQ model in a hesitant fuzzy environment

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ABSTRACT

This article describes an inventory model with several attributes. The primary purpose of an economic order quantity (EOQ) model is to select the best alternative in the face of uncertainty and other considerations. Unlike an intuitionistic fuzzy set (IFS), a hesitant fuzzy set (HFS) has an emergent implication in current decision-making problems. A membership function class is assumed, and a hesitant fuzzy decision matrix with elements that are membership grades is constructed. Using these values, the scores are derived with the help of hesitant fuzzy weighted geometric (HFWG), hesitant fuzzy geometric (HFG), hesitant fuzzy Einstein weighted geometric (HFEWG) and hesitant fuzzy Einstein ordered weighted geometric (HFEOWG) operators. Finally, a decision is made using the scores of each alternative.


## Introduction

A hesitant fuzzy set (HFS) is the generalization of all types of fuzzy sets. Due to the versatile nature of an HFS, other types of set that are similar to HFSs, namely, dual hesitant fuzzy sets (DHFSs) [53], interval-valued HFSs [14], generalized HFSs [29], and HFS linguistic term sets [32] have been developed by researchers in recent years. However, in operational terms, numerous aggregation operators have been presented in the literature, including the hesitant fuzzy weighted averaging (HFWA), hesitant fuzzy averaging (HFA), hesitant fuzzy Bonferroni weighted geometric (HFBWG), hesitant fuzzy Choquet integral (HFCI), HFEWG, and HFEOWG operators [46]. Farhadinia [25] has studied a novel method for ranking HFS decision-making problems, and Broumi [11] has developed some new operational laws for interval-valued intuitionistic hesitant fuzzy sets. Based on the nature of each problem, researchers in different disciplines have been investigating these operators and their scores and accuracy in minimizing the amount of conflict in the decision-making process.

The intuitionistic fuzzy set (IFS) was introduced by Atannasov [5,6]. Since then, a large number of research articles on inventory management problems that consider this topic have been published. In such problems, the membership and non-membership functions are used to determine a score. De and Sana [19] have developed a backlogging model using IFSs with the score of the objective function. De et al. [18] have studied an EOQ model with backorder that considers the interpolation bypass technique as an alternative to the Pareto optimality technique for intuitionistic fuzzy sets. On decision-making problems, a trapezoidal-valued IFS has been studied by Beg and Rashid [8]; interval-valued intuitionistic fuzzy sets have been developed by Wei et al. Wei et al. [44], and geometric aggregation rules have been analysed by Wei [45]. Authors such as Takeuti and Tinani [36], Atanassov and Gargov [4], Dabois et al. [16], Dymova and Sevastjanov [24], Angelov [3] have studied current issues in decision-making problems using IFSs in inventory management. In an intuitionistic fuzzy environment, De [20] has investigated an EOQ model in which a natural idle time (the duration of the general closing time each day) is considered.

In a fuzzy inventory, after the studies of by Zadeh [51] and Bellman and Zadeh [9]; numerous articles about ranking L-R fuzzy numbers were analysed by Wang et al. [43], the centroid method was used by Voskoglou [41], and Allahviranloo and Saneifard [2] have already been able to rank fuzzy numbers using the centre of gravity method. Ramli and Mohamad [30] performed a

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comparative analysis of the centroid method. A novel approach called the genetic-algorithm-based fuzzy weighted average had been carefully studied by Deep et al. [22]. Another approach, the $\alpha$-cut method, has been discussed by De and Goswami [17]. De and Sana [21] have used $\alpha$-cuts and Yager's [50] ranking index to solve the fuzzy order quantity inventory model problem with a fuzzy quantity shortage and fuzzy promotional index. Banerjee and Roy [7] have described a fuzzy probability model.

Numerous research articles on supply chains have been published recently. Cárdenas-Barrón and Sana [13] have studied a two-echelon supply chain model for sales teams' initiatives' dependent demand rate. A model to honour Ford Whitman Harris was developed by Cárdenas-Barrón et al. [12]. Recently, Sana and Cárdenas-Barrón [35] have studied a multi-item economic production quantity (EPQ) model for promotional-effort-sensitive demand. Recently, the vendor-managed EPQ model that includes deterioration and the defective-items-related vendor-buyer integrated model were analysed by Taleizadeh et al. [37] and Treviño-Garza et al. [38], respectively.

A hesitant fuzzy set can cover almost all of the fields in the physical sciences and social sciences. It can be applied to the study of human behaviour, emotions, natural instincts and crime. Using this process, we can remember the contributions of Wang and Jiang [42] to the noble knowledge of international justice. We can apply hesitant fuzzy set-theoretic decision analysis to the punctual research developed by the researchers such as Dudycha [23], Harsanyi and Selten [27], Jean [39], and Hoff and Stiglitz [28]. The uncertainties in the stock market have been studied by Rizvi and Arshad [31]. The aspect-oriented programming (AOP) method for efficient services has been introduced by Choi [15]. However, a methodological study in professional fields has been improved by Akhayan and Kleshcheva [1] and by Bernavskaya [10] alone.

In our present study, which refers to the model developed by De [20]; which has also been described by Sana [33,34], a profit function is considered. On the basis of different qualitative characteristics of a single item, attributes are first identified and then, re-arranged according to a normal standard. Then, assuming different criteria on the basis of different inputs to the model, a fuzzy membership function is developed. As for a fuzzy set, we have used the nearest interval number [26] to create a triangular fuzzy number. Then, considering a class of bell-shaped membership functions, we take the membership grades at the lower, centre and upper objective values of the objective functions. In this way, we create a hesitant fuzzy decision matrix. Finally, we use several aggregation operators to determine the scores and rank them.

