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# New observables in longitudinal single-spin asymmetries in semi-inclusive DIS

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**Abstract**

We analyze longitudinal beam and target single-spin asymmetries in semi-inclusive deep inelastic scattering and in jet deep inelastic scattering, including all possible twist-3 contributions as well as quark mass corrections. We take into account the path-ordered exponential in the soft correlators and show that it leads to the introduction of a new distribution and a new fragmentation function contributing to the asymmetries.


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## 1. Introduction

Longitudinal beam and target single-spin asymmetries have been at the center of the attention lately, since they have been measured by the HERMES and CLAS experimental Collaborations [1–4] and more measurements are planned. They were originally believed to be signals of the so-called T-odd fragmentation functions [5], in particular, of the Collins function [6–12]. However, both types of asymmetry can receive contributions also from T-odd distribution functions [13–16], a fact that has often been neglected in analyses. An exhaustive treatment of the contributions of T-odd distribution functions has not been carried out completely so far, especially up to subleading order in an expansion in $1/Q$, $Q^2$ being the virtuality of the incident photon and the only hard scale of the process, and including quark mass corrections. It is the purpose of the present work to describe the longitudinal beam and target spin asymmetries in a complete way in terms of leading and subleading twist distribution and fragmentation functions. We consider both single-particle inclusive DIS, $e + p \to e' + h + X$, and single-jet inclusive DIS, $e + p \to e' + \text{jet} + X$. We assume factorization holds for these processes, even though at present there is no

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factorization proof for observables containing subleading-twist transverse-momentum dependent functions (only recently proofs for the leading-twist case have been presented in Refs. [17,18]).

We devote particular attention to the claims presented in Ref. [19], where it was suggested that the decomposition of the quark correlator should contain more terms than the ones considered in Refs. [14,20]. The inclusion of the gauge link in the proper definition of the correlator, in fact, introduces a dependence on the light cone vector, $n_-$, that defines the direction along which the path-ordered exponential is running. The inclusion of this new degree of freedom spoils the Lorentz-invariance relations among distribution functions as pointed out in Refs. [14,20], but the study in Ref. [19] is incomplete as an extra term in the decomposition of the unpolarized correlator has been neglected. This gives rise to a *new* distribution function and a *new* fragmentation function. We take these new terms into account and study their effect on the longitudinal asymmetries. Evidence—either from experiments or from model calculations—for the existence of these new functions could support the necessity of introducing the gauge-link direction in the decomposition of the correlator.

## 2. Unpolarized target

We adopt the point of view of Ref. [19] and complete the treatment presented there. We introduce first of all the four-momentum of the target, $P$, and that of the quark, $p$, and their decomposition in terms of light-cone vectors

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu, \qquad p^\mu = xP^+ n_+^\mu + p^- n_-^\mu + p_T^\mu. \tag{1}$$

To construct the hadronic tensor and consequently the cross sections, we start from the distribution correlation function (for the moment being we shall consider the target to be unpolarized)

$$\Phi^{[+]}(x, p_T) = \int \mathrm{d}p^-\, \Phi^{[+]}(P, p, n_-), \tag{2}$$

where $\Phi^{[+]}(x, p_T)$ includes the transverse link [21,22]

$$\Phi_{ij}^{[+]}(x, p_T) = \int \frac{\mathrm{d}\xi^- \,\mathrm{d}^2\xi_T}{(2\pi)^3} e^{+\mathrm{i}p\cdot\xi} \langle P|\bar{\psi}_j(0)\, \mathcal{L}_{[0^-,\infty^-]}\mathcal{L}_{[0_T,\xi_T]}\mathcal{L}_{[\infty^-,\xi^-]}\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}. \tag{3}$$

The notation $\mathcal{L}_{[a,b]}$ indicates a straight gauge link running from $a$ to $b$.

