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Physics Letters B

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Graviton as a Goldstone boson: Nonlinear sigma model for tensor field gravity

J.L. Chkareuli [a,b], J.G. Jejelava [a,b,*], G. Tatishvili [b]

[a] *E. Andronikashvili Institute of Physics, 0177 Tbilisi, Georgia*
[b] *Center for Elementary Particle Physics, ITP, Ilia State University, 0162 Tbilisi, Georgia*

A B S T R A C T

Spontaneous Lorentz invariance violation (SLIV) realized through a nonlinear tensor field constraint $H_{\mu\nu}^2 = \pm M^2$ (M is the proposed scale for Lorentz violation) is considered in tensor field gravity theory, which mimics linearized general relativity in Minkowski space–time. We show that such a SLIV pattern, due to which the true vacuum in the theory is chosen, induces massless tensor Goldstone modes some of which can naturally be associated with the physical graviton. When expressed in terms of the pure Goldstone modes, this theory looks essentially nonlinear and contains a variety of Lorentz and *CPT* violating couplings. Nonetheless, all SLIV effects turn out to be strictly canceled in all the lowest order processes considered, provided that the tensor field gravity theory is properly extended to general relativity (GR). So, as we generally argue, the measurable effects of SLIV, induced by elementary vector or tensor fields, are related to the accompanying gauge symmetry breaking rather than to spontaneous Lorentz violation. The latter appears by itself to be physically unobservable, only resulting in a non-covariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. However, while Goldstonic vector and tensor field theories with exact local invariance are physically indistinguishable from conventional gauge theories, there might appear some principal distinctions if this local symmetry were slightly broken at very small distances in a way that could eventually allow one to differentiate between them observationally.

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1. Introduction

It is no doubt an extremely challenging idea that spontaneous Lorentz invariance violation (SLIV) could provide a dynamical approach to quantum electrodynamics [1], gravity [2] and Yang–Mills theories [3] with photon, graviton and non-Abelian gauge fields appearing as massless Nambu–Goldstone (NG) bosons [4] (for some later developments see [5–9]). This idea has recently gained new impetus in the gravity sector—as for composite gravitons [10], so in the case when gravitons are identified with the NG modes of the symmetric two-index tensor field in a theory preserving a diffeomorphism (diff) invariance, apart from some noninvariant potential inducing spontaneous Lorentz violation [11,12].

We consider here an alternative approach which has had a long history, dating back to the model of Nambu [13] for QED in the framework of nonlinearly realized Lorentz symmetry for the underlying vector field. This may indeed appear through the "length-fixing" vector field constraint

$$A_\mu^2 = n^2 M^2, \quad n^2 \equiv n_\nu n^\nu = \pm 1 \tag{1}$$

(where n_ν is a properly oriented unit Lorentz vector, while M is the proposed scale for Lorentz violation) much as it works in the nonlinear σ-model [14] for pions, $\sigma^2 + \pi^2 = f_\pi^2$, where f_π is the pion decay constant. Note that a correspondence with the nonlinear σ model for pions may appear rather suggestive in view of the fact that pions are the only presently known Goldstone particles whose theory, chiral dynamics [14], is given by the nonlinearly realized chiral $SU(2) \times SU(2)$ symmetry rather than by an ordinary linear σ model.[1] The constraint (1) means in essence that the vector field A_μ develops some constant background value

* Corresponding author at: Center for Elementary Particle Physics, ITP, Ilia State University, 0162 Tbilisi, Georgia.
E-mail address: jjejelava@yahoo.com (J.G. Jejelava).

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[1] Another motivation for the constraint (1) might be an attempt to avoid an infinite self-energy for the electron in classical electrodynamics, as was originally suggested by Dirac [15] (and extended later to various vector field theories [16]) in terms of the Lagrange multiplier term, $\frac{1}{2}\lambda(A_\mu^2 - M^2)$, due to which the constraint (1) appears as an equation of motion for the auxiliary field $\lambda(x)$. Recently, there was also discussed in the literature a special quadratic Lagrange multiplier potential [17], $\frac{1}{4}\lambda(A_\mu^2 - M^2)^2$, leading to the same constraint (1) after varying the action, while the auxiliary λ field completely decouples from the vector field dynamics rather than acting as a source of some extra current density, as it does in the Dirac model. Formally, numbers of independent degrees of freedom in these models appear different from those in the Nambu model [13], where the SLIV constraint is proposed to be substituted into the action prior to varying the action. However, in their ghost-free and stability (positive Hamiltonian) phase space areas [17] both of them are physically equivalent to the Nambu model with the properly chosen initial condition.

$$\langle A_\mu(x) \rangle = n_\mu M \tag{2}$$

and the Lorentz symmetry $SO(1,3)$ formally breaks down to $SO(3)$ or $SO(1,2)$ depending on the time-like ($n^2 > 0$) or space-like ($n^2 < 0$) nature of SLIV. The point is, however, that, in sharp contrast to the nonlinear σ model for pions, the nonlinear QED theory, due to the starting gauge invariance involved, ensures that all the physical Lorentz violating effects turn out to be nonobservable. It was shown [13], while only in the tree approximation and for the time-like SLIV ($n^2 > 0$), that the nonlinear constraint (1) implemented into the standard QED Lagrangian containing a charged fermion field $\psi(x)$

$$L_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}(i\gamma\,\partial + m)\psi - e A_\mu \overline{\psi}\gamma^\mu\psi \tag{3}$$

as a supplementary condition appears in fact as a possible gauge choice for the vector field A_μ, while the S-matrix remains unaltered under such a gauge convention. Really, this nonlinear QED contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of the pure Goldstonic photon modes (a_μ) according to the constraint condition (1)

$$A_\mu = a_\mu + \frac{n_\mu}{n^2}\big(M^2 - n^2 a^2\big)^{\frac{1}{2}}, \qquad n_\mu a_\mu = 0 \quad \big(a^2 \equiv a_\mu a^\mu\big) \tag{4}$$

(for definiteness, one takes the positive sign for the square root when expanding it in powers of a^2/M^2). However, the contributions of all these Lorentz violating couplings to physical processes completely cancel out among themselves. So, SLIV is shown to be superficial as it affects only the gauge of the vector potential A_μ at least in the tree approximation [13].

