An alternative optimization technique for interval objective constrained optimization problems via multiobjective programming

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Abstract An alternative optimization technique via multiobjective programming for constrained optimization problems with interval-valued objectives has been proposed. Reduction of interval objective functions to those of noninterval (crisp) one is the main ingredient of the proposed technique. At first, the significance of interval-valued objective functions along with the meaning of interval-valued solutions of the proposed problem has been explained graphically. Generally, the proposed problems have infinitely many compromise solutions. The objective is to obtain one of such solutions with higher accuracy and lower computational effort. Adequate number of numerical examples has been solved in support of this technique.

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1. Introduction

In the context of present day socio-economic scenario, the uncertainty handling optimization techniques are most powerful to increase the productivity of business companies and public organizations. The existence of impreciseness is inevitable in real world data most of which are collected from some insufficient information. While formulating mathematical models, the impreciseness may also come into the existence due to decision-making under uncertain situations. At present, it is a burning question to the researchers: How to model this impreciseness properly to handle the complicated uncertain situations arisen in reality and also how to develop the appropriate solution methodologies? Stochastic [1–3], fuzzy [4–6], or grey optimization techniques [7,8] are some conventional and very familiar approaches to tackle these problems. Each of these methods has some advantages and shortcomings. Alternatively, to deal with the ambiguity of the available data or the impreciseness of any parameter, one may replace those by intervals. An interval can bound the uncertainty/impreciseness...
within its upper and lower limits. Sengupta and Pal [9] have explained the advantages of using intervals to represent uncertain or imprecise parameters over fuzzy set theoretic or probabilistic approaches for solving real world decision-making problems. The main privilege of using interval-oriented techniques is that one has to calculate only the bounds of the intervals which specify the limits of uncertainty. By using intervals and interval-oriented techniques, one can handle the uncertainty/imprecision in a deterministic way [10]. Several researchers [11–17] have used intervals successfully to represent impreciseness and also modeled many real world application problems in interval form. According to Ishibuchi and Tanaka [18], if the imprecise data are represented by intervals, then the expected value of the data can be specified by the centers of the intervals and the uncertainty can be measured by the widths. However, in most of the interval-oriented techniques, there arise some important questions regarding the ranking of arbitrary interval numbers during the implementation. Sometimes, it becomes the key factor to measure the efficiency of the technique. Regarding interval ranking, a pioneering work has been done by Moore [19]. After Moore [19], a number of interval ordering definitions [11,18,20–22] have been developed in different ways to serve various purposes. Detailed survey of these ranking definitions has been given in [9,23] with their advantages and shortcomings. The primary goal of these definitions is to develop reliable solution technique for interval optimization problems with the help of interval ranking. The primary developments of the concept of interval numbers and their analytical characteristics along with the applications of different branches of mathematics have been provided by Moore [19]. Recently, Moore et al. [24] have given an extensive version of their previous works with the application of INTLAB software in interval analysis. There exist various approaches to solve interval-oriented optimization problems. Some of these approaches ensure the guarantee to enclose the set of all optimal solutions covering all possibilities [25–28]. In the second approach, the aim is to give some approximations of compromise solution [9,18,29,30]. Many of the optimization techniques are developed on the basis of Branch and Bound (B&B) algorithms. On the other hand, several simple prototype algorithms for noninterval constrained/unconstrained optimization problems were given by [19,31–33]. Jaulin et al. [12] and Kearfott [13] have provided an illustrative overview of the state-of-the-art of rigorous interval analysis with its applications in optimization problems for global optimality. However, most of the interval-oriented algorithms have been applied to solve noninterval-valued optimization problems. Ratschek and Rokne [34,35] have given some valuable discussions about the interval tools for global optimality including the accelerating devices (i.e., by modifying the algorithm) for rapid convergence. Previously, many researchers developed different types of interval-oriented algorithms/optimization techniques [16,18,20,36–39] for interval linear systems. Ishibuchi and Tanaka [18] proposed a method for linear optimization problems with interval objective functions by converting those into multiobjective optimization problem. An interval-oriented approach of obtaining rigorous solution of linear programming problems with uncertain data has been given in [25]. The solution set, in this case, defines very sharp and guaranteed error bounds and also the method permits a rigorous sensitivity analysis. Chanas and Kuchta [20] have generalized the works of [18] with the help of $t_0, t_1$-cut of the intervals and developed the general method using multiobjective programming for interval linear programming. A different technique for interval linear optimization problems with interval objective function was proposed by Inuiguchi and Sakawa [36] by introducing the minimax regret criterion. The repeated use of the well known simplex method is the basis of this method from a starting reference solution set. Another approach by using an efficient interval ordering (Acceptability index method [11]) for an interval linear programming problem (ILPP) has been given in [9]. Some previous developments in the solution methodology of ILPP have been given by Fiedler et al. [40]. Recently, Hladik [27] and Gabrel et al. [41] have introduced two different methods for interval linear programming problems. Suprajitno and Bin Mohd [42] have used the modified simplex method for interval linear programming problems. An optimization technique has been proposed by Allahdadi and Nehi [43] to determine the optimal solution set of the ILPP by using the best and worst case (BWC) methods. Hladik [44] proposed a novel algorithm for testing basis stability for ILP. Besides these, the survey work of Hladik [45] contains detailed discussions of the state-of-the-art for the recent developments of ILPP.