The most general form of the correlation function $\Phi^{[+]}$ complying with Hermiticity and parity constraints reads

$$\begin{aligned}\Phi^{[+]}(P, p, n_-) = {} & MA_1 + \not{P}A_2 + \not{p}A_3 + \frac{\mathrm{i}}{2M}[\not{P}, \not{p}]A_4 + \frac{M^2}{P\cdot n_-}\not{n}_- B_1 \\ & + \frac{\mathrm{i}M}{2P\cdot n_-}[\not{P}, \not{n}_-]B_2 + \frac{\mathrm{i}M}{2P\cdot n_-}[\not{p}, \not{n}_-]B_3 + \frac{1}{P\cdot n_-}\gamma_5\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu n_{-\rho} p_\sigma B_4.\end{aligned} \tag{4}$$

The last term was neglected in Ref. [19]. It is a T-odd and chiral-even structure.

Keeping only the leading and subleading terms in $1/P^+$ we obtain

$$\begin{aligned}\Phi^{[+]}(x, p_T) &\equiv \int \mathrm{d}p^-\, \Phi^{[+]}(P, p; n_-) \\ &= \frac{1}{2}\left\{f_1\not{n}_+ + \mathrm{i}h_1^\perp \frac{[\not{p}_T, \not{n}_+]}{2M}\right\} + \frac{M}{2P^+}\left\{e + f^\perp\frac{\not{p}_T}{M} + \mathrm{i}h\frac{[\not{n}_+, \not{n}_-]}{2} + g^\perp\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M}\right\},\end{aligned} \tag{5}$$

where the new function $g^\perp$ was introduced. The functions on the right-hand side depend on $x$ and $p_T^2$ and they are explicitly

$$f_1\big(x, p_T^2\big) = 2P^+\int \mathrm{d}p^-\,(A_2 + xA_3), \qquad h_1^\perp\big(x, p_T^2\big) = 2P^+\int \mathrm{d}p^-\,(-A_4),$$

$$e(x, p_T^2) = 2P^+ \int \mathrm{d}p^-\, A_1, \qquad f^\perp(x, p_T^2) = 2P^+ \int \mathrm{d}p^-\, A_3,$$
$$h(x, p_T^2) = 2P^+ \int \mathrm{d}p^- \left(\frac{p\cdot P - xM^2}{M^2} A_4 + B_2 + xB_3\right), \qquad g^\perp(x, p_T^2) = 2P^+ \int \mathrm{d}p^-\, B_4.$$

The last function has never been discussed in the literature so far, but it could correspond to the object calculated in the framework of the diquark model in Refs. [23,24], as we shall see after we study the expression for the asymmetry.

The structure of the fragmentation correlator $\Delta$ is analogous to that of $\Phi$, including in particular the presence of a new fragmentation function $G^\perp$. The complete expression up to subleading twist is

$$\begin{aligned}\Delta^{[-]}(z, k_T) &\equiv \int \mathrm{d}k^+\, \Delta^{[-]}(P_h, k; n_+)\\ &= z\left\{D_1 \not{n}_- + \mathrm{i} H_1^\perp \frac{[\not{k}_T, \not{n}_-]}{2M_h}\right\} + \frac{zM_h}{P_h^-}\left\{E + D^\perp \frac{\not{k}_T}{M_h} + \mathrm{i}H \frac{[\not{n}_-, \not{n}_+]}{2} + G^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\right\}. \end{aligned}\tag{6}$$

The transverse gauge link leads to full color gauge invariant expressions at leading and next-to-leading order for the hadronic tensor. The tree level result at leading and next-to-leading order was given by Ref. [20], Eq. (73). In that paper the need to consider transverse gluon fields at infinity was mentioned but the transverse gauge link was not taken into account. Taking this link into account, which allows T-odd distribution functions including $g^\perp$, does not change the procedure of obtaining the hadronic tensor (compare the expressions for the hadronic tensor given in Ref. [25] with the ones in Ref. [20]). The main difference is the inclusion of the new distribution and fragmentation functions. We obtain (using a notation similar to that of Ref. [25])

$$\begin{aligned}2MW^{\mu\nu} = &\int \mathrm{d}^2p_T\, \mathrm{d}^2k_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\\ &\times \mathrm{Tr}\bigg[\Phi^{[+]}(x, p_T)\gamma^\mu \Delta^{[-]}(z, k_T)\gamma^\nu - \gamma_\alpha \frac{\not{n}_+}{Q\sqrt{2}}\gamma^\nu \Phi^{[+]}_{\partial^{-1}G}{}^{\alpha}(x, p_T)\gamma^\mu \Delta(z, k_T)\\ &- \gamma^\alpha \frac{\not{n}_-}{Q\sqrt{2}}\gamma^\mu \Delta^{[-]}_{\partial^{-1}G}{}^{\alpha}(z, k_T)\gamma^\nu \Phi^{[+]}(x, p_T) + (\mu \leftrightarrow \nu)^*\bigg],\end{aligned}\tag{7}$$

