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The Construction of Large Sets of Idempotent Quasigroups

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The maximum number of idempotent quasigroups of order n which pairwise agree on the main diagonal only is $n - 2$. Such a collection is called a large set of idempotent quasigroups of order n. The main result in this paper is the construction of a large set of idempotent quasigroups of order n for *every* $n \geqslant 3$ *except* $n = 6$, for which no such collection exists, and $n = 14$ and 62. Additionally, the known spectrum for large sets of Mendelsohn quasigroups is improved.

1. INTRODUCTION

Let $(Q, \circ)$ be a quasigroup of order n and define an $n^2 \times 3$ array A by (x, y, z) is a row of A if and only if $x \circ y = z$. Then A is an orthogonal array. That is, if we run our fingers down any two columns of A we get each ordered pair belonging to $Q \times Q$ exactly once. Conversely, if A is any $n^2 \times 3$ orthogonal array (defined on a set Q) and we define a binary operation '$\circ$' on Q by $x \circ y = z$ if and only if (x, y, z) is a row of A, then $(Q, \circ)$ is a quasigroup. Hence we can think of a quasigroup of order n as an $n^2 \times 3$ orthogonal array and conversely. If $\alpha \in S_3$ (the symmetric group on $\{1, 2, 3\}$) and A is an $n^2 \times 3$ orthogonal array we will denote by $A\alpha$ the orthogonal array obtained by permuting the columns of A according to α. Two orthogonal arrays are *equal* if and only if they define the same quasigroup. In other words, if we *disregard* the level at which the rows occur, the two orthogonal arrays contain exactly the same rows. Two orthogonal arrays A and B are said to be *conjugate* provided there is at least one $\alpha \in S_3$ such that $A\alpha = B$. Two quasigroups are said to be conjugate provided their corresponding orthogonal arrays are conjugate. The *conjugate invariant subgroup* H of an orthogonal array A is defined by

$$H = \{\alpha \in S_3 \mid A\alpha = A\}.$$

The quasigroup $(Q, \circ)$ is said to be *idempotent* provided it satisfies the identity $x^2 = x$; i.e., $a \circ a = a$ for all $a \in Q$. The corresponding orthogonal array A has the property that $(a, a, a) \in A$ for every $a \in Q$. Hence the $n(n - 1)$ non-idempotent rows of A consist of 3 distinct elements. Trivially, any $n(n - 1) \times 3$ partial orthogonal array (based on a set of size n) in which all of the rows consist of 3 distinct elements can be enlarged to an idempotent orthogonal array by adding n rows of the form (a, a, a). If H is a subgroup of S_3, the *idempotent spectrum* of H is set of all n such there is an *idempotent* quasigroup of order n with conjugate invariant subgroup containing H. The information presented in Table 1 is extremely well-known. See [3] for details.

In view of the preceding remarks, the following problem arises quite naturally.

THE GENERAL PROBLEM *Denote by $T(n)$ the set of $n(n - 1)(n - 2)$ ordered triples of elements from the set $\{1, 2, 3, \ldots, n\}$ with the property that the 3 coordinates of each ordered triple are distinct. Let H be a subgroup of S_3. For which n belonging to the idempotent spectrum of H is it possible to partition $T(n)$ into $n - 2$ $n(n - 1) \times 3$ partial orthogonal arrays $A_1, A_2, \ldots, A_{n-2}$ such that each of $A_1, A_2, \ldots A_{n-2}$ is invariant under conjugation by H?*

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Table 1

Subgroup of S_3	Idempotent spectrum	Characterization
$\langle 1 \rangle$	all n except 2	quasigroup satisfying $x^2 = x$
$\langle (12) \rangle$	all odd $n \geqslant 1$	quasigroup satisfying $x^2 = x$, $xy = yx$
$\langle (13) \rangle$	all odd $n \geqslant 1$	quasigroup satisfying $x^2 = x$, $(yx)x = y$
$\langle (23) \rangle$	all odd $n \geqslant 1$	quasigroup satisfying $x^2 = x$, $x(xy) = y$
$\langle (123) \rangle$	all $n \equiv 0$ or $1 \pmod 3$ *except* $n = 6$	quasigroup satisfying $x^2 = x$, $x(yx) = y$ Mendelsohn quasigroup *equivalent* to a Mendelsohn triple system
S_3	all $n \equiv 1$ or $3 \pmod 6$	quasigroup satisfying $x^2 = x$, $(yx)x = y$, $xy = yx$ Steiner quasigroup *equivalent* to a Steiner triple system