This paper has been organized as follows: Section 1 presents an introduction to the literature, Section 2 describes Assumptions and notation considerations, Section 3 lists the Preliminaries of the model, Section 4 formulates a crisp model, Section 5 formulates a fuzzy mathematical model and determines its implications for an HFS, Section 6 illustrates the model with numerical examples, and a selection of the best alternatives and conclusions are presented in Section 7.

## Assumptions and notation

The following notation and assumptions are used to develop the model.

### *Notation*

$q$: Quantity ordered per cycle
$t_1$: Length of opening (day)
$t_2$: Length of closing/natural idle time (day)
$c_1$: Inventory holding cost per unit per day ($)
$b$: Set up cost per cycle ($)
$i_c$: Average natural idle time cost per unit idle time ($).
$p$: Profit ($) per item.
$t$: Cycle length ($t = n+1$) (days)
$T$: Time horizon (days)
z: Total average profit per cycle ($)

### *Assumptions*

1. Inventory is taken at opening time and is maintained during the natural idle time; it is take again at closing time on the last day of the cycle.
2. Replenishments are instantaneous.
3. The time horizon is infinite (days)
4. The sum of the open and closed periods is unity.
5. Shortages are not allowed.
6. The demand rate is $d = d_0 e^{\lambda(n+1)t_1}$, where the initial demand rate is $d_0$, $(n+1)$ is the cycle length, $t_1$ is the amount of time the business is open, and $\lambda$ is a shape parameter.
7. The holding cost is uniform throughout the cycle.
8. The average natural idle time (leisure/pause) cost is constant per unit idle time.
9. The security charge, telephone charge, transportation cost (if any), etc., beyond the working hours are included in the idle time costs.

## Preliminaries

*Concepts: a hesitant fuzzy set (HFS) is defined in terms of the functions that return a set of membership values for each element in the domain.*

**Definition 1**. [40]: *Let X be a reference set; an HFS on X is a function h that maps a subset of values in* [0,1];

$$h : X \to \phi([0,1]). \tag{1}$$

**Definition 2**. [40]: *Let* $M = \{\mu_1, \mu_2, \ldots \mu_n\}$ *be a set of n membership functions. Then, the HFS associated with M,* $h_M$ *is defined as*

$$h_M : X \to \phi([0,1]) \text{ and } h_M(x) = \bigcup_{\mu\in M} \{\mu(x)\} \text{ where } x \in X. \tag{2}$$

**Definition 3**. [46,47]: *Let h be an HFE; then, the score function s of h is defined as*

$$s(h) = \frac{1}{n(h)} \sum_{\gamma\in h} \gamma, \tag{3}$$

*where* $n(h)$ *is the number of elements in h. For two HFEs (hesitant fuzzy elements)* $h_1$ and $h_2$, *if* $s(h_1) > s(h_2)$ *then* $h_1 > h_2$, *and* $h_1 = h_2$ *if* $s(h_1) = s(h_2)$.

**Definition 4**. [46]: *Let h be an HFE; then, the accuracy function k of h is defined as*

$$k(h) = 1 - \sqrt{\frac{1}{n(h)} \sum_{\gamma\in h} (\gamma - s(h))^2}, \tag{4}$$

*where n(h) is the number of elements in h and s(h) is the score function. For two HFEs* $h_1$ and $h_2$, $k(h_1) < k(h_2) \rightarrow h_1 < h_2$ *and* $k(h_1) = k(h_2) \rightarrow h_1 = h_2$.

**Definition 5.** [46]: *Let* $h_i$ $(i = 1, 2, \ldots n)$ *be a collection of HFEs; then, the hesitant fuzzy weighted geometric (HFWG) operator is a mapping* $H^n \rightarrow H$ *such that*

$$HFWG(h_1, h_2, \ldots\ldots h_n) = \bigcup_{\gamma_1 \in h_1,\ \gamma_2 \in h_2,\ \ldots,\gamma_n \in h_n} \left\{ \prod_{j=1}^{n} \gamma_i^{w_i} \right\}, \tag{5}$$

*where* $w = (w_1, w_2, \ldots w_n)^T$ *is the weighting vector with* $w_i \in [0, 1]$ *and* $\sum_{i=1}^{n} w_i = 1$; *if* $w = \left(\frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$, *then the HFWG operator reduces to the hesitant fuzzy geometric (HFG) operator*

$$HFG(h_1, h_2, \ldots h_n) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \ldots, \gamma_n \in h_n} \left\{ \prod_{j=1}^{n} \gamma_i^{\frac{1}{n}} \right\}. \tag{6}$$

**Definition 6.** [52]: *Einstein operations on a hesitant fuzzy set.*
*Let* $\alpha > 0$, $h_i$ $(i = 1, 2, 3)$ *be three HFEs; then,*

i) $h_1 \oplus_E h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \frac{\gamma_1 + \gamma_2}{1 + \gamma_1 \gamma_2} \right\}$

ii) $h_1 \oplus_E h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \frac{\gamma_1 \gamma_2}{1 + (1 - \gamma_1)(1 - \gamma_2)} \right\}$

iii) $h_3^{\hat{}\alpha} = \bigcup_{\gamma \in h_3} \left\{ \frac{2\gamma^{\alpha}}{(2 - \gamma)^{\alpha} + \gamma^{\alpha}} \right\}$