where the $(\mu \leftrightarrow \nu)^*$ acts on the last two terms only and

$$\begin{aligned}&\big(\Phi^{[\pm]}_{\partial^{-1}G}\big)^\alpha_{ij}(x, p_T)\\ &\quad= \int \mathrm{d}p^- \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} e^{\mathrm{i}p\xi} \langle P, S|\bar{\psi}_j(0) \int_{\pm\infty}^{\xi^-} \mathrm{d}\eta^-\, U^{[\pm]}(0,\eta) G^{+\alpha}(\eta) U^{[\pm]}(\eta, \xi)\psi_i(\xi)|P, S\rangle\bigg|_{\substack{\eta^+=\xi^+\\ \eta_T=\xi_T}},\end{aligned}\tag{8}$$

$$\Phi^{[+]}_{\partial^{-1}G}{}^{\alpha}(x, p_T) = \Phi_D^{[+]\alpha}(x, p_T) - \Phi_\partial^{[+]\alpha}(x, p_T),\tag{9}$$

$$\Delta^{[-]}_{\partial^{-1}G}{}^{\alpha}(z, k_T) = \Delta_D^{[-]\alpha}(z, k_T) - \Delta_\partial^{[-]\alpha}(z, k_T).\tag{10}$$

Note that in the derivation of the last two equations we made use of identities which also relates the Qiu–Sterman mechanism to the Sivers effect [25–27].

Certain traces of correlation functions which contain a covariant derivative can be related to distribution and fragmentation functions by using the equations of motion. Including T-odd and longitudinal target polarization we obtain for the distribution functions

$$\frac{1}{2}\mathrm{Tr}\big[\Phi^{[+]}_{\partial^{-1}G}{}^{\alpha}\sigma_\alpha^{\ +}\big] = \mathrm{i}(Mxe - mf_1 - \mathrm{i}Mxh) - \frac{p_T^2}{M}h_1^\perp,\tag{11}$$

$$\frac{1}{2}\,\mathrm{Tr}\big[\Phi^{[+]}_{\partial^{-1}G}{}^{\alpha}\mathrm{i}\sigma^{+}_{\alpha}\gamma_5\big] = -mS_L g_{1L} + \mathrm{i}MxS_L e_L + MxS_L h_L - \frac{p_T^2}{M}S_L h_{1L}^{\perp}, \tag{12}$$

$$\begin{aligned}\frac{1}{2}\,\mathrm{Tr}\big[\Phi^{[+]}_{\partial^{-1}G}{}^{\alpha}\gamma^{+}\big] &= \frac{1}{2}\mathrm{i}\epsilon_T^{\alpha\beta}\,\mathrm{Tr}\big[\Phi^{[+]}_{\partial^{-1}G\,\beta}\gamma^{+}\gamma_5\big] + p_T^{\alpha}\left(xf^{\perp} + \mathrm{i}\frac{m}{M}h_1^{\perp} + \mathrm{i}xg^{\perp} - f_1\right)\\ &\quad - \epsilon_T^{\alpha\beta}p_{T\beta}\left(xS_L f_L^{\perp} - \mathrm{i}\frac{m}{M}S_L h_{1L}^{\perp} + \mathrm{i}xS_L g_L^{\perp} - \mathrm{i}S_L g_{1L}\right).\end{aligned} \tag{13}$$