With good reason, the collection $A_1, A_2, \ldots, A_{n-2}$ is called a *large set* of pairwise disjoint $n(n - 1) \times 3$ partial orthogonal arrays invariant under conjugation by H. If we add the n idempotent rows to each A_i, the resulting orthogonal arrays have only these idempotent rows in common. These orthogonal arrays will be called a *large set of idempotent* orthogonal arrays invariant under conjugation by H. The corresponding quasigroups are called a *large set of idempotent quasigroups invariant under conjugation by* H. A tremendous amount of work has been done on the problem of constructing large sets of idempotent quasigroups invariant under conjugation by $H = S_3$ (= Steiner quasigroups). In particular, it is now known that a large set of Steiner quasigroups of order n exists for every n in the spectrum (= all $n \equiv 1$ or $3 \pmod 6$) except for 7 and *possibly* six other cases [5]. To date, the only other attack on the large set problem that the authors are aware of is for $H = \langle (123) \rangle$ [2] (= Mendelsohn quasigroup), where large sets of Mendelsohn quasigroups of order n are constructed for an infinite number of n in the spectrum (= all $n \equiv 0$ or $1 \pmod 3$, except $n = 6$). However, this problem remains far from settled.

The main purpose of this paper is to give a complete solution of the large set problem for $H = \langle 1 \rangle$. In particular, we construct a large set of idempotent quasigroups of order n for *every* $n \geqslant 3$, except $n = 6$, for which no such collection exists, and possibly $n = 14$ and 62. Additionally, we improve the known spectrum for large sets of Mendelsohn quasigroups.

Like a lot of solutions to design problems in combinatorics, we give a construction that works for *all* but a handful of cases, followed by an assortment of constructions for the remaining cases.

2. The Main Construction

In what follows, we will denote the (partial) orthogonal array A by (Q, R), where Q is the set on which A is based, and R is the set of rows of A. The *number* $|Q|$ is, of course, called the *order* of (Q, R). A *transversal* of the orthogonal array (Q, R) of order n is any collection of n rows $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots, (x_n, y_n, z_n)$ such that $\{x_1, x_2, \ldots, x_n\} = \{y_1, y_2, \ldots, y_n\} = \{z_1, z_2, \ldots, z_n\} = Q$. The orthogonal array (Q, R) is said to be *regular* provided that R contains *exactly one* idempotent row *and* each of

$$\begin{cases} T_1 = \{(x, x, y) \mid \text{all } x \in Q\}, \\ T_2 = \{(x, y, x) \mid \text{all } x \in Q\}, \text{ and} \\ T_3 = \{(y, x, x) \mid \text{all } x \in Q\} \text{ is a transversal.} \end{cases}$$

Let $A \cap X = \varnothing$, and let $(A \cup X, R)$ be a (partial) orthogonal array. If (y_1, y_2, y_3) is any ordered triple, define $R(y_1, y_2, y_3)$ to be the collection of rows obtained from R as follows: for each row $(x_1, x_2, x_3) \in R$, replace x_i with (x_i, y_i) if and only if $x_i \in X$, *and* place the resulting row in $R(y_1, y_2, y_3)$.