Some time ago, this result was extended to the one-loop approximation and for both the time-like ($n^2 > 0$) and space-like ($n^2 < 0$) Lorentz violation [18]. All the contributions to the photon–photon, photon–fermion and fermion–fermion interactions violating physical Lorentz invariance happen to exactly cancel among themselves in the manner observed long ago by Nambu for the simplest tree-order diagrams. This means that the constraint (1), having been treated as a nonlinear gauge choice at the tree (classical) level, remains as a gauge condition when quantum effects are taken into account as well. So, in accordance with Nambu's original conjecture, one can conclude that physical Lorentz invariance is left intact at least in the one-loop approximation, provided we consider the standard gauge invariant QED Lagrangian (3) taken in flat Minkowski space–time. Later this result was also confirmed for spontaneously broken massive QED [19] and non-Abelian theories [20] (some interesting aspects of the SLIV conditioned nonlinear QED were also considered in [21]).

Actually, we here use a similar nonlinear constraint for a symmetric two-index tensor field

$$H_{\mu\nu}^2 = \mathfrak{n}^2 M^2, \quad \mathfrak{n}^2 \equiv \mathfrak{n}_{\mu\nu}\mathfrak{n}^{\mu\nu} = \pm 1 \tag{5}$$

(where $\mathfrak{n}_{\mu\nu}$ is now a properly oriented 'unit' Lorentz tensor, while M is the proposed scale for Lorentz violation) which fixes its length in a similar way to the vector field case above. Also, in analogy to the nonlinear QED case [13] with its gauge invariant Lagrangian (3), we propose the linearized Einstein–Hilbert kinetic term for the tensor field, which by itself preserves a diff invariance. We show that such a SLIV pattern (5), due to which the true vacuum in the theory is chosen, induces massless tensor Goldstone modes some of which can naturally be collected in the physical graviton. The linearized theory we start with becomes essentially nonlinear, when expressed in terms of the pure Goldstone modes, and contains a variety of Lorentz (and CPT) violating couplings. However, all SLIV effects turn out to be strictly canceled in physical processes once the tensor field gravity theory (being considered as

the weak-field limit of general relativity (GR)) is properly extended to GR. So, this formulation of SLIV seems to amount to the fixing of a gauge for the tensor field in a special manner making the Lorentz violation only superficial just as in the nonlinear QED framework [13]. From this viewpoint, both conventional QED and GR theories appear to be generic Goldstonic theories in which some of the gauge degrees of freedom of these fields are condensed (thus eventually emerging as a noncovariant gauge choice), while their massless NG modes are collected in photons or gravitons in such a way that the physical Lorentz invariance is ultimately preserved. However, there might appear some principal distinctions between conventional and Goldstonic theories if, as we argue later, the underlying local symmetry were slightly broken at very small distances in a way that could eventually allow us to differentiate between them in an observational way.

The Letter is organized in the following way. In Section 2 we formulate the model for tensor field gravity and find massless NG modes some of which are collected in the physical graviton. Then in Section 3 we derive general Feynman rules for the basic graviton–graviton and graviton–matter (scalar) field interactions in the Goldstonic gravity theory. In essence the model contains two perturbative parameters, the inverse Planck and SLIV mass scales, $1/M_P$ and $1/M$, respectively, so that the SLIV interactions are always proportional to some powers of them. Some lowest order SLIV processes, such as graviton–graviton scattering and graviton scattering off the massive scalar field, are considered in detail. We show that all these Lorentz violating effects, taken in the tree approximation, in fact turn out to vanish so that physical Lorentz invariance is ultimately restored. Finally, in Section 4 we present a resume and conclude.

2. The model

According to our philosophy, we propose to consider the tensor field gravity theory which mimics linearized general relativity in Minkowski space–time. The corresponding Lagrangian for one real scalar field ϕ (representing all sorts of matter in the model)

$$\mathcal{L}(H_{\mu\nu},\phi) = \mathcal{L}(H) + \mathcal{L}(\phi) + \mathcal{L}_{int} \tag{6}$$

consists of the tensor field kinetic terms of the form

$$\mathcal{L}(H) = \frac{1}{2}\partial_\lambda H^{\mu\nu}\partial^\lambda H_{\mu\nu} - \frac{1}{2}\partial_\lambda H_{tr}\partial^\lambda H_{tr} \\ - \partial_\lambda H^{\lambda\nu}\partial^\mu H_{\mu\nu} + \partial^\nu H_{tr}\partial^\mu H_{\mu\nu} \tag{7}$$

(H_{tr} stands for the trace of $H_{\mu\nu}$, $H_{tr} = \eta^{\mu\nu}H_{\mu\nu}$) which is invariant under the diff transformations

$$\delta H_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu, \qquad \delta x^\mu = \xi^\mu(x), \tag{8}$$

together with the free scalar field and interaction terms

$$\mathcal{L}(\phi) = \frac{1}{2}\big(\partial_\rho\phi\partial^\rho\phi - m^2\phi^2\big), \qquad \mathcal{L}_{int} = \frac{1}{M_P} H_{\mu\nu} T^{\mu\nu}(\phi). \tag{9}$$

Here $T^{\mu\nu}(\phi)$ is the conventional energy–momentum tensor for a scalar field

$$T^{\mu\nu}(\phi) = \partial^\mu\phi\partial^\nu\phi - \eta^{\mu\nu}\mathcal{L}(\phi), \tag{10}$$

and the coupling constant in $\mathcal{L}_{int}$ is chosen to be the inverse of the Planck mass M_P. It is clear that, in contrast to the tensor field kinetic terms, the other terms in (6) are only approximately invariant under the diff transformations (8), as they correspond to the weak-field limit in GR. Following the nonlinear σ-model for QED [13], we propose the SLIV condition (5) as some tensor field length-fixing constraint which is supposed to be substituted into the total Lagrangian $\mathcal{L}(H_{\mu\nu},\phi)$ prior to the variation of the action.