Using these identities we can calculate the hadronic tensor in Eq. (7) by using FORM [28]. We obtain the unpolarized parts of Eq. (77) and Eq. (78) of Mulders and Tangerman [20] (denoted by $2MW_{\mathrm{U}}^{[\mathrm{MT}]\mu\nu}$) together with some extra terms

$$\begin{aligned}2MW_{\mathrm{U}}^{\mu\nu} &= 2MW_{\mathrm{U}}^{[\mathrm{MT}]\mu\nu}\\ &\quad + 2z_h\int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\\ &\quad \times\bigg\{-\big(g_{\perp}^{\mu\nu}\boldsymbol{k}_{\perp}\cdot\boldsymbol{p}_{\perp} + k_{\perp}^{\{\mu}p_{\perp}^{\nu\}}\big)\frac{1}{MM_h}h_1^{\perp}H_1^{\perp} + p_{\perp}^{\{\mu}t^{\nu\}}\frac{2\boldsymbol{k}_T^2}{MM_hQ}h_1^{\perp}H_1^{\perp}\\ &\quad + p_{\perp}^{\{\mu}t^{\nu\}}\frac{2M_h}{z_hMQ}h_1^{\perp}H + k_{\perp}^{\{\mu}t^{\nu\}}\frac{2xM}{M_hQ}hH_1^{\perp} - t^{[\mu}p_{\perp}^{\nu]}\frac{2\mathrm{i}m}{MQ}h_1^{\perp}D_1\\ &\quad + t^{[\mu}p_{\perp}^{\nu]}\frac{2\mathrm{i}M_h}{z_hMQ}h_1^{\perp}E - t^{[\mu}p_{\perp}^{\nu]}\frac{2\mathrm{i}}{Q}xg^{\perp}D_1 - t^{[\mu}k_{\perp}^{\nu]}\frac{2\mathrm{i}}{Q}f_1\frac{G^{\perp}}{z}\bigg\}.\end{aligned} \tag{14}$$

Notice that the hadronic tensor we obtain is electromagnetic gauge invariant ($q_\mu W^{\mu\nu} = 0$). Gauge invariance is insured thanks to the contribution of the quark–gluon–quark correlator $\Phi_{\partial^{-1}G}$.

Unpolarized T-odd distribution functions can be measured for instance in beam single-spin asymmetries. The polarization of the beam forms an antisymmetric structure that has to be contracted with the antisymmetric part of $W^{\mu\nu}$. This part consists of either T-odd distribution functions with T-even fragmentation functions or vice versa.

We find that the $A_{LU}$ asymmetry is given by[1]

$$A_{LU} = \frac{(L_{\mu\nu}^{\lambda_e=1} - L_{\mu\nu}^{\lambda_e=-1})2MW_{\mathrm{U}}^{\mu\nu}}{\int \mathrm{d}^2P_h^{\perp}\,(L_{\mu\nu}^{\lambda_e=1} + L_{\mu\nu}^{\lambda_e=-1})2MW_{\mathrm{U}}^{\mu\nu}} = \frac{2y\sqrt{1-y}}{(1-y+y^2/2)f_1D_1}\sin\phi_h\frac{M}{Q}\mathcal{A}, \tag{15}$$

where

$$\mathcal{A} = \mathcal{I}\left\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_{\perp}}{M_h}\left[\left(xe - \frac{m}{M}f_1\right)H_1^{\perp} + \frac{M_h}{M}f_1\frac{G^{\perp}}{z}\right] - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_{\perp}}{M}\left[\frac{M_h}{M}h_1^{\perp}\left(\frac{E}{z} - \frac{m}{M_h}D_1\right) - xg^{\perp}D_1\right]\right\}. \tag{16}$$

Here we introduced the symbol $\hat{\boldsymbol{h}} = \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$ and the shorthand notation

$$\mathcal{I}\{\cdots\} \equiv \int \mathrm{d}^2\boldsymbol{p}_T\,\mathrm{d}^2\boldsymbol{k}_T\,\delta^{(2)}\left(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_T\right)\{\cdots\}. \tag{17}$$

Preliminary measurements of this asymmetry have been presented by the CLAS and HERMES Collaborations [4,29]. The interpretation of such asymmetry has to take into account the possible contribution of $g^{\perp}$ and $G^{\perp}$.

From now on we will avoid writing explicitly the charge weighted summation over the quark flavors and omit the flavor indices of the functions.

[1] We use the same definition of azimuthal angles as in Ref. [20].