THE $nv + 2$ CONSTRUCTION. Let (Q, R_1) be a regular orthogonal array of order v, $(\{\infty_1, \infty_2\} \cup X, R_2)$ an indempotent orthogonal array of order $n + 2$, (X, R_3) an orthogonal array of order n, and finally $(\{\infty\} \cup X, R_4)$ an orthogonal array of order $n + 1$ such that $(a, a, \infty) \in R_4$ for all $a \in \{\infty\} \cup X$. Denote by R_5 the set of $n^2 + n$ rows obtained from R_4 by *deleting* all rows of the form (a, a, ∞), and *replacing* each row of the form (∞, a, b) by (∞_1, a, b) and each row of the form (a, ∞, b) by (a, ∞_2, b). Then $(\{\infty_1, \infty_2\} \cup X, R_5)$ is a partial orthogonal array of order $n + 2$. Now set $S = \{\infty_1, \infty_2\} \cup (X \times Q)$ and define a collection R of rows of S as follows:
(1) $(x, x, x) \in R$ for every $x \in S$;
(2) for the *unique* idempotent row (i, i, i) belonging to R_1, place the non-independent rows of $R_2(i, i, i)$ in R;
(3) for each row of the form $(a, a, b) \in R_1$, $a \neq b$, place the rows of $R_5(a, a, b)$ in R;
(4) for each row of the form $(b, a, a) \in R_1$, $a \neq b$, place the rows of $(R_5(123))(b, a, a)$ in R ($R_5(123)$ is the (123) conjugate of R_5);
(5) for each row of the form $(a, b, a) \in R_1$, $a \neq b$, place the rows of $(R_5(213))(a, b, a)$ in R ($R_5(213)$ is the (213) conjugate of R_5); and finally,
(6) for each row $(a, b, c) \in R_1$, where a, b, and c are distinct, place the rows of $R_3(a, b, c)$ in R.

It is straight forward to see that (S, R) is an idempotent orthogonal array of order $nv + 2$.

LEMMA 2.1. *If there exists a regular orthogonal array of order v* and *a large set of idempotent orthogonal arrays of order $n + 2$, then there exists n idempotent orthogonal arrays of order $nv + 2$ having only the idempotent rows in common.*

PROOF. Let $(\{\infty_1, \infty_2\} \cup X, R_2^1)$, $(\{\infty_1, \infty_2\} \cup X, R_2^2)$, ..., $(\{\infty_1, \infty_2\} \cup X, R_2^n)$ be a large set of idempotent orthogonal arrays of order $n + 2$. Further, let α be a cycle of length n on X and define

$$\begin{cases} R_3^{\alpha^i} = \{(x, y, z\alpha^i) \mid \text{all } (x, y, z) \in R_3\}, \text{ and} \\ R_5^{\alpha^i} = \{(x, y, z\alpha^i) \mid \text{all } (x, y, z) \in R_5\}. \end{cases}$$

Then the n orthogonal arrays $(X, R_3^{\alpha^1})$, $(X, R_3^{\alpha^2})$, ..., $(X, R_3^{\alpha^n})$ are pairwise disjoint, and so are the partial orthogonal arrays

$$(\{\infty_1, \infty_2\} \cup X, R_5^{\alpha^1}), (\{\infty_1, \infty_2\} \cup X, R_5^{\alpha^2}), \ldots (\{\infty_1, \infty_2\} \cup X, R_5^{\alpha^n}).$$

Now, denote by (S, R^i) the idempotent orthogonal array constructed by the $nv + 2$ construction using (Q, R_1), $(\{\infty_1, \infty_2\} \cup X, R_2^i)$, $(X, R_3^{\alpha^i})$, and $(\{\infty_1, \infty_2\} \cup X, R_5^{\alpha^i})$. It is straight forward to see that the n idempotent orthogonal arrays (S, R^1), (S, R^2), ... (S, R^n) have only their idempotent rows in common.

LEMMA 2.2. *If there exists v pairwise disjoint regular orthogonal arrays of order v* and *a large set of idempotent orthogonal arrays of order $n + 2$, then there exists a large set of idempotent orthogonal arrays of order $nv + 2$.*

PROOF. Let (Q, T_1) and (Q, T_2) be a pair of disjoint regular orthogonal arrays of order v. Since $T_1 \cap T_2 = \emptyset$, if (i, i, i) is the unique idempotent row of T_1 and (j, j, j) is the unique idempotent row of T_2, we must have $i \neq j$. It follows that each of the n idempotent orthogonal arrays constructed using (Q, T_1) in Lemma 2.1 intersects each of the n idempotent orthogonal arrays constructed using (Q, T_2) in precisely the idempotent rows. The statement of the Lemma follows.

