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Determination of the constants of damage models

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Abstract

Damage models are basic elements in numerical simulation materials failure in particular at high strain rates. Many damage models can be found in the literature. However, a few of them such as GNT (Gurson, Tvergaard and Needleman) and Johnson-Cook models have gained wide application in the simulations. The models involve a number of constants to be determined normally by experiment in which void development in the specimen must be considered. This is usually a hard task and the results are always questionable. In this investigation a combined experimental, numerical and optimization technique is employed for identification of the constants of Johnson-Cook material and damage model. The experiments are conducted at low to high strain rate regimes using standard testing devices such Instron and high rate apparatuses such as "Flying Wedge". The experiments are simulated using the same specimen geometries and the apparatus. The simulations are carried out using the commercial codes, Ls-dyna. The differences between the deformed shapes of the specimens from the experiments and those predicted from the numerical simulations are taken as the objective function for optimization purposes. The optimum constants are obtained using generic algorithm.

Keywords: Constants of damage model, Genetic algorithm, Simulation, Johnson-Cook model

1. Introduction

The capabilities of the numerical codes such as ANSYS, ABAQUS, NASTRAN, etc have reached to the point that they can predict the sever deformation of materials with high geometry complexity under various loadings. However, these codes still lack comprehensive failure models which are required in design of engineering components. The reason is that failure is physically a very complicated process. The mechanisms of ductile and brittle fracture are quite different and depend on various parameters such as strain rate, temperature, etc. The failure of some materials may change from ductile fracture under quasi-static loading to a brittle one at high strain rates. Therefore, material failure models have drawn the attention of many researchers over past decades and a number of models have been developed by different authors. In this work, only the models which deal only with ductile fracture are studied. The models can basically be divided into two categories: microscopic models based on continuum mechanics and macroscopic constitutive models based purely on experimental data. The macroscopic constitutive failure models such as maximum shear stress, constant fracture strain, Wilkins et al [1], Johnson-Cook

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Keywords: Constants of damage model, Genetic algorithm, Simulation, Johnson-Cook model
[2], Cockcroft-Latham [3] and Xue-Wierzbicki [4] fracture models have been evaluated numerically and experimentally by Wierzbicki et al [5] and Teng & Wierzbicki [6]. Teng & Wierzbicki implemented six fracture models into ABAQUS/Explicit and applied to model the failure processes of steel and an aluminum target plate impacted by a projectile. A few of the failure models are reviewed in more details in section 2.

2. Failure models

As stated in section 1, a variety of damage models are available in the literature. However, only a few of them have gained more application in the numerical codes. In this section, a few of macroscopic models such as Johnson-Cook, Cockcroft-Latham, Bao-Wierzbicki and Wilkins models and an analytical approach such as GNT model are described.

2.1. Johnson-Cook failure model

Johnson-Cook model is described as follows [2]:

\[
\varepsilon_f = \left( D_1 + D_2 \exp \left( D_3 (P / \sigma_y) \right) \right) \left( 1 + D_4 \ln \varepsilon \right) \left( 1 + D_5 T^* \right)
\]

(1)

In which \( P / \sigma_y \) is stress triaxiality parameter and \( \varepsilon \) is strain rate. The constants \( D_1 \) through \( D_5 \) are material constants and obtained from experiment. The quantity described by \( D = \sum \frac{\Delta \varepsilon}{\varepsilon_f} \) called Damage parameter is a function of strain rate and stress triaxiality coefficient. In this relationship \( \Delta \varepsilon = 2 \left[ \sum (d \varepsilon, -d \varepsilon) \right]^{1/2} \) is the plastic strain increment in each repetition and \( \varepsilon_f \) is the fracture strain. When Damage parameter reaches unity failure will occur and the failed element will vanish.

2.2. GNT analytical failure model

GNT (Gurson, Tvergaard and Needleman) is defined as follows [7]:

\[
\phi = \left( \frac{\sigma}{\sigma_0(\bar{\varepsilon})} \right)^2 + 2q_f \cosh \left[ \frac{3 \sigma_n}{2 \sigma} \right] \left( 1 + q_i \bar{\varepsilon}^2 \right) = 0
\]

(2)