This eliminates, as we will see, a massive Higgs mode in the final theory thus leaving only massless Goldstone modes, some of which are then collected in the physical graviton.

Let us first turn to the spontaneous Lorentz violation itself, which is caused by the constraint (5). This constraint can be written in the more explicit form

$$H_{\mu\nu}^2 = H_{00}^2 + H_{i=j}^2 + \left(\sqrt{2}H_{i\neq j}\right)^2 - \left(\sqrt{2}H_{0i}\right)^2$$
$$= \mathfrak{n}^2 M^2 = \pm M^2 \tag{11}$$

(where the summation over all indices $(i, j = 1, 2, 3)$ is imposed) and means in essence that the tensor field $H_{\mu\nu}$ develops the vacuum expectation value (vev) configuration

$$\langle H_{\mu\nu}(x)\rangle = \mathfrak{n}_{\mu\nu} M \tag{12}$$

determined by the matrix $\mathfrak{n}_{\mu\nu}$. The initial Lorentz symmetry $SO(1, 3)$ of the Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ given in (6) then formally breaks down at a scale M to one of its subgroups. If one assumes a "minimal" vacuum configuration in the $SO(1, 3)$ space with the vev (12) developed on a single $H_{\mu\nu}$ component, there are in fact the following three possibilities

(a)　$\mathfrak{n}_{00} \neq 0,$　　$SO(1, 3) \rightarrow SO(3),$

(b)　$\mathfrak{n}_{i=j} \neq 0,$　　$SO(1, 3) \rightarrow SO(1, 2),$

(c)　$\mathfrak{n}_{i\neq j} \neq 0,$　　$SO(1, 3) \rightarrow SO(1, 1)$　　(13)

for the positive sign in (11), and

(d)　$\mathfrak{n}_{0i} \neq 0,$　　$SO(1, 3) \rightarrow SO(2)$　　(14)

for the negative sign. These breaking channels can be readily derived by counting how many different eigenvalues the vev matrix $\mathfrak{n}$ has for each particular cases (a)–(d). Accordingly, there are only three Goldstone modes in the cases (a), (b) and five modes in the cases (c)–(d).[2] In order to associate at least one of the two transverse polarization states of the physical graviton with these modes, one could have any of the above-mentioned SLIV channels except for the case (a). Indeed, it is impossible for the graviton to have all vanishing spatial components, as one needs for the Goldstone modes in case (a). Therefore, no linear combination of the three Goldstone modes in case (a) could behave like the physical graviton (see more detailed consideration in [12]). Apart from the minimal vev configuration, there are many others as well. A particular case of interest is that of the traceless vev tensor $\mathfrak{n}_{\mu\nu}$

$$\mathfrak{n}_{\mu\nu}\eta^{\mu\nu} = 0 \tag{15}$$

in terms of which the Goldstonic gravity Lagrangian acquires an especially simple form (see below). It is clear that the vev in this case can be developed on several $H_{\mu\nu}$ components simultaneously, which in general may lead to total Lorentz violation with all six Goldstone modes generated. For simplicity, we will use this form of vacuum configuration in what follows, while our arguments can be applied to any type of vev tensor $\mathfrak{n}_{\mu\nu}$.

Aside from the pure Lorentz Goldstone modes, the question of the other components of the symmetric two-index tensor $H_{\mu\nu}$ naturally arises. Remarkably, they turn out to be Pseudo-Goldstone modes (PGMs) in the theory. Indeed, although we only propose Lorentz invariance of the Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$, the SLIV constraint (5) formally possesses the much higher accidental symmetry $SO(7, 3)$ of the constrained bilinear form (11), which manifests itself when considering the $H_{\mu\nu}$ components as the "vector" ones under $SO(7, 3)$. This symmetry is in fact spontaneously broken, side by side with Lorentz symmetry, at the scale M. Assuming again a minimal vacuum configuration in the $SO(7, 3)$ space, with the vev (12) developed on a single $H_{\mu\nu}$ component, we have either time-like $(SO(7, 3) \rightarrow SO(6, 3))$ or space-like $(SO(7, 3) \rightarrow SO(7, 2))$ violations of the accidental symmetry depending on the sign of $\mathfrak{n}^2 = \pm 1$ in (11). According to the number of broken $SO(7, 3)$ generators, just nine massless NG modes appear in both cases. Together with an effective Higgs component, on which the vev is developed, they complete the whole ten-component symmetric tensor field $H_{\mu\nu}$ of the basic Lorentz group. Some of them are true Goldstone modes of the spontaneous Lorentz violation, others are PGMs since, as was mentioned, an accidental $SO(7, 3)$ symmetry is not shared by the whole Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ given in (6). Notably, in contrast to the scalar PGM case [14], they remain strictly massless being protected by the starting diff invariance[3] which becomes exact when the tensor field gravity Lagrangian (6) is properly extended to GR. Owing to this invariance, some of the Lorentz Goldstone modes and PGMs can then be gauged away from the theory, as usual.