We now establish the existence of v pairwise disjoint regular orthogonal arrays of order v for all but a handful of cases.

LEMMA 2.3. *There exists an idempotent orthogonal array of order v having either* 3 *or* 6 *distinct conjugates that are pairwise orthogonal for every $v \geqslant 5$* except possibly *for $v \in Z = \{6, 10, 12, 14, 15, 18, 20, 21, 22, 24, 26, 28, 30, 33, 34, 38, 39, 42, 44, 46, 48, 52, 54, 60\}$.*

PROOF. To begin with, Frank Bennett [1] has constructed such orthogonal arrays for every order $v \geqslant 61$. Now there are idempotent orthogonal arrays of orders 5 and 7 having 3 distinct conjugates, which are pairwise orthogonal [1], and an idempotent orthogonal array of order v having 6 distinct conjugates, which are pairwise orthogonal for every $v = p^\alpha \geqslant 8$ (p a prime) [4]. The numbers $v \leqslant 60$ and $v \notin Z$ that are not powers of a prime are obtained using routine (and obvious) PBD constructions. (See [1] for example.)

LEMMA 2.4. *If $v \geqslant 5$* and *$v \notin Z$, there exists a pair of idempotent orthogonal arrays (Q, R) and (Q, T) such that both (Q, R) and $(Q, R(23))$ are orthogonal to (Q, T).*

PROOF. Let (Q, R) be an idempotent orthogonal array having either 3 or 6 distinct conjugates that are pairwise orthogonal. If (Q, R) has 6 distinct conjugates, then $(Q, R(23))$ and (Q, R) are both orthogonal to $(Q, R(12))$. If (Q, R) has 3 distinct conjugates, one of two things is true: either (23) belongs to the conjugate invariant subgroup H of (Q, R) or not. If $(23) \in H$, then $(12) \notin H$, and so $(Q, R) = (Q, R(23))$ is orthogonal to $(Q, R(12))$. If $(23) \notin H$, choose any $\alpha \notin H$ such that (Q, R), $(Q, R(23))$, and $(Q, R\alpha)$ are distinct. Combining the above cases completes the proof.

LEMMA 2.5. *If $v \geqslant 5$* and *$v \notin Z$, there exists v* disjoint *regular orthogonal arrays of order v.*

PROOF. Let (Q, S) and (Q, T) be a pair of idempotent orthogonal arrays such that both (Q, S) and $(Q, S(23))$ are orthogonal to (Q, T). Define v collections of rows $R_1, R_2, \ldots, R_v$ as follows: $(a, b, c) \in R_i$ if and only if for some $x \in Q$, $(x, a, b) \in S$ *and* $(x, c, i) \in T$. It is a straight forward matter to see that each of $(Q, R_1), (Q, R_2), \ldots, (Q, R_v)$ is an orthogonal array of order v. The only difficulty in showing that each is regular is in showing that $\{(b, a, a) \mid \text{all } a \in Q\}$ is a transversal. So suppose in (Q, R_i) that (b, a, a) *and* $(b, c, c) \in R_i$. Then for some $x \in Q$, $(x, b, a) \in S$ and $(x, a, i) \in T$ *and* for some $y \in Q$, $(y, b, c) \in S$ and $(y, c, i) \in T$. Hence $(x, b, a), (y, b, c) \in S$ and $(x, a, i), (y, c, i) \in T$. But then (x, a, b) and $(y, c, b) \in S(23)$ and (x, a, i) and $(y, c, i) \in T$, which cannot happen since $(Q, S(23))$ and (Q, T) are orthogonal. Hence, each (Q, R_i) is regular, which completes the proof.