In which \( \sigma \) is the equivalent stress, \( \sigma_0(\bar{\varepsilon}) \) is the macroscopic yield stress depending on the microscopic equivalent plastic strain \( \bar{\varepsilon} \) through the hardening law: \( \sigma_0 = \sigma_0(\bar{\varepsilon}) \). \( q_i \) is the material parameter to be determined by experiment. \( \sigma_n \) is the mean hydrostatic pressure. \( f \) defines the void fraction which is considered as an internal variable of the model. Tvergaard and Needleman developed a failure model in which void volume is considered as an independent parameter to indicate the material failure. This model is expressed as follows:

\[
\dot{f} = \dot{f}_n + \dot{f}_g
\]

(3)

Where \( \dot{f} \) is the total rate of void volume growth which is the summation of the \( \dot{f}_n \) as the void nucleation rate and \( \dot{f}_g \) as the void growth rate. Considering the incompressibility of the material, void growth rate could be obtained by the following relationship:

\[
\dot{f}_g = (1 - f) \dot{\eta}
\]

(4)
Where $\eta$ is the plastic volume strain rate. On the other hand, void nucleation rate is determined by the following equation:

$$f_n = A \dot{\varepsilon}$$  \hspace{1cm} (5)

In which $\dot{\varepsilon}$ is the effective plastic strain rate and $A$ is as follows:

$$A = \frac{f_n}{S_N \sqrt{2\pi}} \exp \left[ -1 \left( \frac{\varepsilon - \varepsilon_N}{S_N} \right)^2 \right]$$  \hspace{1cm} (6)

In this relationship $f_n$ and $S_N$ indicate the voids volume at the fracture moment and void nucleation, respectively. Besides that, $\varepsilon$ is the plastic strain and $\varepsilon_N$ which is usually constant is the average strain at the time of nucleation.

2.3. Modified Cockcroft-Latham fracture criterion

Cockcroft and Latham [3] suggested that the critical damage which leads to fracture can be described as the integral of the maximum principal stress $\sigma_i$ with respect to the effective plastic strain.

$$D_\alpha = \int_0^\infty \langle \sigma_i \rangle d\bar{\varepsilon}_{pl}$$  \hspace{1cm} (7)

Where $\langle \rangle$ is the Macauley bracket which can be defined as:

$$\langle S \rangle = \begin{cases} S & \text{if } S > 0 \\ 0 & \text{if } S \leq 0 \end{cases}$$

This criterion named as Cockcroft-Latham (CL) fracture criterion has the energy density dimension. CL criterion was later modified by normalizing the maximum principal stress by the equivalent stress [6].

$$D_\alpha = \int_0^\infty \frac{\langle \sigma_i \rangle}{\sigma} d\bar{\varepsilon}_{pl}$$  \hspace{1cm} (8)

As it is observable from the last equation, modified CL fracture model is based on only one constant to indicate the fracture properties of a material. So there are many choices to use for calibration such as upsetting, shear or tension tests. Consequently, the errors increase in prediction of residual velocities for both types of high and low ductile materials results in lack of valid meaning for high velocity impacts applications.

2.4. Bao-Wierzbicki fracture criterion

Bao and Wierzbicki fracture criterion states that if the stress triaxiality coefficient is less than $1/3$, fracture will not occur. This statement that proved later by experiment is the distinguishing feature of this criterion [6].

2.5. Wilkins fracture model

Wilkins fracture model as a damage cumulative criterion is defined as the integral function of the weighted effective plastic strain $\bar{\varepsilon}_{pl}$ [1]:

$$D = \int \bar{\varepsilon}_{pl} w \nu d\bar{\varepsilon}_{pl}$$  \hspace{1cm} (9)
Which should be calculated in critical volume, $R_{cr}$. $w_1$ is related to the pressure and is defined as follows:

$$w_1 = (1/1 + \alpha p)\gamma$$

In which $\alpha$ and $\gamma$ are two material constants. Since $w_1$ is dimensionless, the unit of $\alpha$ should be $Pa^{-1}$. $w_2$ that is related to the ratio of deviatoric principal stresses, is determined by the next equation.

$$w_2 = (2 - A)^\beta$$

In which $\beta$ and $A$ are material constant and the ratio of deviatoric principal stresses, respectively.

$$A = \max \left( \frac{s_2}{s_3}, \frac{s_2}{s_1}, \frac{s_2}{s_1} \right), \ s_3 \leq s_2 \leq s_1$$

It is notable that $A = 1$ for at the symmetry axis of a round bar and $A = 0$ for a round bar under torsion or a plane strain specimen. The critical value of damage $D_{cr}$ which causes the material to fracture only depends on material characteristics and independent of loading condition, geometry or size of the specimen.