*Decision making with a hesitant fuzzy set*

Let $y = \{y_1, y_2, \ldots y_m\}$ be a discrete set of alternatives. Let $A = \{A_1, A_2, \ldots A_n\}$ be a collection of attributes and let $w = (w_1, w_2, \ldots w_n)$ be the weight vector of $A_j (j = 1, 2, \ldots n)$, with $w_j > 0, j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$. If the decision makers provide several values for the alternatives $y_j (j = 1, 2, \ldots, m)$ for the attribute $A_j (j = 1, 2, \ldots n)$ with anonymity, these values can be considered HFEs, $h_{ij}$. If two decision makers provide the same value, the value emerges only once in the $h_{ij}$. Suppose the decision matrix $H = (h_{ij})_{m \times n}$ is a hesitant fuzzy decision matrix where the $h_{ij}$ $(i = 1, 2, \ldots m, j = 1, 2, \ldots n)$ are in the form of HFEs. Then, to determine the best alternatives, we use the following operators:

$$h_i = HFEWG(h_{i1}, h_{i2}, \ldots h_{in}) = \bigcup_{\gamma_{i1} \in h_{i1},\ \gamma_{i2} \in h_{i2}, \ldots, \gamma_{in} \in h_{in}} \left\{ \frac{2\prod_{j=1}^{n} (\gamma_{ij})^{w_j}}{\prod_{j=1}^{n} (2 - \gamma_{ij})^{w_j} + \prod_{j=1}^{n} (\gamma_{ij})^{w_j}} \right\} \tag{7}$$

Or $h_i = HFEOWG(h_{i1}, h_{i2}, \ldots h_{in})$

$$= \bigcup_{\gamma_{i\sigma(j)} \in h_{i\sigma(j)},\ j = 12, \ldots n} \left\{ \frac{2\prod_{j=1}^{n} \gamma i\sigma(j)^{w_j}}{\prod_{j=1}^{n} \left(2 - \gamma_{i\sigma(j)}\right)^{w_j} + \prod_{j=1}^{n} \gamma_{i\sigma(j)}{}^{w_j}} \right\} \tag{8}$$

Now, we select $w_j > 0, j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$ using the normal distribution-based method [49] and, applying definitions 3, 4 and 5, we rank them and determine the best alternatives.

*Converting a fuzzy number to its nearest interval number*

Let $\widetilde{A} = (a_1, a_2, a_3)$ be an arbitrary triangular fuzzy number with linear membership function

$$\mu_A(x) = \left\{ \begin{array}{ll} \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 < x < a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 < x \le a_3 \\ 0 & \text{for } elsewhere \end{array} \right\}. \tag{9}$$

The $\alpha$-cut of the membership function of $A$ can be written as $[A_L(\alpha), A_R(\alpha)]$.

Now, per Grzegorzewski [26]; the nearest interval is

$$[C_L, C_R] \text{ where } C_L = \int_0^1 A_L(\alpha) d\alpha = \frac{a_1 + a_2}{2} \quad \text{and } C_R = \int_0^1 A_R(\alpha) d\alpha = \frac{a_2 + a_3}{2}. \tag{10}$$

*Basic interval arithmetic*

Let $A = [a_1, a_2], B = [b_1, b_2]$ and the usual operations $\{+, -, \times, \div\}$, namely addition, subtraction, multiplication and division be as follows:

$A + B = [a_1 + b_1, a_2 + b_2]$, $A - B = [a_1 - b_2, a_2 - b_1]$

$A \cdot B = [\min(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2), \max(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2)]$ $A/B = [\min(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)]$ $\delta A = [\delta a_1, \delta a_2]$ if $\delta \ge 0$ and $\delta A = [\delta a_2, \delta a_1]$ if $\delta < 0$.

*Optimizing the interval environment*

We have the problem of finding interval-valued coefficients for the variables in the non-linear objective function.

To solve it, the first step is to minimize $Z(X) = \sum_{i=1}^{n} [a_{Li}, a_{Ri}] \prod_{j=1}^{k} x_j^{r_j}$ subject to $x_j > 0$, $j = 1, 2, \ldots n$, $r_j \in Q$ (the set of rational numbers) and $x \in F \in R^+$, where $F$ is a feasible region of $x$. Then, we can split $Z(X)$ into the form

$Z(X) = [Z_L(X), Z_R(X)]$,

where

$$Z_L(X) = \sum_{i=1}^{n} a_{Li} \prod_{j=1}^{k} x_j^{r_j} \tag{11}$$

$$Z_R(X) = \sum_{i=1}^{n} a_{Ri} \prod_{j=1}^{k} x_j^{r_j}, \tag{12}$$

and the centre of the objective function is

$$Z_C(X) = \frac{1}{2}[Z_L(X) + Z_R(X)]. \tag{13}$$

## The crisp mathematical model

Inventory (Fig. 1) is taken at opening time, $t_1$ and meets the demand of $d$ items per unit time. Then, it remains steady up until