LEMMA 2.6. *There exists a large set of idempotent orthogonal arrays of order v for every $v \geqslant 3$ and $v \notin \{6, 14, 22, 30, 46, 54, 62\}$.*

PROOF. To begin with, it is well-known that there are $n - 1$ pairwise orthogonal quasigroups of order n for every $n = p^\alpha \geqslant 3$ (p a prime). Such a collection is, of course, equivalent to $n - 2$ pairwise orthogonal idempotent quasigroups of order n, which is, among other things, a large set of idempotent quasigroups of order n. Hence, there is a large set of idempotent orthogonal arrays of order v for $v \in N = \{3, 4, 5, 8, 17\}$. It is a trivial matter to see that if $7 \leqslant v \leqslant 62$ *and* $v \notin \{14, 22, 30, 46, 54, 62\}$, then v can be written in at least one way in the form $v = 2 + n \cdot u$, where $n + 2 \in N$ and $u \notin Z$. If $v \geqslant 63$, then $v - 2 \geqslant 61 \notin Z$, and so we can always take $n = 1$ and write $v = 2 + 1 \cdot (v - 2)$. In any case, Lemma 2.2 guarantees a large set of idempotent orthogonal arrays of order $nv + 2$.

3. THE REMAINING CASES

It is a trivial matter to construct a pair of idempotent orthogonal arrays of order 6 that intersect in the 6 idempotent rows only. However, a brute force computer search shows that no such pair can be extended to a large set. Hence, there does not exist a large set of idempotent orthogonal arrays of order 6. Of the remaining cases, the authors can handle 22, 30, 46, and 54 only. After much valiant effort, the authors relunctantly leave the cases $v = 14$ and 62 unsettled.

The cases 22, 30, 46, and 54 are handled with a trivial modification of the constructions used in [2] to construct large sets of Mendelsohn quasigroups (= Mendelsohn arrays).

THE $3v$ CONSTRUCTION. Let Q be a set of size $v \neq 6$, (Q, T) any idempotent orthogonal array, and α any cycle of length v on Q. Let $S = Q \times \{1, 2, 3\}$. In [2], $2v$ Mendelsohn arrays $(S, R_1), (S, R_2), \ldots, (S, R_{2v})$ of order $3v$ are constructed that pairwise intersect in their idempotent rows and such that if (a, b, c) is a non-idempotent row of any R_i, then

$$\{a, b, c\} \neq \begin{cases} \{(x, 1), (y, 2), (z\alpha^{1+i}, 3)\} \text{ where} \\ (x, y, z) \in T \text{ and } i = 1, 2, \ldots, v - 2, \textit{or} \\ \{(x, j), (y, j), (z, j)\}, j = 1, 2, \text{ or } 3. \end{cases}$$

If $(Q, L_1), (Q, L_2), \ldots, (Q, L_{v-2})$ is any large set of Mendelsohn arrays of order v, construct $v - 2$ idempotent orthogonal arrays $(S, R_{2v+1}), (S, R_{2v+2}), \ldots, (S, R_{3v-2})$ as follows:

(1) $(x, x, x) \in R_{2v+i}$, for every $x \in S$;

(2) for each $i = 1, 2, 3$, $((x, i), (y, i), (z, i)) \in R_{2v+i}$ if and only if $(x, y, z) \in L_i$, and x, y, z are distinct; and

(3) if $(x, y, z) \in T$, the six rows $((x, 1), (y, 2), (z\alpha^{1+i}, 3))$, $((y, 2), (z\alpha^{1+i}, 3), (x, 1))$, $((z\alpha^{1+i}, 3), (x, 1), (y, 2))$, $((y, 2), (x, 1), (z\alpha^{1+i}, 3))$, $((x, 1), (z\alpha^{1+i}, 3), (y, 2))$, $((z\alpha^{1+i}, 3), (y, 2), (x, 1))$ belong to R_{2v+i}.

Then (S, R_{2v+i}) and (S, R_{2v+j}) intersect in their idempotent rows only and, of course, $(S, R_1), (S, R_2), \ldots, (S, R_{3v-2})$ is a large set of idempotent arrays.

LEMMA 3.1. *There exists a large set of idempotent orthogonal arrays of order v for $v = 30$ and 54.*

PROOF. Write $30 = 3 \cdot 10$ and $54 = 3 \cdot 18$.