To deconvolute the $\sin\phi_h$ asymmetry in a clean manner, it is necessary to introduce a unit vector $\hat{\boldsymbol{a}}$ (fixed with respect to the lepton scattering plane), weight the asymmetry with $\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}$ and integrate over $P_{h\perp}$. Defining

$$A_{\cdots}^{\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}} = \int \mathrm{d}^2 P_{h\perp}\,(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}})A\cdots, \tag{18}$$

we find that

$$\begin{aligned} A_{LU}^{\boldsymbol{P}_h^{\perp}\cdot\hat{\boldsymbol{a}}} &= \frac{2y\sqrt{1-y}}{(1-y+y^2/2)\,f_1D_1}\sin\phi_a \\ &\quad\times\frac{MM_h}{Q}\left[\frac{m}{M}zf_1H_1^{\perp(1)} - \frac{M_h}{M}f_1G^{\perp(1)} - xzeH_1^{\perp(1)} + \frac{m}{M_h}zh_1^{\perp(1)}D_1 - h_1^{\perp(1)}E + \frac{M}{M_h}xzg^{\perp(1)}D_1\right]. \end{aligned} \tag{19}$$

The distribution and fragmentation functions on the right-hand side depend only on $x$ and $z$, respectively. The asymmetry is maximized by choosing $\hat{\boldsymbol{a}}$ perpendicular to the lepton scattering plane, one obtains ($\phi_a = \pi/2$)

$$\begin{aligned} A_{LU}^{|\boldsymbol{P}_h^{\perp}|\sin\phi_h} &= \frac{2y\sqrt{1-y}}{(1-y+y^2/2)\,f_1D_1}\frac{MM_h}{Q} \\ &\quad\times\left[\frac{m}{M}zf_1H_1^{\perp(1)} - \frac{M_h}{M}f_1G^{\perp(1)} - xzeH_1^{\perp(1)} + \frac{m}{M_h}zh_1^{\perp(1)}D_1 - h_1^{\perp(1)}E + \frac{M}{M_h}xzg^{\perp(1)}D_1\right]. \end{aligned} \tag{20}$$

Apart from the presence of $g^{\perp}$, $G^{\perp}$, the terms with quark masses, and a factor 2 difference in the definition, the expression for the weighted asymmetry corresponds to Eq. (21) of Ref. [16] (the different sign is due to a different definition of the azimuthal angle). A similar result was also obtained in Ref. [15].

In jet semi-inclusive DIS with massless quarks $H_1^{\perp}$, $E$ and $G^{\perp}$ vanish and $D_1$ reduces to $\delta(1-z_h)$. The asymmetries are in that case directly proportional to the T-odd distribution function $g^{\perp}$.

$$A_{LU,j}^{|\boldsymbol{P}_{h\perp}|\sin\phi_h} = \frac{M^2}{Q}\frac{2y\sqrt{1-y}}{(1-y+y^2/2)}\frac{xg^{\perp(1)}}{f_1}. \tag{21}$$

In Refs. [23,24] model calculations of this jet asymmetry have been studied. Without the introduction of the function $g^{\perp}$ this asymmetry would vanish, therefore suggesting a connection between the model calculations of the asymmetry and the function $g^{\perp}$. An experimental study of this asymmetry in jet semi-inclusive DIS (e.g., at ZEUS, H1 or at a future facility as eRHIC) would be important to establish if the functions $g^{\perp}$ exists. Its measurement would allow also a cleaner study of the terms containing the functions $e$ and $h_1^{\perp}$ in the asymmetry of Eq. (20). Note that perturbative contributions to this asymmetry have also to be taken into account [30,31].

## 3. Target polarized along the virtual photon

So far, no complete study has been performed including the T-odd distribution function $f_L^{\perp}$ and $G^{\perp}$. When taking longitudinal target polarization into account, the use of the vector $n_-$ in this case generates no other structures than the ones already presented in Ref. [14], even though it changes the relation between the distribution functions and the amplitudes, invalidating Lorentz invariance relations.