3. Johnson-cook material model

In addition to material damage model, material model is also an essential requirement in any simulation of deformation. The model of most of metals depends on deformation temperature and strain rate. In 1983 Johnson and Cook using Hancock and Mackenzie experiments on variety of metals developed an experimental relationship which states the influences of temperature, strain and strain rate on Von Mises stress. This relationship which has many applications in numerical simulation of material behavior is expressed in the following [8]:

$$\sigma = (A + B \varepsilon_p^m) \left( 1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \left[ 1 - \left( \frac{T^*}{T_m} \right)^n \right]$$

(10)

Where $\varepsilon_p$ is the effective plastic strain, $B$, $C$, $m$, $n$ and $A$ are five material constants, and can be obtained from experiment. $\dot{\varepsilon}$ and $\dot{\varepsilon}_0$ are the current and reference strain rates. $T^*$ is the homologous temperature is defined by $T^* = \frac{T_0 - T_r}{T_m - T_r}$, in which $T_r$ and $T_m$ are room and material melting temperatures, respectively. The first term at the right hand side of equation 10 shows the relation between semi-static stress and strain at the room temperature. The second term states the effect of strain rate and the last term illustrates the effect of temperature.

4. Determination of the constants

There are various techniques for determination of the constants of material failure models. Majzoobi et al. [9 &10] obtained the variation of fracture strain versus stress triaxiality coefficient for steel and copper specimens of different notch radii. The results were used for determining the coefficients $D_i$ to $D_3$ in Johnson-Cook failure model [2]. They also obtained the variation of fracture strain versus $\ln \dot{\varepsilon}$ under dynamic test conditions. The results were employed for determining the coefficient $D_i$ in Johnson-Cook failure model. Ochewit et al. [11] used a different approach for identification of the constants of Gurson model. They used the elongation of tensile specimens as obtained from experiment and numerical simulation. The constants $q$, $S_n$, $\alpha$, $f_0$ and $f_n$ were adjusted to provided the same elongation in the simulations as measured from the experiment. The constants $f_i$ and $f_f$ were obtained by optimizing the difference between the experimental and numerical elongation of the specimens. In
another investigation, Kunna and Springmann [12] obtained the constants of Gurson model using load-displacement diagram measured from the experiments and a non-linear optimization technique. They considered the difference between the experimental and numerical load-displacement diagram at some specific points on the diagram as the objective function for optimization purpose as follows:

$$\phi(p) = \frac{1}{2} \sum_{i=1}^{n_i} \left[ f_i(p) - \bar{f}_i(p) \right]^2$$

In which $n_i$ is the number of the points, $\phi(p)$ is the objective function and $f_i(p)$ and $\bar{f}_i(p)$ are the experimental and numerical loads, respectively. In another investigation, Springmann and Kunna [13] adopted a different technique for identification of the constants of Gurson model. They conducted some tensile tests and measured the displacement of some specific points on the specimen at different loading stages. They also obtained the displacement of the same points by simulation for different sets of the constants. The constants were determined by optimizing the experimental and the numerical displacements by defining the objective function as follows:

$$\phi(p) = \frac{1}{2} \sum_{i=1}^{n_i} \sum_{j=1}^{n_m} \left[ u_{ij}(p) - (u_{ij})_p \right]^2$$

In which $n_i$ is the number of the loading stages, $n_m$ is the number of points, $\phi(p)$ is the objective function and $u_{ij}(p)$ and $(u_{ij})_p$ are the experimental and numerical displacement, respectively. In this work, a similar approach is adopted for determination of the constants of Johnson-Cook model.

5. Tests and Simulation procedures

Five constants are going to be determined for Johnson-cook failure model. The first constants are related to quasi-static conditions. The 4th and 5th constants define the effects of strain rate and temperature. Only the constants $C_1$-$C_4$ are identified and the effect of temperature is neglected in this work. The quasi-static related constants are obtained by a combined experimental/numerical/optimization technique using quasi-static tensile tests. The stress-strain curves are used for identification of Johnson-Cook material model and the deformed profiles of the specimens are employed for determining the constants of failure model.

6. Quasi-static tests

Quasi-static tests were carried out on an Instron tensile testing machine. Specimen geometries are illustrated in Fig. 1. Two types of specimen geometries are used for the experiments. The plain specimen is used for determination of the constants of Johnson-Cook material model and the notched specimens are used for identification of Johnson-Cook failure model.