THE $3v + 1$ CONSTRUCTION. Let Q be a set of size v, (Q, T) any orthogonal array having an orthogonal mate, and α any cycle of length v on Q. Set $S = \{\infty\} \cup (Q \times \{1, 2, 3\})$. In [2], $2v$ Mendelsohn arrays $(S, R_1), (S, R_2), \ldots, (S, R_{2v})$ of order $3v + 1$ are constructed that pairwise intersect in their idempotent rows, and such that if (a, b, c) is a non-idempotent row of any R_i, then

$$\{a, b, c\} \neq \begin{cases} \{\infty, (y, i), (z, i)\} \text{ or} \\ \{(x, j), (y, j), (z, j)\}, j = 1, 2, 3; \end{cases}$$

and

$$\{a, b, c\} \not\subseteq \begin{cases} \{\infty, (x, 1), (x, 2), (z\alpha^i, 3)\}, \text{ where} \\ (x, y, z) \in T \text{ and } i = 1, 2, \ldots, v - 1. \end{cases}$$

Now let $(\{\infty\} \cup Q, L_1), (\{\infty\} \cup Q, L_2\}, \ldots, (\{\infty\} \cup Q, L_{v-1})$ be a large set of Mendelsohn arrays of order $v + 1$, and construct $v - 1$ idempotent orthogonal arrays $(S, R_{2v+1}), \ldots, (S, R_{3v-1})$ as in the $3v$ construction. Then the $3v - 1$ idempotent orthogonal arrays $(S, R_1), (S, R_2), \ldots, (S, R_{3v-1})$ are a large set of idempotent orthogonal arrays of order $3v + 1$.

LEMMA 3.2. *There exists a large set of idempotent orthogonal arrays of order v for* $v = 22$ *and* 46.

PROOF. Write $22 = 3 \cdot 7 + 1$ and $46 = 3 \cdot 15 + 1$.

Combining Lemmas 2.6, 3.1, and 3.2 gives the following result, which is the main result in this paper.

THEOREM 3.3. *There exists a large set of idempotent orthogonal arrays of order v for every* $v \geqslant 3$ except $v = 6$ (*for which no such collection exists*) *and possibly* $v = 14$ *and* 62.

4. REMARKS

The $nv + 2$ construction can be used to construct large sets of Mendelsohn arrays as follows: let $v \equiv 1$ or 5 (mod 6), $Q = \{0, 1, 2, \ldots, v - 1\}$, and for *each* $a \in Q$ define a collection of rows R_a of Q by $(x, y, z) \in R_a$ if and only if $x + y + z \equiv a \pmod{v}$. Then $(Q, R_0), (Q, R_1), \ldots, (Q, R_{v-1})$ is a collection of pairwise disjoint totally symmetric regular orthogonal arrays. Now in the $nv + 2$ construction, take $(\{\infty_1, \infty_2\} \cup X, R_2)$ to be a Mendelsohn array, and replace (6) by: Since (Q, R_1) is totally symmetric, $\langle(123)\rangle$ acting as a permutation group on R_1 (among other things) partitions the rows of R_1 with 3 distinct coordinates into orbits that look like $\{(a, b, c), (b, c, a), (c, a, b)\}$. For each such orbit, choose a row, say (a, b, c), and place the rows of $R_3(a, b, c) \cup ((R_3(a, b, c))(123) \cup (R_3(a, b, c))(132)$ in R. The result is a Mendelsohn array.

The following theorem is immediate.

THEOREM 4.1. *If* $v \equiv 1$ *or* 5(mod 6) *and a large set of Mendelsohn arrays of order* $n + 2$ *exists, then there exists a large set of Mendelsohn arrays of order* $nv + 2$.

So, for example, since $16 = 2 + 2 \cdot 7$, $7 \equiv 1 \pmod 6$, and there is a large set of 2 Mendelsohn arrays of order 4, Theorem 4.1 produces a large set of Mendelsohn arrays of order 16 (an order that was previously in doubt).

The constructions in [2] produced large sets of Mendelsohn arrays of order $3 \leqslant v \leqslant 100$ for every admissible v except possibly $v = 16, 18, 22, 24, 40, 42, 46, 48, 52, 54, 58, 60, 64, 66, 70, 72, 76, 78, 85, 94$, and 96. A straight-forward check shows that the constructions in [2] *coupled* with Theorem 4.1 reduce the exceptions $\leqslant 100$ to $v = 18, 22, 54, 66, 78$, and 94.

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