Now, one can rewrite the Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ in terms of the Goldstone modes explicitly using the SLIV constraint (5). For this purpose, let us take the following handy parameterization for the tensor field $H_{\mu\nu}$

$$H_{\mu\nu} = h_{\mu\nu} + \frac{\mathfrak{n}_{\mu\nu}}{\mathfrak{n}^2}(\mathfrak{n} \cdot H) \quad (\mathfrak{n} \cdot H \equiv \mathfrak{n}_{\mu\nu} H^{\mu\nu}) \tag{16}$$

where $h_{\mu\nu}$ corresponds to the pure Goldstonic modes[4] satisfying

$$\mathfrak{n} \cdot h = 0 \quad (\mathfrak{n} \cdot h \equiv \mathfrak{n}_{\mu\nu} h^{\mu\nu}) \tag{17}$$

while the effective "Higgs" mode (or the $H_{\mu\nu}$ component in the vacuum direction) is given by the scalar product $\mathfrak{n} \cdot H$. Substituting this parameterization (16) into the tensor field constraint (5), one comes to the equation for $\mathfrak{n} \cdot H$

$$\mathfrak{n} \cdot H = \left(M^2 - \mathfrak{n}^2 h^2\right)^{\frac{1}{2}} = M - \frac{\mathfrak{n}^2 h^2}{2M} + O\left(1/M^2\right) \tag{18}$$

taking, for definiteness, the positive sign for the square root and expanding it in powers of h^2/M^2, $h^2 \equiv h_{\mu\nu} h^{\mu\nu}$. Putting then the parameterization (16) with the SLIV constraint (18) into the Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ given in (6), (7), (9), one comes to the truly Goldstonic tensor field gravity Lagrangian $\mathcal{L}(h_{\mu\nu}, \phi)$ containing an infinite series in powers of the $h_{\mu\nu}$ modes. For the traceless vev tensor $\mathfrak{n}_{\mu\nu}$ (15) it takes, without loss of generality, the especially simple form

$$\mathcal{L}(h_{\mu\nu}, \phi) = \frac{1}{2}\partial_\lambda h^{\mu\nu}\partial^\lambda h_{\mu\nu} - \frac{1}{2}\partial_\lambda h_{tr}\partial^\lambda h_{tr}$$
$$- \partial_\lambda h^{\lambda\nu}\partial^\mu h_{\mu\nu} + \partial^\nu h_{tr}\partial^\mu h_{\mu\nu}$$
$$+ \frac{1}{2M}h^2\big[-2\mathfrak{n}^{\mu\lambda}\partial_\lambda\partial^\nu h_{\mu\nu} + \mathfrak{n}^2(\mathfrak{n}\partial\partial)h_{tr}\big]$$

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[2]　Indeed, the vev matrices in the cases (a), (b) look, respectively, as $\mathfrak{n}^{(a)} = diag(1, 0, 0, 0)$ and $\mathfrak{n}^{(b)} = diag(0, 1, 0, 0)$, while in the cases $(c$-$d)$ these matrices, taken in the diagonal bases, have the forms $\mathfrak{n}^{(c)} = diag(0, 1, -1, 0)$ and $\mathfrak{n}^{(d)} = diag(1, -1, 0, 0)$, respectively (for certainty, we fixed $i = j = 1$ in the case (b), $i = 1$ and $j = 2$ in the case (c), and $i = 1$ in the case (d)). The groups of invariance of these vev matrices are just the surviving Lorentz subgroups indicated on the right-handed sides in (13) and (14). The broken Lorentz generators determine then the numbers of Goldstone modes mentioned above.

[3]　For nonminimal vacuum configuration when vevs are developed on several $H_{\mu\nu}$ components, thus leading to a more substantial breaking of the accidental $SO(7, 3)$ symmetry, some extra PGMs are also generated. However, they are not protected by a diff invariance and acquire masses of the order of the breaking scale M.

[4]　It should be particularly emphasized that the modes collected in the $h_{\mu\nu}$ are in fact the Goldstone modes of the broken accidental $SO(7, 3)$ symmetry of the constraint (5), thus containing the Lorentz Goldstone modes and PGMs put together.

$$+ \frac{1}{8M^2} h^2 \left[-\mathfrak{n}^2 \partial^2 + 2(\partial \mathfrak{n} \mathfrak{n} \partial) \right] h^2$$

$$+ \mathcal{L}(\phi) + \frac{M}{M_P} \mathfrak{n}^2 \left[\mathfrak{n}_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \right] + \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}$$

$$+ \frac{1}{2MM_P} h^2 \left[-\mathfrak{n}_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \right] \tag{19}$$

written in the $O(h^2/M^2)$ approximation in which, besides the conventional graviton bilinear kinetic terms, there are also three- and four-linear interaction terms in powers of $h_{\mu\nu}$ in the Lagrangian. Some of the notations used are collected below

$$h^2 \equiv h_{\mu\nu} h^{\mu\nu}, \qquad h_{tr} \equiv \eta^{\mu\nu} h_{\mu\nu},$$

$$\mathfrak{n} \partial \partial \equiv \mathfrak{n}_{\mu\nu} \partial^\mu \partial^\nu, \qquad \partial \mathfrak{n} \mathfrak{n} \partial \equiv \partial^\mu \mathfrak{n}_{\mu\nu} \mathfrak{n}^{\nu\lambda} \partial_\lambda. \tag{20}$$

The bilinear scalar field term

$$\frac{M}{M_P} \mathfrak{n}^2 \left[\mathfrak{n}_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \right] \tag{21}$$

in the third line in the Lagrangian (19) merits special notice. This term arises from the interaction Lagrangian $\mathcal{L}_{int}$ (9) after application of the tracelessness condition (15) for the vev tensor $\mathfrak{n}_{\mu\nu}$. It could significantly affect the dispersion relation for the scalar field ϕ (and any other sort of matter as well) thus leading to an unacceptably large Lorentz violation if the SLIV scale M were comparable with the Planck mass M_P. However, this term can be gauged away by an appropriate redefinition (going to new coordinates $x^\mu \to x^\mu + \xi^\mu$) of the scalar field derivative according to

$$\partial^\mu \phi \to \partial^\mu \phi + \partial_\rho \xi^\mu \partial^\rho \phi. \tag{22}$$

In fact with the following choice of the parameter function $\xi^\mu(x)$

$$\xi^\mu(x) = \frac{M}{2M_P} \mathfrak{n}^2 \mathfrak{n}^{\mu\nu} x_\nu,$$

the term (21) is canceled by an analogous term stemming from the scalar field kinetic term in $\mathcal{L}(\phi)$ given in (9).[5] On the other hand, since the diff invariance is an approximate symmetry of the Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ we started with (6), this cancellation will only be accurate up to the linear order corresponding to the tensor field theory. Indeed, a proper extension of this theory to GR with its exact diff invariance will ultimately restore the usual dispersion relation for the scalar (and other matter) fields. Taking this into account, we will henceforth omit the term (21) in $\mathcal{L}(h_{\mu\nu}, \phi)$ thus keeping the "normal" dispersion relation for the scalar field in what follows.