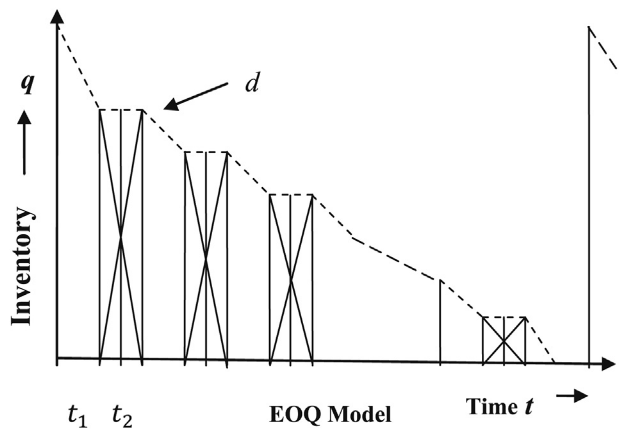


**Fig. 1.** A logistic diagram of the inventory system.

closing time, $t_2$. Then, it starts at the next day's opening time; this process continues for up to $(n + 1)$ days.

Therefore, the inventory holding cost ($HC$) is

$$\begin{aligned} HC &= c_1\left[\frac{1}{2}(2q-dt_1)t_1+\frac{1}{2}(2q-3dt_1)t_1+\frac{1}{2}(2q-5dt_1)t_1 \right.\\ &\quad \left. +\ldots n\ times+\frac{1}{2}dt_1^2\right]+c_1[(q-dt_1)t_2+(q-2dt_1)t_2 \\ &\quad +(q-3dt_1)t_2+\ldots n\ times] \\ &= c_1\left[qt_1(1+1+\ldots .n\ times)-\frac{1}{2}dt_1^2(1+3+5+\ldots+2n \right.\\ &\quad \left. -1)+\frac{1}{2}dt_1^2\right]+c_1[q(1+1+\ldots n\ times)t_2-dt_1t_2(1+2 \\ &\quad +3+\ldots+n)] \\ &= c_1\left[n\,qt_1-\frac{1}{2}dt_1^2\left(n^2-1\right)+n\,q\,t_2-\frac{1}{2}n\,(n+1)dt_1t_2\right] \\ &= \frac{(n+1)c_1\,d\,t_1}{2}[n(t_1+t_2)+t_1]. \end{aligned} \tag{14}$$

where

$$q=(n+1)dt_1 \tag{15}$$

and

$$(t_1+t_2)=1. \tag{16}$$

The cost of idle time is

$$PC=i_c(n+1)t_2. \tag{17}$$

The length of a cycle is

$$t=(n+1)(t_1+t_2)\ =(n+1). \tag{18}$$

From equations (14–18), the total average profit per cycle is given by

$$\begin{aligned} z &= [Revenue-(\ Holding\ cost+Idle\ time\ cost \\ &\quad +Setup\ cost)]\times Numberof\ cycles \\ &= [pq-(\ HC+PC+b)]\frac{T}{t}=\left[p(n+1)dt_1\right. \\ &\quad \left.-\left\{\frac{(n+1)c_1\,dt_1}{2}\{n+t_1\}+(n+1)i_ct_2+b\right\}\right]\frac{T}{n+1} \\ &= \left[pdt_1-\left\{\frac{n\,c_1\,dt_1}{2}+\frac{c_1\,dt_1^2}{2}+i_ct_2+\frac{b}{n+1}\right\}\right]T \end{aligned} \tag{19}$$

Now, our objective is to maximize Z such that

$$Maximize\ z=\left[pdt_1-\left\{\frac{n\,c_1\,dt_1}{2}+\frac{c_1\,dt_1^2}{2}+i_ct_2+\frac{b}{n+1}\right\}\right]T \tag{20}$$

subject to the following conditions: $q=(n+1)d\,t_1,\ (t_1+t_2)=1.$

**Special Case**: If we assume $t_2\rightarrow 0$, *then* $i_c=0$; therefore, equation (19) reduces to

$$z=pqT-\left[\frac{h\,q}{2}+\frac{b\,d}{q}\right]T. \tag{21}$$

This is the classical EOQ model.

## The fuzzy mathematical model

We consider the cost vector and the demand rate fuzzy numbers. Then, the objective function (20), after fuzzification, is as follows:

$$\begin{aligned} Maximize\ \widetilde{z} &= \left[\widetilde{p}\widetilde{d_0}e^{\widetilde{\lambda}\ (n+1)t_1}\ t_1-\left\{\frac{n\,\widetilde{c_1}\,\widetilde{d_0}e^{\widetilde{\lambda}\ (n+1)t_1}\ \ t_1}{2}\right.\right. \\ &\quad \left.\left.+\frac{\widetilde{c_1}\ \widetilde{d_0}e^{\widetilde{\lambda}\ (n+1)t_1}\ t_1^2}{2}+\widetilde{i_c}t_2+\frac{\widetilde{b}}{n+1}\right\}\right]\widetilde{T} \end{aligned} \tag{22}$$