However, most of these techniques are restricted only to linear programming problems with inequality constraints. Consideration of nonlinearity in the structure of model formulation is inevitable for most of the engineering, financial or managerial decision-making problems. Liu and Wang [26] have investigated the solution methodology for Quadratic programming problems (QPP) with interval coefficients. In this case, the problem is transformed into a pair of two-level programming problems and applying the duality theorem and the variable transformation technique, the pair of two-level mathematical programming problem is transformed to the conventional one-level QPP. Recently, Jiang et al. [30] prescribed an optimization technique for nonlinear programming problems with interval coefficients by using genetic algorithm (GA) and multiobjective optimization technique. Hladik [28] has proposed a technique to determine the optimal bounds for nonlinear programming problems with interval data that ensures the exact bounds to enclose the set of all optimal solutions. Bhurjee and Panda [46] have introduced a technique for general interval optimization problems. The interval-valued problem is transformed into interval free problem for finding the efficient solutions of the original problem. Parametric representation of interval-valued functions and its important analytic properties are studied and it is used to the newly developed optimization technique.

It is already stated that most of the techniques developed for solving classical/interval-valued constrained/bound-constrained/unconstrained optimization problems are based on Branch and Bound (B&B) algorithm which consists of the following four steps: (i) branching of the prescribed search region, (ii) bounding of interval objective values, (iii) comparison of a continuum of interval values, and (iv) choosing of an optimum value. Two different multiplet splitting techniques for global solution of nonlinear bound-constrained optimization problems have been introduced by Karmakar et al. [47] and they suggested that the multisection division technique is more
acceptable between the two division techniques. Later on, Karmakar and Bhunia [48–50] have applied the multisect technique successfully to solve noninterval (or degenerate interval) constrained global optimization problems and bound constrained/constrained problems with uncertain coefficients in interval form. In interval-oriented $\&B$ algorithms, the method of ranking of intervals plays a vital role to estimate the efficiency of the techniques. In spite of getting better results by using interval $\&B$ algorithms in comparison with the other existing methods, some disadvantages have been encountered there as follows,

(i) The computational time is generally very high for higher dimensional problems. Computational time and complexity also depend on the number of subdivided boxes.
(ii) The efficiency of the algorithm depends on the interval ranking definition used, whereas we know that there exists no complete interval ordering.

In this paper, an alternative technique for constrained optimization problems with interval-valued objective function has been proposed. Generally, this type of problem has infinitely many compromise solutions. The aim of this technique is to obtain one of such solutions with higher accuracy and lower computational cost. In this technique, at first, the interval-valued problem is reduced to a noninterval multiobjective optimization problem. The reduced problem has been solved by the well known Global Criterion Method (GCM) to obtain the Pareto Optimal solution (or efficient solution). Finally, to demonstrate the effectiveness of the proposed technique, some numerical examples have been solved and the results have been compared with the same to some existing techniques available in the literature.