Together with the Lagrangian one must also specify other supplementary conditions for the tensor field $h^{\mu\nu}$ (appearing eventually as possible gauge fixing terms in the Goldstonic tensor field gravity) in addition to the basic Goldstonic "gauge" condition $\mathfrak{n}_{\mu\nu} h^{\mu\nu} = 0$ given above (17). The point is that the spin 1 states are still left in the theory and are described by some of the components of the new tensor $h_{\mu\nu}$. This is certainly inadmissible.[6] Usually, the spin 1 states (and one of the spin 0 states) are excluded by the conventional Hilbert–Lorentz condition

$$\partial^\mu h_{\mu\nu} + q \partial_\nu h_{tr} = 0 \tag{23}$$

(q is an arbitrary constant, giving for $q = -1/2$ the standard harmonic gauge condition). However, as we have already imposed the constraint (17), we cannot use the full Hilbert–Lorentz condition (23) eliminating four more degrees of freedom in $h_{\mu\nu}$. Otherwise, we would have an "over-gauged" theory with a nonpropagating graviton. In fact, the simplest set of conditions which conform with the Goldstonic condition (17) turns out to be

$$\partial^\rho (\partial_\mu h_{\nu\rho} - \partial_\nu h_{\mu\rho}) = 0. \tag{24}$$

This set excludes only three degrees of freedom[7] in $h_{\mu\nu}$ and, besides, it automatically satisfies the Hilbert–Lorentz spin condition as well. So, with the Lagrangian (19) and the supplementary conditions (17) and (24) lumped together, one eventually comes to a working model for the Goldstonic tensor field gravity. Generally, from ten components of the symmetric two-index tensor $h_{\mu\nu}$ four components are excluded by the supplementary conditions (17) and (24). For a plane gravitational wave propagating in, say, the z direction another four components are also eliminated, due to the fact that the above supplementary conditions still leave freedom in the choice of a coordinate system, $x^\mu \to x^\mu + \xi^\mu(t - z/c)$, much as it takes place in standard GR. Depending on the form of the vev tensor $\mathfrak{n}_{\mu\nu}$, caused by SLIV, the two remaining transverse modes of the physical graviton may consist solely of Lorentz Goldstone modes or of Pseudo-Goldstone modes, or include both of them.

3. The lowest order SLIV processes

The Goldstonic gravity Lagrangian (19) looks essentially nonlinear and contains a variety of Lorentz and *CPT* violating couplings when expressed in terms of the pure tensor Goldstone modes. However, as we show below, all violation effects turn out to be strictly canceled in the lowest order SLIV processes. Such a cancellation in vector-field theories, both Abelian [13,18,19] and non-Abelian [20], and, therefore, their equivalence to conventional QED and Yang–Mills theories, allows one to conclude that the nonlinear SLIV constraint in these theories amounts to a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. It seems that a similar conclusion can be made for tensor field gravity, i.e. the SLIV constraint (5) corresponds to a special gauge choice in a diff and Lorentz invariant theory. This conclusion certainly works for the diff invariant free tensor field part (7) in the starting Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$. On the other hand, its matter field sector (9), possessing only an approximate diff invariance, might lead to an actual Lorentz violation through the deformed dispersion relations of the matter fields involved. However, as was mentioned above, a proper extension of the tensor field theory to GR with its exact diff invariance ultimately restores the dispersion relations for matter fields and, therefore, the SLIV effects vanish. Taking this into account, we omit the term (21) in the Goldstonic gravity Lagrangian $\mathcal{L}(h_{\mu\nu}, \phi)$ thus keeping the "normal" dispersion relation for the scalar field representing all the matter in our model.

We are now going to consider the lowest order SLIV processes, after first establishing the Feynman rules in the Goldstonic gravity theory. We use for simplicity, both in the Lagrangian $\mathcal{L}$ (19) and forthcoming calculations, the traceless vev tensor $\mathfrak{n}_{\mu\nu}$, while our results remain true for any type of vacuum configuration caused by SLIV.

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[5] In the general case, with the vev tensor $\mathfrak{n}_{\mu\nu}$ having a nonzero trace, this cancellation would also require the redefinition of the scalar field itself as $\phi \to \phi (1 - \mathfrak{n}_{\mu\nu} \eta^{\mu\nu} \frac{M}{M_P})^{-1/2}$.

[6] Indeed, spin 1 must be necessarily excluded as the sign of the energy for spin 1 is always opposite to that for spin 2 and 0.

[7] The solution for a gauge function $\xi_\mu(x)$ satisfying the condition (24) can generally be chosen as $\xi_\mu = \Box^{-1} (\partial^\rho h_{\mu\rho}) + \partial_\mu \theta$, where $\theta(x)$ is an arbitrary scalar function, so that only three degrees of freedom in $h_{\mu\nu}$ are actually eliminated.

3.1. Feynman rules

The Feynman rules stemming from the Lagrangian $\mathcal{L}$ (19) for the pure graviton sector are as follows:

(i) The first and most important is the graviton propagator which only conforms with the Lagrangian (19) and the gauge conditions (17) and (24)

$$
\begin{aligned}
-i D_{\mu\nu\alpha\beta}(k) = {} & \frac{1}{2k^2}(\eta_{\beta\mu}\eta_{\alpha\nu} + \eta_{\beta\nu}\eta_{\alpha\mu} - \eta_{\alpha\beta}\eta_{\mu\nu}) \\
& - \frac{1}{2k^4}(\eta_{\beta\nu}k_\alpha k_\mu + \eta_{\alpha\nu}k_\beta k_\mu \\
& + \eta_{\beta\mu}k_\alpha k_\nu + \eta_{\alpha\mu}k_\beta k_\nu) \\
& - \frac{1}{k^2(\mathfrak{n}kk)}(k_\alpha k_\beta \mathfrak{n}_{\mu\nu} + k_\nu k_\mu \mathfrak{n}_{\alpha\beta}) \\
& + \frac{1}{k^2(\mathfrak{n}kk)^2}\left[\mathfrak{n}^2 - \frac{2}{k^2}(k\mathfrak{n}\mathfrak{n}k)\right]k_\mu k_\nu k_\alpha k_\beta \\
& + \frac{1}{k^4(\mathfrak{n}kk)}(\mathfrak{n}_{\mu\rho}k^\rho k_\nu k_\alpha k_\beta + \mathfrak{n}_{\nu\rho}k^\rho k_\mu k_\alpha k_\beta \\
& + \mathfrak{n}_{\alpha\rho}k^\rho k_\nu k_\mu k_\beta + \mathfrak{n}_{\beta\rho}k^\rho k_\nu k_\alpha k_\mu)
\end{aligned}
\tag{25}
$$

(where $(\mathfrak{n}kk) \equiv \mathfrak{n}_{\mu\nu}k^\mu k^\nu$ and $(k\mathfrak{n}\mathfrak{n}k) \equiv k^\mu \mathfrak{n}_{\mu\nu}\mathfrak{n}^{\nu\lambda}k_\lambda$). It automatically satisfies the orthogonality condition $\mathfrak{n}^{\mu\nu}D_{\mu\nu\alpha\beta}(k) = 0$ and on-shell transversality $k^\mu k^\nu D_{\mu\nu\alpha\beta}(k, k^2 = 0) = 0$. This is consistent with the corresponding polarization tensor $\epsilon_{\mu\nu}(k, k^2 = 0)$ of the free tensor fields, being symmetric, traceless ($\eta^{\mu\nu}\epsilon_{\mu\nu} = 0$), transverse ($k^\mu \epsilon_{\mu\nu} = 0$), and also orthogonal to the vacuum direction, $\mathfrak{n}^{\mu\nu}\epsilon_{\mu\nu}(k) = 0$. Apart from that, the gauge invariance allows us to write the polarization tensor in the factorized form [22], $\epsilon_{\mu\nu}(k) = \epsilon_\mu(k)\epsilon_\nu(k)$, and to proceed with the above-mentioned tracelessness and transversality expressed as the simple conditions $\epsilon_\mu \epsilon^\mu = 0$ and $k^\mu \epsilon_\mu = 0$ respectively. In the following we will use these simplifications. As one can see, only the standard terms given by the first bracket in (25) contribute when the propagator is sandwiched between conserved energy–momentum tensors of matter fields, and the result is always Lorentz invariant.

(ii) Next is the 3-graviton vertex with graviton polarization tensors (and 4-momenta) given by $\epsilon^{\alpha\alpha'}(k_1)$, $\epsilon^{\beta\beta'}(k_2)$ and $\epsilon^{\gamma\gamma'}(k_3)$

$$
\begin{aligned}
& -\frac{i}{2M}P^{\alpha\alpha'}(k_1)(\eta^{\beta\gamma}\eta^{\beta'\gamma'} + \eta^{\beta\gamma'}\eta^{\beta'\gamma}), \\
& -\frac{i}{2M}P^{\beta\beta'}(k_2)(\eta^{\alpha\gamma}\eta^{\alpha'\gamma'} + \eta^{\alpha\gamma'}\eta^{\alpha'\gamma}), \\
& -\frac{i}{2M}P^{\gamma\gamma'}(k_3)(\eta^{\beta\alpha}\eta^{\beta'\alpha'} + \eta^{\beta\alpha'}\eta^{\beta'\alpha})
\end{aligned}
\tag{26}
$$

where the momentum tensor $P^{\mu\nu}(k)$ is

$$
P^{\mu\nu}(k) = -\mathfrak{n}^{\nu\rho}k_\rho k^\mu - \mathfrak{n}^{\mu\rho}k_\rho k^\nu + \eta^{\mu\nu}\mathfrak{n}^{\rho\sigma}k_\rho k_\sigma.
\tag{27}
$$

Note that all 4-momenta at the vertices are taken ingoing throughout.

(iii) Finally, the 4-graviton vertex with the graviton polarization tensors (and 4-momenta) $\epsilon^{\alpha\alpha'}(k_1)$, $\epsilon^{\beta\beta'}(k_2)$, $\epsilon^{\gamma\gamma'}(k_3)$ and $\epsilon^{\delta\delta'}(k_4)$

$$
\begin{aligned}
& iQ_{\mu\nu}(\eta^{\alpha\beta}\eta^{\alpha'\beta'} + \eta^{\alpha\beta'}\eta^{\alpha'\beta})(\eta^{\gamma\delta}\eta^{\gamma'\delta'} + \eta^{\gamma\delta'}\eta^{\gamma'\delta}) \\
& \quad \times (k_1 + k_2)^\mu (k_1 + k_2)^\nu \\
& + iQ_{\mu\nu}(\eta^{\alpha\gamma}\eta^{\alpha'\gamma'} + \eta^{\alpha\gamma'}\eta^{\alpha'\gamma})(\eta^{\beta\delta}\eta^{\beta'\delta'} + \eta^{\beta\delta'}\eta^{\beta'\delta}) \\
& \quad \times (k_1 + k_3)^\mu (k_1 + k_3)^\nu \\
& + iQ_{\mu\nu}(\eta^{\alpha\delta}\eta^{\alpha'\delta'} + \eta^{\alpha\delta'}\eta^{\alpha'\delta})(\eta^{\gamma\beta}\eta^{\gamma'\beta'} + \eta^{\gamma\beta'}\eta^{\gamma'\beta}) \\
& \quad \times (k_1 + k_4)^\mu (k_1 + k_4)^\nu.
\end{aligned}
\tag{28}
$$