Using equations (9–10) and basic interval arithmetic, we have the following:

$$\begin{aligned} Z_L(n,\ t_1) &= \left[T_Lp_Lt_1d_{0L}e^{\lambda_L\ (n+1)t_1}-T_R\left\{\frac{n\,c_{1R}\,d_{0R}e^{\lambda_R\ (n+1)t_1}t_1}{2}\right.\right. \\ &\quad \left.\left.+\frac{c_{1R}\,d_{0R}e^{\lambda_R\ (n+1)t_1}t_1^2}{2}+i_{cR}t_2+\frac{b_R}{n+1}\right\}\right], \end{aligned} \tag{23}$$

$$\begin{aligned} Z_R(n,\ t_1) &= \left[T_Rp_Rt_1d_{0R}e^{\lambda_R\ (n+1)t_1}-T_L\left\{\frac{n\,c_{1L}\,d_{0L}e^{\lambda_L\ (n+1)t_1}t_1}{2}\right.\right. \\ &\quad \left.\left.+\frac{c_{1L}\,d_{0L}e^{\lambda_L\ (n+1)t_1}t_1^2}{2}+i_{cL}t_2+\frac{b_L}{n+1}\right\}\right] \end{aligned} \tag{24}$$

and

$$Z_C(n,\ t_1)=\frac{1}{2}[Z_L(n\ ,,t_1)+Z_R(n\ ,,t_1)\ ]. \tag{25}$$

*The implications of HFS theory for inventory management problems*

HFS theory is a subject in fuzzy set theory that is growing in popularity, and it covers all of the fields in the decision-making and support systems in operations research (OR). Because inventory management is a part of OR, it is quite natural to study inventory problems in a hesitant fuzzy environment. In inventory decision-making processes, we usually observe that any commodity is available from several companies at the different cost and selling prices and different levels of quality, for which the demand varies. Consequently, the decision makers (DMs) in any business management team should use information gathered from surveys of the global market to sustain their business in the future. To handle such information, researchers rely usually on stochastic models, including fuzzy stochastic models. However, in the last decades, before the invention of HFS theory, there was no tool as powerful (although some aggregation operators Xia et al. [47,48] are present in IFS theory, their application to inventory models is critical, more complex and not verified by other methods) was present to help with the selection of the best alternatives without going through the complete solution process of the inventory system.

Generally, the attributes of any inventory model are qualitative in nature; examples include {getup/packing, durability, availability}, {fresh, nutritious, tasty} and {purity, usefulness, availability}. As the quality varies, the parameters associated with it may change and, therefore, we have a sequence of models, namely, a model set. When several membership functions or a family of membership functions are considered, we create a hesitant fuzzy decision matrix.

*The hesitant fuzzy mathematical model*

Suppose we are interested in studying the model's maxima, at which the characteristic attributes of the items are qualitative, namely, $\{Purity, Food\ value, Tasty\}$ or $\{getup, durability, availability\}$, etc. We define this type of attribute as $A = \{A_1, A_2, A_3\}$. Suppose a fruit seller has four different fruits and he/she has to choose one of them to focus on to maximize the average profit. The only available information is that the set of membership values of the different alternatives with various attributes are known here.

Now, the general form of a hesitant fuzzy decision matrix is

$$H = (h_{ij})_{m\times n} = \begin{bmatrix} h_{11} & h_{12} & \ldots h_{1n} \\ h_{21} & h_{22} & \ldots h_{2n} \\ h_{31} & h_{32} & \ldots h_{3n} \\ \ldots & \ldots & \ldots \\ h_{m1} & h_{m2} & \ldots h_{mn} \end{bmatrix}, \text{ where } h_{ij} = \{\gamma_{ij}\}, \quad i = 1, 2, \ldots m \text{ and } j = 1, 2, \ldots n.$$

We define the membership value $\gamma_{ij}$ from the family of bell-shaped membership functions as follows:

$$\gamma_{ij}^z = \begin{cases} 1 & if\ Z_{ij} \le L_{ij} \\ \theta_{ij} e^{-\frac{1}{2}\left(\frac{Z_{ij}-L_{ij}}{U_{ij}-L_{ij}}\right)^2} & if\ L_{ij} \le Z_{ij} \le U_{ij} \\ 0 & if\ Z_{ij} \ge U_{ij} \end{cases} \tag{26}$$

These functions are continuous on $[L_{ij}, U_{ij}]$ and $0 < \theta_{ij} < 1$ for $i = 1, 2, \ldots m$ and $j = 1, 2, ..n$.

In our model, three attributes and four alternatives (Table 1) are present; they are defined as follows:where $\gamma_i^z = \{\gamma_{i1}^z, \gamma_{i2}^z, \ldots \gamma_{in}^z\}$ is obtained from (26). With these criteria, the fuzzy objectives are shown in Table 2.

**Table 1**
Criteria under fuzzification

| Criteria | Fuzzy number | Fuzzy membership |
|---|---|---|
| $X_1$ | Holding cost, idle time cost and setup cost | $\gamma_1^z$ |
| $X_2$ | Unit profit | $\gamma_2^z$ |
| $X_3$ | Demand rate | $\gamma_3^z$ |
| $X_4$ | All of the above | $\gamma_4^z$ |

**Table 2**
Probable fuzzy alternatives.

| Criterion | Profit function |
|---|---|
| $X_1$ | $Maximize\ \tilde{z} = \left[pdt_1 - \left\{\frac{n\ d\ \tilde{c}_1\ t_1}{2} + \frac{d\tilde{c}_1\ t_1^2}{2} + \tilde{i}_c t_2 + \frac{\tilde{b}}{n+1}\right\}\right]T$ |
| $X_2$ | $Maximize\ \tilde{z} = \left[\tilde{p}t_1 d - \left\{\frac{n\ c_1\ d\ t_1}{2} + \frac{c_1\ d\ t_1^2}{2} + i_c t_2 + \frac{b}{n+1}\right\}\right]T$ |
| $X_3$ | $Maximize\ \tilde{z} = \left[\tilde{d}pt_1 - \left\{\frac{n\ c_1\tilde{d}\ \ t_1}{2} + \frac{c_1\ \tilde{d}\ t_1^2}{2} + i_c t_2 + \frac{b}{n+1}\right\}\right]T$ |
| $X_4$ | $Maximize\ \tilde{z} = \left[\tilde{p}\ \tilde{d}t_1 - \left\{\frac{n\tilde{c}_1\tilde{d}t_1}{2} + \frac{\tilde{c}_1\tilde{d}t_1^2}{2} + \tilde{i}_c t_2 + \frac{\tilde{b}}{n+1}\right\}\right]\tilde{T}$ |