In the next section, we have given a brief survey of some interval ordering definitions. Section 3 provides the statement of the problem, concept of optimal solutions, and the geometrical interpretation of the meaning of interval objective function and the optimal solutions in terms of decision-makers’ choice. In Section 4, we have given the details of proposed solution technique. We have explained the technique for different functional forms with supported numerical examples. Section 5 includes more numerical experiments taken from the existing literature and a detailed comparative discussion is given.

2. Order relations of interval numbers

In this section, we shall discuss the order relations of closed intervals. Let $A = [a_L, a_R]$ and $B = [b_L, b_R]$ be two closed intervals. This pair of intervals may be one of the following three types:

Type I: Nonoverlapping, i.e., when $a_L > b_R$ or $b_L > a_R$.
Type II: Partially overlapping, i.e., when $b_L \leq a_L \leq b_R < a_R$ or $a_L \leq b_L < a_R < b_R$.
Type III: Completely overlapping, i.e., when $a_L \leq b_L < b_R \leq a_R$ or $b_L \leq a_R < a_R \leq b_R$.

Over the last few decades, many researchers proposed several definitions for interval order relations in different angles. In this area, Moore [19] first proposed two transitive order relations, which are given by

Definition 2.1.

(i) The first order relation ‘$<$’ is applicable only for Type – I intervals. It is not a partial order. Second relation ‘$\subseteq$’ is the generalization of the definition of subsets for intervals. It follows the properties of partial order relation as the traditional set operation ‘$\subseteq$’ is partial order.

Ishibuchi and Tanaka [18] defined the order relations of intervals $A = [a_L, a_R] = \langle a_L, a_R \rangle$ and $B = [b_L, b_R] = \langle b_C, b_W \rangle$ for minimization problems in the following ways:

Definition 2.3. $A \leq_{LR} B$ iff $a_L \leq b_L$ and $a_R \leq b_R$.

Definition 2.4. $A \leq_{CB} B$ iff $a_C \leq b_C$ and $a_W \leq b_W$.

Definition 2.5. The acceptability function (or acceptability index or value judgment index) $\lambda(A, B) : I \times I \rightarrow [0, \infty)$ for the intervals $A$ and $B$ with $b_C \geq a_C$ is defined as

\[ A(A, B) = \frac{b_C - a_C}{b_W + a_W} \text{ where } b_W + a_W \neq 0. \]

$A(A, B)$ may be regarded as a grade of acceptability of the ‘first interval to be inferior to the second’. If $A(A, B) = 0$ then for minimization problem, the interval $A$ cannot be accepted. If $0 < A(A, B) < 1$, $A$ can be accepted with the grade of acceptability $\frac{b_C - a_C}{b_W + a_W}$. Finally, if $A(A, B) \geq 1$, $A$ is accepted with full satisfaction.

According to them, the acceptability index is only a value based ranking index and the definition can be applied partially to select the best alternative interval from the pessimistic point of view of the decision-maker. Therefore, only the optimistic decision-makers can use it completely.
In the second approach, Sengupta and Pal [11] introduced
the fuzzy preference for the ranking of a pair of intervals on
the real line with respect to a pessimistic decision-maker’s
point of view. They defined a nonlinear membership function,
which lies in the interval [0, 1], when the value of this mem-
bership function lies within the interval [0,333, 0,666], this de-
definition fails to find out the order relations.

Hu and Wang [21] also proposed a modified version of or-
der relations of interval numbers. Introducing new ap-
proaches, they have tried to fulfill the shortcomings of the
previous definitions. They also introduced some novel interval
arithmetic operations and proved that their ranking definitions
satisfy some basic properties like reflexivity, anti-symmetricity
eq. with the help of newly developed arithmetic operations.
The interval ranking relation ‘\(<_\)’ is defined as follows:

**Definition 2.6.** For any two intervals \(A = [a_L, a_R] = (a_C, a_W)\)
and \(B = [b_L, b_R] = (b_C, b_W)\)
\[A <_\ B \iff \begin{cases}
    a_C < b_C & \text{whenever } a_C \neq b_C \\
    a_W \geq b_W & \text{whenever } a_C = b_C
\end{cases}
\]
and \(A < B \iff A <_\ B \text{ and } A \neq B\).