Here we have used the self-evident identities for all ingoing momenta ($k_1 + k_2 + k_3 + k_4 = 0$), such as

$$
\begin{aligned}
& (k_1 + k_2)^\mu (k_1 + k_2)^\nu + (k_3 + k_4)^\mu (k_3 + k_4)^\nu \\
& \quad = 2(k_1 + k_2)^\mu (k_1 + k_2)^\nu
\end{aligned}
$$

and so on, and denoted by $Q_{\mu\nu}$ the expression

$$
Q_{\mu\nu} \equiv -\frac{1}{4M^2}(-\mathfrak{n}^2\eta_{\mu\nu} + 2\mathfrak{n}_{\mu\rho}\mathfrak{n}_\nu^\rho).
\tag{29}
$$

Coming now to the gravitational interaction of the scalar field, one has two more vertices:

(iv) The standard graviton–scalar–scalar vertex with the graviton polarization tensor $\epsilon^{\alpha\alpha'}$ and the scalar field 4-momenta p_1 and p_2

$$
-\frac{i}{M_p}(p_1^\alpha p_2^{\alpha'} + p_2^\alpha p_1^{\alpha'}) + \frac{i}{M_p}\eta^{\alpha\alpha'}[(p_1 p_2) + m^2]
\tag{30}
$$

where $(p_1 p_2)$ stands for the scalar product.

(v) The contact graviton–graviton–scalar–scalar interaction caused by SLIV with the graviton polarization tensors $\epsilon^{\alpha\alpha'}$ and $\epsilon^{\beta\beta'}$ and the scalar field 4-momenta p_1 and p_2

$$
\frac{i}{MM_p}(g^{\alpha\beta}g^{\alpha'\beta'} + g^{\alpha\beta'}g^{\alpha'\beta})(\mathfrak{n}_{\mu\nu}p_1^\mu p_2^\nu).
\tag{31}
$$

Just the rules (i)–(v) are needed to calculate the lowest order processes mentioned above.

3.2. Graviton–graviton scattering

The matrix element for this SLIV process to the lowest order $1/M^2$ is given by the contact h^4 vertex (28) and the pole diagrams with longitudinal graviton exchange between two Lorentz violating h^3 vertices (26). There are three pole diagrams in total, describing the elastic graviton–graviton scattering in the s- and t-channels respectively, and also the diagram with an interchange of identical gravitons. Remarkably, the contribution of each of them is exactly canceled by one of three terms appearing in the contact vertex (28). Actually, for the s-channel pole diagrams with ingoing gravitons with polarizations (and 4-momenta) $\epsilon_1(k_1)$ and $\epsilon_2(k_2)$ and outgoing gravitons with polarizations (and 4-momenta) $\epsilon_3(k_3)$ and $\epsilon_4(k_4)$ one has, after some evident simplifications related to the graviton propagator $D_{\mu\nu}(k)$ (25) inside the matrix element

$$
i\mathcal{M}_{pole}^{(1)} = i\frac{1}{M^2}(\epsilon_1 \cdot \epsilon_2)^2(\epsilon_3 \cdot \epsilon_4)^2(-\mathfrak{n}^2 k^2 + 2k^\mu \mathfrak{n}_{\mu\nu}\mathfrak{n}^{\nu\lambda}k_\lambda).
\tag{32}
$$

Here $k = k_1 + k_2 = -(k_3 + k_4)$ is the momentum running in the diagrams listed above, and all the polarization tensors are properly factorized throughout, $\epsilon_{\mu\nu}(k) = \epsilon_\mu(k)\epsilon_\nu(k)$, as was mentioned above. We have also used that, since ingoing and outgoing gravitons appear transverse ($k_a^\mu \epsilon_\mu(k_a) = 0$, $a = 1, 2, 3, 4$), only the third term in the momentum tensors $P^{\mu\nu}(k_a)$ (27) in the h^3 couplings (26) contributes to all pole diagrams. Now, one can readily confirm that this matrix element is exactly canceled with the first term in the contact SLIV vertex (28), when it is properly contracted with the graviton polarization vectors. In a similar manner, two other terms in the contact vertex provide the further one-to-one cancellations with the remaining two pole matrix elements $i\mathcal{M}_{pole}^{(2,3)}$. So, the Lorentz violating contribution to graviton–graviton scattering is absent in Goldstonic gravity theory in the lowest $1/M^2$ approximation.

3.3. Graviton scattering on a massive scalar

This SLIV process appears in the order $1/MM_p$ (in contrast to the conventional $1/M_p^2$ order graviton–scalar scattering). It is directly related to two diagrams one of which is given by the contact graviton–graviton–scalar–scalar vertex (31), while the other corresponds to the pole diagram with longitudinal graviton exchange between the Lorentz violating h^3 vertex (26) and the ordinary graviton–scalar–scalar vertex (30). Again, since ingoing and outgoing gravitons appear transverse ($k_a^\mu \epsilon_\mu(k_a) = 0$, $a = 1, 2$), only the third term in the momentum tensors $P^{\mu\nu}(k_a)$ (27) in the h^3 coupling (26) contributes to this pole diagram. Apart from that, the most crucial point is that, due to the scalar field energy–momentum tensor conservation, the terms in the inserted graviton propagator (25) other than the standard ones (first bracket in (25)) give a vanishing result. Keeping all this in mind together with the momenta satisfying $k_1 + k_2 + p_1 + p_2 = 0$ ($k_{1,2}$ and $p_{1,2}$ are the graviton and scalar field 4-momenta, respectively), one readily comes to a simple matrix element for the pole diagram

$$i\mathcal{M}_{pole} = -\frac{2i}{MM_p}\phi(p_2)(\varepsilon_1 \cdot \varepsilon_2)^2\big(\mathfrak{n}_{\mu\nu}p_1^\mu p_2^\nu\big)\phi(p_1). \qquad (33)$$

This pole term is precisely canceled by the contact term, $i\mathcal{M}_{con}$, when the SLIV vertex (31) is properly contracted with the graviton polarization vectors and the scalar boson wave functions. Again, we may conclude that physical Lorentz invariance is left intact in graviton scattering on a massive scalar, provided that its dispersion relation is supposed to be recovered when going from the tensor field Lagrangian $\mathcal{L}$ (19) to general relativity, as was argued above.