![Fig. 1: Notched and plain Specimens for quasi-static tests](image-url)

A typical stress-strain curve is depicted in Fig. 2. The figure shows the engineering and true stress-strain curves on the same graph. The tests were performed at a velocity of 0.01 mm/sec. The first three constants of Johnson-
Cook material model, \((A, B, \text{ and } n)\) were obtained by fitting the stress-strain curve to the first term in the model. These are: \(A=400, B=920, n=0.15\).

![Fig. 2: A typical stress-strain curve from quasi-static tests](image)

7. Numerical simulations of quasi static tests

Numerical simulations are carried out using Ls-dyna hydrocode [14]. Because the plastic deformation is localized in the notch area and due to the symmetry of the notch, only \(\frac{1}{4}\) of the notch gauge length is considered in the simulations. The finite element model of the notch is shown in Fig. 3.

![Fig. 3: Finite element model of the specimens for numerical simulations](image)

The simulations are carried out for 10 sets of constants given in Table 1. From the simulations, the final notch gauge length and root radius is measured for optimization purposes.

Table 1. A number of different sets of constants used for simulations

<table>
<thead>
<tr>
<th>(D_1)</th>
<th>(D_2)</th>
<th>(D_3)</th>
<th>(D_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>-0.5</td>
<td>1.5925</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>-0.1</td>
<td>-0.5425</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>-0.15</td>
<td>-0.2975</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4</td>
<td>-0.1</td>
<td>0.2125</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1</td>
<td>-0.1</td>
<td>-0.2575</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>-0.05</td>
<td>0.6475</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>-0.1</td>
<td>0.2175</td>
</tr>
<tr>
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<td>0.5</td>
<td>-0.1</td>
<td>0.4325</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>-0.75</td>
<td>0.4475</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>-0.05</td>
<td>1.0275</td>
</tr>
</tbody>
</table>
8. Optimization

In order to identify the constants of the failure model, the reduction of the notch root radius, $\Delta d$, and the elongation of the notch gauge length, $\Delta L$, were measured after fracture of the specimens using a projector. The objective function is defined as follows:

$$OBJ = \frac{OBJ^1 + OBJ^2}{2}$$

$$OBJ^1 = \Delta L_{\text{experimental}} - \Delta L_{\text{numerical}}$$

$$OBJ^2 = \Delta d_{\text{experimental}} - \Delta d_{\text{numerical}}$$

In which $\Delta L_{\text{experimental}}$, $\Delta L_{\text{numerical}}$, $\Delta d_{\text{experimental}}$ and $\Delta d_{\text{numerical}}$ are given by:

$$\Delta L_{\text{experimental}} = L_f - L_i$$

$$\Delta L_{\text{numerical}} = L_f(p) - L_i$$

$$\Delta d_{\text{experimental}} = d_f - d_i$$

$$\Delta d_{\text{numerical}} = d_f(p) - d_i$$

Where $L_f$ and $L_f(p)$ are the notch final gauge length from the experiment and the simulation, respectively. $L_i$ is the initial gauge length of the notch. $d_f(p)$ and $d_i$ have similar definitions where $d$ denotes the notch root diameter. A non-linear function is assumed for the objective function as follows:

$$OBJ(x) = a_0 + \sum_{i=1}^{N} a_i x_i + \sum_{i,j=1}^{N} b_{ij} x_i x_j$$

(11)

In which $x_i$ is the unknowns vector (the constants of failure model which are D1-D3 in this case) and the coefficients $a_i$, $a_i$ and $b_{ij}$ must be evaluated. For three unknowns, the expansion of Eq. (10) becomes:

$$OBJ = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_1^2 + a_5 x_2^2 + a_6 x_1 x_2 + a_7 x_1 x_3 + a_8 x_2 x_3$$

(12)

As it is seen, Eq. (11) requires 10 constants to be evaluated. Therefore, 10 simulations and consequently 10 set of the constants of the failure model, as given in Table 1, are needed for the evaluation. Substituting these sets of constants into Eq. (12) yields:

<table>
<thead>
<tr>
<th>1</th>
<th>D_1^{(1)}</th>
<th>D_2^{(1)}</th>
<th>D_3^{(1)}</th>
<th>...</th>
<th>D_1 D_2^{(1)}</th>
<th>D_1 D_3^{(1)}</th>
<th>D_2 D_3^{(1)}</th>
<th>a_0</th>
<th>OBJ^{(1)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D_1^{(2)}</td>
<td>D_2^{(2)}</td>
<td>D_3^{(2)}</td>
<td>...</td>
<td>D_1 D_2^{(2)}</td>
<td>D_1 D_3^{(2)}</td>
<td>D_2 D_3^{(2)}</td>
<td>a_1</td>
<td>OBJ^{(2)}</td>
</tr>
<tr>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>...</td>
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<tr>
<td>1</td>
<td>D_1^{(10)}</td>
<td>D_2^{(10)}</td>
<td>D_3^{(10)}</td>
<td>...</td>
<td>D_1 D_2^{(10)}</td>
<td>D_1 D_3^{(10)}</td>
<td>D_2 D_3^{(10)}</td>
<td>A_0</td>
<td>OBJ^{(10)}</td>
</tr>
</tbody>
</table>