We find that the longitudinal polarized parts of Eq. (77) and Eq. (78) of Ref. [20] (denoted by $2MW_{\mathrm{L}}^{[\mathrm{MT}]\mu\nu}$)

$$\begin{aligned} 2MW_{\mathrm{L}}^{\mu\nu} &= 2MW_{\mathrm{L}}^{[\mathrm{MT}]\mu\nu} - \frac{4}{Q}S_L\epsilon_{\perp}^{\rho\{\mu}k_{\perp\rho}t^{\nu\}}g_{1L}G^{\perp} + \frac{4xz}{Q}S_L\epsilon_{\perp}^{\rho\{\mu}p_{\perp\rho}t^{\nu\}}f_L^{\perp}D_1 \\ &\quad + \frac{4\mathrm{i}xzM}{M_hQ}S_L\epsilon_{\perp}^{\rho[\nu}k_{\perp\rho}t^{\mu]}e_LH_1^{\perp}. \end{aligned} \tag{22}$$

The asymmetry reads

$$A_{UL} = \frac{L^{\mathrm{U}}_{\mu\nu}(2MW^{\mu\nu}_{S_L=1} - 2MW^{\mu\nu}_{S_L=-1})}{\int \mathrm{d}^2 P_h^\perp\, L^{\mathrm{U}}_{\mu\nu}(2MW^{\mu\nu}_{S_L=1} + 2MW^{\mu\nu}_{S_L=-1})}$$
$$= \frac{1}{(1-y+y^2/2)\, f_1 D_1}\left[(1-y)\sin 2\phi_h\, \mathcal{B} + 2(2-y)\sqrt{1-y}\,\frac{M}{Q}\sin\phi_h\, \mathcal{C}\right], \tag{23}$$

where

$$\mathcal{B} = \mathcal{I}\left\{\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp) - \boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{MM_h} h_{1L}^\perp H_1^\perp\right\}, \tag{24}$$

$$\mathcal{C} = \mathcal{I}\left\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M_h}\left[\left(xh_L - \frac{m}{M}g_{1L}\right)H_1^\perp + \frac{M_h}{M}g_{1L}\frac{G^\perp}{z}\right] + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M}\left[\frac{M_h}{M}h_{1L}^\perp\frac{\tilde{H}}{z} - xf_L^\perp D_1\right]\right\}, \tag{25}$$

where we introduced the function $\tilde{H} = H + H_1^\perp z\boldsymbol{k}_\perp^2/M_h^2$.

Following the same steps as described in the previous section to deconvolute the $\sin\phi_h$ asymmetry, we find

$$A_{UL}^{\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}} = \frac{2(2-y)\sqrt{1-y}}{(1-y+y^2/2)\, f_1 D_1}\sin\phi_a$$
$$\times \frac{MM_h}{Q}\left[\frac{m}{M}zg_1 H_1^{\perp(1)} - \frac{M_h}{M}g_1 G^{\perp(1)} - xzh_L H_1^{\perp(1)} + h_{1L}^{\perp(1)}\tilde{H} - \frac{M}{M_h}xzf_L^{\perp(1)}D_1\right]. \tag{26}$$

Again, the asymmetry is maximized by choosing $\phi_a = \pi/2$. For this particular $\hat{\boldsymbol{a}}$ the weight $\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}$ reduces to $|\boldsymbol{P}_{h\perp}|\sin\phi_h$. Neglecting quark masses, the $\sin\phi_h$ asymmetry for jet production reduces to

$$A_{UL,j}^{|\boldsymbol{P}_{h\perp}|\sin\phi_h} = -\frac{M^2}{Q}\frac{2(2-y)\sqrt{1-y}}{(1-y+y^2/2)}\frac{xf_L^{\perp(1)}}{f_1}. \tag{27}$$

This is the situation studied in the model calculations of Ref. [24].

To deconvolute the $\sin 2\phi_h$ term we introduce a new unit vector $\hat{\boldsymbol{b}}$ and weight with $(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}})(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{b}})$. We obtain

$$A_{UL}^{(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}})(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{b}})} = MM_h\frac{2z^2(1-y)\sin(\phi_a+\phi_b)}{(1-y+y^2/2)\, f_1 D_1}h_{1L}^{\perp(1)}H_1^{\perp(1)}. \tag{28}$$