*The procedure for solving the hesitant fuzzy mathematical model*

The procedure for solving this problem in a hesitant fuzzy environment is as follows:

Step 1 Find the lower, centre and upper value of each alternative (stated in Table 2) using the operations and arithmetic relations defined in Sections 2.3 and 2.4 and equations (23), (24) and (25).

Step 2 Construct a set of membership functions, assign the lower and upper values of each alternative and determine the variation factor $\theta_{ij}$ for each attribute.

Step 3 Using steps 1 and 2, calculate the membership grade of each alternative with respect to each attribute to create a hesitant fuzzy decision matrix (HFDM).

Step 4 Apply several of the aggregation operators described in Definitions 5, 6, 7 and 8 carefully. In case of ordered fuzzy numbers, find a score for each element of the HFDM, rank them and define a new HFDM. Then, apply the aggregation rules. During the process of computing different aggregate values, the normal standard method [49] can be used.

Step 5 Once the aggregate values of each alternative have been determined, calculate their scores using Definition 3. If any two or more scores are found to be equal, calculate the accuracy using Definition 4 and rearrange them accordingly.

Step 6 Rank the scores and make a decision. If the problems have benefit criteria then select the highest ranked; otherwise, consider the final rank.

## Numerical examples

**Example 1.** Let the inventory holding cost = $\$c_1 3.0$, the idle time cost per day $i_c = \$8.0$, the time horizon T = 30 days, the initial demand $d_0 = 50$ units, the profit per unit $p = \$20.0$, the set up cost $b = \$300$, and $\lambda = 0.3$. The optimal solution is given below (Table 3).

**Table 3**
The optimal crisp solution.

| $n^*$ | $t_1^*$ | $t_2^*$ | $t^*$ | $q^*$ | $z^*$ ($) |
|---|---|---|---|---|---|
| 6 | 0.5 | 0.5 | 7 | 500.089 | 20562.48 |

**Table 4**
The optimal solution for a multi-decision function with ($t_1^* = 0.5 = t_2^*$).

| Criterion | Decision variable | $z_{L-max}$ | $z_{C-max}$ | $z_{R-max}$ | $L = z_{min} U = z_{max}$ |
|---|---|---|---|---|---|
| $X_1$ | $q^*$ | 264.625 | 500.089 | 1431.919 | $sL = 199.02$ |
| | $z^*$ | 16273.36 | 20562.48 | 31119.11 | $U = 31119.11$ |
| | $n^*$ | 4 | 6 | 10 | |
| | $z'$ | 19117.69 | 15216.19 | 199.0223 | |
| | $z''$ | 21962.02 | 25908.76 | 15659.07 | |
| $X_2$ | $q^*$ | 368.941 | 500.089 | 867.921 | $L = 12622.08$ |
| | $z^*$ | 15443.5 | 20562.48 | 27087.42 | $U = 27087.42$ |
| | $n^*$ | 5 | 6 | 8 | |
| | $z'$ | 20055.25 | 15204.38 | 12622.08 | |
| | $z''$ | 24667.01 | 25920.57 | 19854.75 | |
| $X_3$ | $q^*$ | 91.115 | 396.903 | 547.819 | $L = 381.058$ |
| | $z^*$ | 9465.914 | 21698.03 | 43015.01 | $U = 43015.01$ |
| | $n^*$ | 2 | 5 | 5 | |
| | $z'$ | 16497.02 | 381.058 | 381.05 | |
| | $z''$ | 23528.13 | 43015.00 | 21698.0 | |
| $X_4$ | $q^*$ | 91.115 | 120.720 | 150.325 | $L = 1632.38$ |
| | $z^*$ | 1632.383 | 18186.06 | 34739.73 | $U = 34739.73$ |
| | $n^*$ | 2 | 2 | 2 | |
| | $z'$ | 18186.06 | 1632.383 | 1632.383 | |
| | $z''$ | 34739.73 | 34739.73 | 18186.06 | |

**Table 5**
The hesitant fuzzy decision matrix.

| Criterion | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $X_1$ | {.121, .161,.175 } | {.364, .483, .524} | {.121, .161,.175 } |
| $X_2$ | {.121,.172,.196 } | {.364, .516, .589} | {.121,.172,.196 } |
| $X_3$ | {.121, .176, .196} | {.364, .529, .587} | {.121, .176, .196} |
| $X_4$ | {.121, .176, .200} | {.364, .529, .600} | {.121, .176, .200} |

**Table 6**
The hesitant fuzzy decision matrix (based on EOP).

| Criterion | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $X_1$ | {.202,.268, .291 } | {.202,.268, .291 } | {.202,.268, .291 } |
| $X_2$ | {.202,.287,.327} | {.202,.287,.327} | {.202,.287,.327} |
| $X_3$ | {.202,.294.326} | {.202,.294.326} | {.202,.294.326} |
| $X_4$ | {.202,.294,.333} | {.202,.294,.333} | {.202,.294,.333} |

Note: EOP = Equal Opportunity.