Having solved the above system of equations, we obtain:

$$OBJ = 1.753961538 + 3.548093542 x_1 + 3.363477208 x_2 + 0.762320513 x_3 - 0.840778727 x_1^2 - 0.791666667 x_2^2 - 0.778319088 x_1^3 - 2.208333333 x_2^3 - 1.112849003 x_1 x_2 + 2.11522792 x_1 x_3 + 2.11522792 x_2 x_3$$

(13)
Minimizing Eq. (12) using an optimization technique such as genetic algorithm, the constants of failure model are obtained as follows:

\[ D_1 = 0.352, \quad D_2 = 0.28, \quad D_3 = -0.113 \]

Two typical experimental and numerical notch profiles are compared in Fig. 4. As the figure indicates, the notch profile predicted by simulation and using the optimized constants, matches well with that obtained from the experiment.

9. Dynamic tests

The dynamic tests were carried out using a high rate testing device “Flying wedge” [15] and [16]. A general view of Flying Wedge is shown in Fig. 5. The specimen geometry and its finite element model used for dynamic testing are shown in Fig. 6.
In this section, the strain rate related constants of Johnson-Cook material and failure models denoted by C and \( D_4 \), respectively in Eqs. 10 and 1 are identified. In this case, the objective function, Eq. (11), requires 6 constants to be evaluated similar to that described for identification of the quasi-static related constants. Six sets of arbitrary constants C and \( D_4 \) are considered as given in Table 2. Having solved the resultant system of equations, the objective function is obtained as follows:

\[-0.16406 \cdot 1.39167 X_1 \cdot 9.41667 X_2^2 + 24.79167 X_1^2 + 91.66667 X_2^2 + 83.33333 X_1 X_2 = 0\]  
(14)

Table 2. A number of different sets of constants used for simulation of dynamic testing

<table>
<thead>
<tr>
<th>C</th>
<th>( D_4 )</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>-0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>0.15</td>
<td>-0.05</td>
<td>0.26</td>
</tr>
<tr>
<td>0.07</td>
<td>-0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.02</td>
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</tr>
<tr>
<td>0.07</td>
<td>-0.04</td>
<td>0.15</td>
</tr>
<tr>
<td>0.07</td>
<td>-0.07</td>
<td>0.56</td>
</tr>
</tbody>
</table>

By optimizing, Eq. (14) using genetic algorithm, we obtain:

\[ C = 0.085 \quad D_4 = -0.024 \]

Therefore, disregarding the effect of temperature, the Johnson-Cook material and failure models for the material considered in this work are obtained as follows:

\[ \varepsilon_f = \left( 0.352 + 0.28 \exp \left( -0.113 \left( \frac{P}{\sigma} \right) \right) \right) \left( 1 - 0.24 \ln \dot{\varepsilon} \right) \]  
(15)

\[ \bar{\sigma} = \left( 400 + 920 \varepsilon^{0.15} \right) \left( 1 + 0.085 \ln \dot{\varepsilon} \right) \]  
(16)

Numerical simulations of dynamic tests are performed using Eqs. 15 and 16 using Ls-dyna code. The experimental and numerical notch profiles for a typical dynamic test are compared in Fig. 7. As the figure suggests, the both profiles match well indicating the correct identification of material and failure constants.
10. Conclusions

From the results given above, the following conclusions may be derived:

1. Strain rate related constants of material and failure models are difficult to obtain by experiment.
2. The experimental/numerical/optimization approach adopted in this work, can substitute conventional pure experimental techniques which are time consuming and expensive.
3. The quasi-static related constants of material and failure models can be obtained simply from tensile stress strain curves (from standard tensile tests) and the deformed profile of the specimens.
4. The deformed profile of specimens tested on a high rate testing machine without resorting to stress-strain curve can be used for identification of dynamic related constants.
5. The constants of Johnson-Cook material and failure models were determined in this work. Having used the models obtained in this work, a good agreement between experimental and numerical predictions of deformed profiles of specimens was observed.

References