Choosing $\hat{\boldsymbol{a}}$ perpendicular and $\hat{\boldsymbol{b}}$ tangent to the lepton scattering plane, one finds the maximal asymmetry

$$A_{UL}^{\boldsymbol{P}_{h\perp}^2\sin(2\phi_h)} = MM_h\frac{4z^2(1-y)}{(1-y+y^2/2)\, f_1 D_1}h_{1L}^{\perp(1)}H_1^{\perp(1)}. \tag{29}$$

## 4. Target polarized along the beam

In experiments the target is polarized along the beam direction and not along the virtual photon direction (we will denote the longitudinal polarization along the beam as $L'$ to distinguish it from that along the virtual photon, $L$). To write the complete $UL'$ asymmetry, therefore, we should include also the leading twist part of the $UT$ asymmetry, which appears with a $1/Q$ suppression [32]. When dealing with the $UT$ asymmetry, we have to check whether the introduction of the $n_-$ vector in the parameterization of the correlator generates new structures or not. It turns out that some new structures appear at subleading twist, and one new structure appears also at leading twist: it is the T-even and chiral-odd structure $[\not{p}_T, \not{n}_+]\epsilon_T^{\rho\sigma} p_\rho S_\sigma$. However, for the $A_{UL'}$ asymmetry this new term is indistinguishable from the transversity and it can absorbed into its definition, leading to no extra distribution functions.

The final answer is

$$
\begin{aligned}
A_{UL'} &= \frac{L^{\mathrm{U}}_{\mu\nu}\big(2MW^{\mu\nu}_{S'_L=1} - 2MW^{\mu\nu}_{S'_L=-1}\big)}{\int \mathrm{d}^2 P_{h\perp}\, L^{\mathrm{U}}_{\mu\nu}\big(2MW^{\mu\nu}_{S'_L=1} + 2MW^{\mu\nu}_{S'_L=-1}\big)} \\
&= \frac{1}{(1-y+y^2/2)\, f_1 D_1}\bigg\{(1-y)\sin 2\phi_h \mathcal{B} \\
&\quad + 2\sqrt{1-y}\frac{M}{Q}\bigg[\sin\phi_h\bigg((2-y)\mathcal{C} - (1-y)\mathcal{D} - \bigg(1-y+\frac{y^2}{2}\bigg)\mathcal{E}\bigg) - \sin 3\phi_h (1-y)\mathcal{F}\bigg]\bigg\}, \qquad (30)
\end{aligned}
$$

where

$$\mathcal{D} = \mathcal{I}\bigg\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M_h} x h_1 H_1^\perp\bigg\}, \qquad (31)$$

$$\mathcal{E} = \mathcal{I}\bigg\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M} x f_{1T}^\perp D_1\bigg\}, \qquad (32)$$

$$\mathcal{F} = \mathcal{I}\bigg\{\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)^2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp) - 2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)(\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp) - \boldsymbol{p}_\perp^2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp)}{2M^2 M_h} x h_{1T}^\perp H_1^\perp\bigg\}. \qquad (33)$$

This asymmetry has been measured by the HERMES Collaboration [1–3]. However, none of the interpretations given so far takes into account the contribution of the function $f_L^\perp$ and $G^\perp$ in $\mathcal{C}$, while only a few discuss the contribution of the Sivers function $f_{1T}^\perp$ in $\mathcal{E}$ [11,12].

The single and double weighted asymmetries read

$$
\begin{aligned}
A_{UL'}^{\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}} &= A_{UL}^{\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}}} + \frac{2\sqrt{1-y}}{(1-y+y^2/2)\, f_1 D_1}\sin\phi_a \\
&\quad \times \frac{MM_h}{Q}\bigg[(1-y)\big(xzh_1 H_1^{\perp(1)}\big) - \bigg(1-y+\frac{y^2}{2}\bigg)\bigg(\frac{M}{M_h} xz f_{1T}^{\perp(1)} D_1\bigg)\bigg], \qquad (34)
\end{aligned}
$$

$$A_{UL'}^{(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}})(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{b}})} = A_{UL}^{(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{a}})(\boldsymbol{P}_{h\perp}\cdot\hat{\boldsymbol{b}})}. \qquad (35)$$