**Example 2.** *Now, to solve the system of alternatives given in*Table 2, *we use the following fuzzy numbers*"

$$\tilde{c}_1 = c_{11}, c_{12}, , c_{13} = (1.5, 3, 4.5),\ \tilde{b} = b_1, b_2, , b_3 = (250, 300, 350),\ \tilde{p} = p_1, p_2, , p_3 = (15, 20, 25),$$

$$\tilde{d}_0 = d_1, d_2, d_3 = (40, 50, 60),\ \tilde{\lambda} = \lambda_1, \lambda_2, , \lambda_3 = (0.1, 0.3, 0.5),\ \tilde{i}_c = i_{c1}, i_{c2}, , i_{c3} = (6, 8, 10),$$

*Using the formulas given in*equation (10)*for these fuzzy numbers, we have*
$(c_{1L}, c_{1C}, c_{1R}) = 2.25, 3, 3.75$,
$(b_L, b_C, b_R) = 275, 300, 325$ $(p_L, p_C, p_R) = 17.5, 20, 22.5 (d_{0L}, d_{0C}, d_{0R}) = 45, 50, 55, \lambda_L, \lambda_C, \lambda_R = (0.2, 0.3, 0.4), i_{cL}, i_{cC}, i_{cR} = (7, 8, 9)$ *and* $T_L, T_C, T_R = (25, 30, 35)$.

*Now, solving*equations (23), (24) and (25)*for each alternative, we find the solutions presented in*Table 4*and the values of the parameters in a fuzzy sense* (Table 5–12).

From Table 12, we see that the score of criterion $X_4$ provides the best solution of all of the alternatives for several aggregation operators because the models have benefit criteria. Therefore, the decision maker (DM) selects the case in which all of the parameters are fuzzy numbers, i.e., the case in which all of the parameters are changed. This indicates that the "all-varied parametric model" provides a better average profit than the others. From Table 4, we also see that the lower profit of $X_4$ is higher than all of the other alternatives. This reveals that each DM expects that the minimum profit be high, disregarding the higher expected profit (the upper value). In some of the other alternatives, the average profit is unexpectedly low. As a result, the decision made using the various operators is an acceptable one.

## Conclusions

In this paper, a new concept of fuzzy set, namely, the HFS, is introduced into the inventory management literature. For the sake of simplicity, we used a single family of membership functions of the objective function of the model set. In addition, for several aggregation operators, we have used the most well-known Einstein weighted aggregation geometric operator. In this study, we observed that the operators used in the present model led to the same decision. However, we have incorporated the concept of a "natural idle time" and made the demand rate exponentially dependent on the total amount of time the business is open during a cycle in the classical EOQ inventory model. Such considerations are introduced because, as we know for any inventory policy, opening the shop/industry/inventory to the public is a basic need to ensure that the goods are ordered as quickly as possible. The limitations of the study in our present article are as follows:

1. The decision is made only on the basis of the HFS scores; this process could be tested using other operators relating to IFS theory or simply fuzzy set theory.

**Table 7**
The decision matrix for the *HFEWG* operator.

| $H$ | $n(h_1) = 19,\ n(h_2) = 27,\ n(h_3) = n(h_4) = 18$ |
|---|---|
| $h_1$ | {.3806, .3762, .3617, .3582, .3540, .3401, .2916, .2880, .2763, .3719, .3575, .3498, .3361, .2845, .2729, .3617, .3435, .3229, .2617} |
| $h_2$ | {.2617, .2756, .3762, .2810, .3566, .3948, .3633, .3394, .4020, .2947, .3864, .3614, .4194 .2890, .2746, .3553, .3731, .3830, .4201, .4126, .3934, .2795, .2931, .2999, .3794, .3999 , .4270} |
| $h_3$ | {.4280, .4218, .4010, .3957, .3898, .3702, .3015, .2968, .2810, .4156, .3950, .3840, .3646, .2920, .2765, .3752, .3460, .2617} |
| $h_4$ | {.4377, .4301, .4090, .3980, .3909, .3713, .3033, .2977, .2819, .4226, .4018, .3840, .3646, .2921, .2765, .3817, .3460, .2617} |

Note: $s(h_1) = .33101, s(h_2) = .35157, s(h_4) = .35838$ , $s(h_3) = .35536$.

**Table 8**
The decision matrix for the *HFEOWG* operator.

| $H$ | $n(h_1) = 26,\ n(h_2) = n(h_3) = n(h_4) = 27$ |
|---|---|
| $h_1$ | {.2108, .2076, .1974, .1994, .1963, .1865, .1647, .1621, .1539, .2082, .2050, .1948, .1968, .1938, .1841, .1625, .1600, .1518, .1993, .1962, .1884, .1855 , .1762, .1554, .1530, .1452} |
| $h_2$ | {.1452, .1550, .1591, .2016, .1964, .1843, .2015, .2146, .2201, .1681, .1637, .1534, .1945, .2072, .2127, .2321, .2263, .2125, .1567, .1672, .1716, .2170 , .2115, .1986, .2169, .2309, .2368} |
| $h_3$ | {.2366, .2319, .2169, .2202, .2158, .2017, .1715, .1567, .1680, .2328, .2281, .2133, .2166, .2122, .1983, .1686, .1651, .1540, .2200, .2156, .2015, .2046 , .2004, .1872, .1590, .1557, .1452} |
| $h_4$ | {.2417, .2206, .2358, .2219, .2165, .2023, .1728, .1685, .1572, .2370, .2313, .2162, .2175, .2122, .1983, .1693, .1651, .1540, .1597, .1557, .1452, .2240, .2186, .2043, .2055, .2004, .1872} |

Note: $s(h_1) = .1554$ , $s(h_2) =$ . 18236, $s(h_4) = .1977$ , $s(h_3) = .1962$.