3.4. Scalar–scalar scattering

This process, due to graviton exchange, appears in the order $1/M_P^2$ and again is given by an ordinary Lorentz invariant amplitude. As was mentioned above, only the standard terms given by the first bracket in the graviton propagator (25) contribute when it is sandwiched between conserved energy–momentum tensors of matter fields. Actually, as one can easily confirm, the contraction of any other term in (25) depending on the graviton 4-momentum $k = p_1 + p_2 = -(p_3 + p_4)$ with the graviton–scalar–scalar vertex (30) gives a zero result.

3.5. Other processes

Many other tree level Lorentz violating processes, related to gravitons and scalar fields (matter fields, in general) appear in higher orders in the basic SLIV parameter $1/M$, by iteration of couplings presented in our basic Lagrangian (19) or from a further expansion of the effective Higgs mode (18) inserted into the starting Lagrangian (6). Again, their amplitudes are essentially determined by an interrelation between the longitudinal graviton exchange diagrams and the corresponding contact multi-graviton interaction diagrams, which appear to cancel each other, thus eliminating physical Lorentz violation in the theory.

Most likely, the same conclusion could be expected for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [18], the corresponding one-loop matrix elements in the Goldstonic gravity theory could either vanish by themselves or amount to the differences between pairs of similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external 4-momenta of the particles involved) which, in the

framework of dimensional regularization, could lead to their total cancellation.

So, the Goldstonic tensor field gravity theory is likely to be physically indistinguishable from conventional general relativity taken in the weak-field limit, provided that the underlying diff invariance is kept exact. This, as we have seen, requires the tensor field gravity to be extended to GR, in order not to otherwise have an actual Lorentz violation in the matter field sector. In this connection, the question arises whether or not the SLIV cancellations continue to work once the tensor field gravity theory is extended to GR, which introduces many additional terms in the starting Lagrangian $\mathcal{L}(H_{\mu\nu}, \phi)$ (6). Indeed, since all the new terms are multi-linear in $H_{\mu\nu}$ and contain higher orders in $1/M_P$, the "old" SLIV cancellations (considered above) will not be disturbed, while "new" cancellations will be provided, as one should expect, by an extended diff invariance. This extended diff invariance follows from the proper expansion of the metric transformation law in GR

$$\delta g_{\mu\nu} = \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} + \xi^\rho \partial_\rho g_{\mu\nu} \qquad (34)$$

up to the order in which the extended tensor field theory, given by the modified Lagrangian $\mathcal{L}_{ext}(H_{\mu\nu}, \phi)$, is considered.

4. Conclusion

We have considered spontaneous Lorentz violation, appearing through the length-fixing tensor field constraint $H_{\mu\nu}^2 = \pm M^2$ (M is the proposed scale for Lorentz violation), in the tensor field gravity theory which mimics general relativity in Minkowski space–time. We have shown that such an SLIV pattern, due to which the true vacuum in the theory is chosen, induces massless tensor Goldstone modes some of which can naturally be associated with the physical graviton. This theory looks essentially nonlinear and contains a variety of Lorentz and *CPT* violating couplings, when expressed in terms of the pure tensor Goldstone modes. Nonetheless, all the SLIV effects turn out to be strictly canceled in the lowest order graviton–graviton scattering, due to the diff invariance of the free tensor field Lagrangian (7) we started with. At the same time, actual Lorentz violation may appear in the matter field interaction sector (9), which only possesses an approximate diff invariance, through deformed dispersion relations of the matter fields involved. However, a proper extension of the tensor field theory to GR, with its exact diff invariance, ultimately restores the normal dispersion relations for matter fields and, therefore, the SLIV effects vanish. So, as we generally argue, the measurable effects of SLIV, induced by elementary vector or tensor fields, can be related to the accompanying gauge symmetry breaking rather than to spontaneous Lorentz violation. The latter appears by itself to be physically unobservable and only results in a noncovariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory.

From this standpoint, the only way for physical Lorentz violation to appear would be if the above local invariance is slightly broken at very small distances. This is in fact a place where the Goldstonic vector and tensor field theories drastically differ from conventional QED, Yang–Mills and GR theories. Actually, such a local symmetry breaking could lead in the former case to deformed dispersion relations for all the matter fields involved. This effect typically appears proportional to some power of the ratio $\frac{M}{M_P}$ (just as we have seen above for the scalar field in our model, see (21)), though being properly suppressed by tiny gauge noninvariance. Remarkably, the higher the SLIV scale M becomes the larger becomes the actual Lorentz violation which, for some value of the scale M, may become physically observable even at low energies. Another basic distinction of Goldstonic theories with

nonexact gauge invariance is the emergence of a mass for the graviton and other gauge fields (namely, for the non-Abelian ones, see [20]), if they are composed from Pseudo-Goldstone modes rather than from pure Goldstone ones. Indeed, these PGMs are no longer protected by gauge invariance and may properly acquire tiny masses, which still do not contradict experiment. This may lead to a massive gravity theory where the graviton mass emerges dynamically, thus avoiding the notorious discontinuity problem [23]. So, while Goldstonic theories with exact local invariance are physically indistinguishable from conventional gauge theories, there are some principal distinctions when this local symmetry is slightly broken which could eventually allow us to differentiate between the two types of theory in an observational way.

One could imagine how such a local symmetry breaking might occur. As was earlier argued [24], only local invariant theories provide the needed number of degrees of freedom for interacting gauge fields once SLIV occurs. Note that a superfluous restriction put on vector or tensor fields would make it impossible to set the required initial conditions in the appropriate Cauchy problem and, in quantum theory, to choose self-consistent equal-time commutation relations [25]. One could expect, however, that quantum gravity could in general hinder the setting of the required initial conditions at extra-small distances. Eventually, this would manifest itself in violation of the above local invariance in a theory through some high-order operators stemming from the quantum gravity influenced area, which could lead to physical Lorentz violation. This attractive point seems to deserve further consideration.

Acknowledgements

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