The $\sin\phi_h$ asymmetry can be rewritten as

$$
\begin{aligned}
A_{UL}^{|\boldsymbol{P}_{h\perp}|\sin\phi_h} &= \frac{2\sqrt{1-y}}{(1-y+y^2/2)\, f_1 D_1}\frac{MM_h}{Q} \\
&\quad \times \bigg[(2-y)\bigg(\frac{m}{M} z g_1 H_1^{\perp(1)} - \frac{M_h}{M} g_1 G^{\perp(1)} - xzh_L H_1^{\perp(1)} + h_{1L}^{\perp(1)}\tilde{H} - \frac{M}{M_h} xz f_L^{\perp(1)} D_1\bigg) \\
&\quad + (1-y)\big(xzh_1 H_1^{\perp(1)}\big) - \bigg(1-y+\frac{y^2}{2}\bigg)\bigg(\frac{M}{M_h} xz f_{1T}^{\perp(1)} D_1\bigg)\bigg]. \qquad (36)
\end{aligned}
$$

Neglecting quark masses, the asymmetry for jet production reads

$$A_{UL',j}^{|\boldsymbol{P}_{h\perp}|\sin\phi_h} = -\frac{M^2}{Q}\bigg[2\sqrt{1-y}\,\frac{x f_{1T}^{\perp(1)}}{f_1} + \frac{2(2-y)\sqrt{1-y}}{(1-y+y^2/2)}\frac{f_L^{\perp(1)}}{f_1}\bigg]. \qquad (37)$$

The measurement of this asymmetry, at facilities where jet DIS can be performed off polarized nucleons (e.g., eRHIC), would allow to determine the size of the terms that contaminate the single-hadron $A_{UL}$ asymmetry. This asymmetry has been interpreted neglecting the contributions of the Sivers function $f_{1T}^\perp$ and of $f_L^\perp$ and $G^\perp$, leading to predictions about the transverse spin asymmetry $A_{UT}$ that are not in good agreement with preliminary data from the HERMES Collaboration [33]. Finally, we point out *en passant* that jet DIS off *transversely* polarized nucleons ($A_{UT,j}$ asymmetry) would be perhaps the best way to pin down the Sivers function.

## 5. Conclusions

In this Letter we presented a complete study of the semi-inclusive DIS beam and target longitudinal spin asymmetries, $A_{LU}$ and $A_{UL}$, up to subleading order in $1/Q$, including transverse momentum dependent and T-odd distribution and fragmentation functions.

In order to be sure to include all contributions, we performed a new analysis of the quark correlation functions, on the basis of what was suggested in Ref. [19], where the necessity to include the direction of the gauge link as an independent degree of freedom in the decomposition of the correlation function was advocated. This revealed the existence of a new distribution function never discussed before, which we named $g^{\perp}$, and the analogous fragmentation function $G^{\perp}$. The new functions are T-odd, depend on transverse momentum, are $1/Q$ suppressed and require no hadron polarization. The very existence of these functions is related to the fundamental importance of the gauge link in the definition of the correlation functions and in particular to observable evidences of the light-cone direction the gauge link runs along.

Both functions turn out to contribute to the beam single spin asymmetry, $A_{LU}$. The present description of such asymmetry [16] is therefore incomplete. In particular, the term containing the function $g^{\perp}$ is the only one that can appear also in jet semi-inclusive DIS, i.e., when the transverse momentum of the jet is observed, instead of the transverse momentum of one hadron. Recent model calculations [23,24] showed the occurrence of nonzero beam longitudinal spin asymmetries in jet DIS. The connection between those model calculations and our formalism has still to be carried out. However, they could possibly constitute a proof the necessity of introducing the function $g^{\perp}$ and thereby corroborating the claims of Ref. [19]. An experimental check of a nonzero jet asymmetry would be of great importance, and could be done at ZEUS and H1, or at a new facility such eRHIC.

For what concerns the target longitudinal single spin asymmetry, $A_{UL}$, we have found two extra terms compared to the existing literature, one containing the function $G^{\perp}$ and one containing the T-odd distribution function $f_L^{\perp}$, whose existence was already known but whose contribution to the $A_{UL}$ was so far neglected [10–12]. This finding has to be taken into account in analyses of the asymmetry, and could provide an explanation for the different behavior of the $A_{UL}$ and preliminary $A_{UT}$ asymmetries observed by the HERMES Collaboration.

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