**Table 9**
The decision matrix for the *HFWG* operator.

| $H$ | $n(h_1) = 26,\quad n(h_2) = 23,\ n(h_3) = n(h_4) = 26$ |
|---|---|
| $h_1$ | {.2080, .2053, .1964, .1855, .1939, .1864, .1615, .1595, .1525, .1439, .1524, .1938, .2027, .2053, .1939, .1914, .1831, .1505, .1574, .1594, .1963, .1937 , .1853, .1830, .1854, .1750} |
| $h_2$ | {.2116, .2160, .1641, .1607, .1521, .1936, .1935, .2045, .1553, .2088, .2283, .2236, .2117, .1675, .2002, .1975, .2087, .2131, .2331, .2161, .1831 , , .1521, .1439} |
| $h_3$ | {.2330, .2292, .2161, .2007, .2164, .2129, .1647, .1553, .2124, .2253, .2290, .2128, .2093, .1973, .1527, .1619, .1646, .1552, .1526, .1439, .1860, .1973 , .2006, .2159, .2124, .2003} |
| $h_4$ | {.2378, .2331, .2198, .2014, .2136, .2178, .1685, .1652, .1558, .2154, .2285, .2013, .2330, .2135, .2093, .1973, .1527, .1619, .1652, .2197, .2154, .1860 , .2031, .1526, .1557, .1439} |

Note: $s(h_1) = .1812$, $s(h_2) = .19387$, $s(h_4) =$ . 1949, $s(h_3) = .1945$.

**Table 10**
The decision matrix for the *HFG* operator.

| $H$ | $n(h_1) = \quad n(h_2) = 18,\ n(h_4) = 15,\quad n(h_3) = 17$ |
|---|---|
| $h_1$ | {.2523, .2455, .2234, .2173, .2388, .2454, .2231, .2171, .1976, .2322, .2386, .2113, .1922, .2111, .2170, .1973, .1920, .1747} |
| $h_2$ | {.1747, .1963, .2051, .2306, .2207, .1964, .2052, .2305, .2409, .2209, .2481, .2593, .2708, .2591, .2307, .2410, .2707, .2829, .} |
| $h_3$ | {.2826, .2729, .2410, .2325, .2633, .2726, .2406, .2324, .2052, .2630, .2540, .2243, .1979, .2242, .2321, .2049, .1747} |
| $h_4$ | {.2885, .2766, .2442, .2340, .2651, .2765, .2440, .2649, .2540, .2243, .1979, .2242, .2338, .2064, .1747} |

Note: $s(h_1) = .21816$, $s(h_2) = .23244$, $s(h_4) = .23942$, $s(h_3) = .23636$.

**Table 11**
The decision matrix for the *HFEWG*$^{EOP}$ operator.

| $H$ | $n(h_1) = 15,\quad n(h_2) = n(h_3) = n(h_4) = 18$ |
|---|---|
| $h_1$ | {.2910, .2873, .2751, .2715, .2599, .2142, .2113, .2020, .2271, .2240, .2836, .2680, .2565, .2211, .2454} |
| $h_2$ | {.3270, .3204, .2930, .2137, .3139, .2260, .2870, .3038, .2776, .2360, .2976, .2820, .2020, .2991, .2184, .2309, .2719, .2574} |
| $h_3$ | {.3260, .3208, .3030, .3038, .2989, .2821, .2183, .2146, .2020, .2357, .2317, .3156, .2981, .2940, .2775, .2278, .2814, .2618} |
| $h_4$ | {.3330, .3266, .3086, .3059, .2999, .2831, .2191, .2146, .2020, .2374, .2326, .3203, .3026, .2940, .2775, .2278, .2857, .2618} |

Note: $s(h_1) = .2492$, $s(h_2) = .26987$, $s(h_4) = .2740$, $s(h_3) = .2718$.

**Table 12**
The decision for several operators.

| Aggregation operator | Ranking of scores | Optimal decision |
|---|---|---|
| *HFEWG* | $s(h_4) > s(h_3) > s(h_2) > s(h_1)$ | $X_4$ |
| *HFEOWG* | $s(h_4) > s(h_3) > s(h_2) > s(h_1)$ | $X_4$ |
| *HFEWG*$^{EOP}$ | $s(h_4) > s(h_3) > s(h_2) > s(h_1)$ | $X_4$ |
| *HFWG* | $s(h_4) > s(h_3) > s(h_2) > s(h_1)$ | $X_4$ |
| *HFG* | $s(h_4) > s(h_3) > s(h_2) > s(h_1)$ | $X_4$ |

2. The decision should be tested by considering several membership functions of the fuzzy parameters instead of selecting a particular family of membership functions.
3. The choices of aggregation operators should have proper justifications.

The above limitations could be relaxed in future studies.

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