The center and the width of the intervals are regarded as the
expected value and the uncertainty of the parameters, respec-
tively, as we have seen previously in Ishibuchi and Tanaka’s
[18] definitions. Therefore, whenever centers of two intervals
are same, they emphasized on the width of the intervals, i.e.,
uncertainty of the parameters and then the decision-maker
defies to have the interval with less uncertainty. Here,
‘\(A <_\ B\)’ indicates that the interval \(A\) is less acceptable to that
of \(B\) for any type of optimization problem.

Mahato and Bhunia [22] proposed another class of defini-
tions of interval order relations that place more importance
on the decision-makers’ preference. There are different types
of decision-making conditions. However, they emphasize on
the optimistic and the pessimistic decision-makings. In opti-
mistic decision-making, the decision-maker selects the best
alternative ignoring the uncertainty. On the other hand, the
pessimistic decision-maker selects the best alternative with less
uncertainty. Naturally, the optimistic decision-maker is more
confident to get the best alternative under uncertain conditions
and the pessimistic decision-maker is less confident to get the
best alternative under such conditions.

Mahato and Bhunia [22] first pointed out the incomple-
teness of the aforementioned interval ranking definitions with
respect to the decision-makers’ point of view. To clarify, let us
consider an example with a pair of intervals of Type-III:

**Example 2.1.** Let \(A = [10,50] = (30, 20)\) and \(B = [25,45] =
(35, 10)\) be two intervals representing the profits in the case
of maximization problems and time/cost intervals in the case
of minimization problems. It is obvious that an optimistic
decision-maker will always prefer the interval \(A\) to \(B\) for both
maximization and minimization problems. However, the job is
not so easy for a pessimistic decision-maker. For maximization
problems, pessimists may choose the interval \(B\) as a most
profitable interval, and for minimization problems, they select
the lower cost/time interval \(A\).

**Optimistic decision-making**

In the context of the optimistic decision-making, Mahato
and Bhunia [22] proposed the following definitions:

**Definition 2.7.** For minimization problems, they defined the
order relation \(\leq_{omin}\) between the intervals \(A = [a_L, a_R]\) and
\(B = [b_L, b_R]\) as follows:

\(A \leq_{omin} B \iff a_L \leq b_L\),
\(A <_{omin} B \iff A \leq_{omin} B \text{ and } A \neq B\).

This implies that \(A\) is superior to \(B\) and \(A\) is accepted. This or-
der relation is not symmetric.

**Definition 2.8.** For maximization problems, the order relation
\(\geq_{omax}\) between the intervals \(A\) and \(B\) is

\(A \geq_{omax} B \iff a_R \geq b_R\),
\(A >_{omax} B \iff A \geq_{omax} B \text{ and } A \neq B\).

This implies that \(A\) is superior to \(B\) and optimistic decision-
maker accepts the profit interval \(A\). Here also, the order relation
\(\geq_{omax}\) is not symmetric.

**Pessimistic decision-making**

In this case, the decision-maker chooses the most preferable
interval according to the principle “Less uncertainty is better
than more uncertainty”. The proposed definitions are as
follows:

**Definition 2.9.** For minimization problems, they defined the
order relation \(\prec_{pmin}\) between the intervals \(A = [a_L, a_R] = (a_C,
a_W)\) and \(B = [b_L, b_R] = (b_C, b_W)\) for a pessimistic decision-
maker as

\(A \prec_{pmin} B \iff a_C < b_C\), for Type – I and Type – II intervals
\(A \prec_{pmin} B \iff a_C \leq b_C\) and \(a_W < b_W\), for some Type – III
intervals

However, for Type – III intervals with \(a_C < b_C\) and
\(a_W > b_W\), a pessimistic decision cannot be taken. In this case,
the optimistic decision can be considered.

**Definition 2.10.** For maximization problems, they defined the
order relation \(\succ_{pmax}\) between the intervals \(A = [a_L, a_R] = (a_C,
a_W)\) and \(B = [b_L, b_R] = (b_C, b_W)\) for a pessimistic decision-
maker as

\(A \succ_{pmax} B \iff a_C > b_C\), for type – I and Type – II intervals
\(A \succ_{pmax} B \iff a_C \geq b_C\) and \(a_W < b_W\), for some Type – III
intervals

However, for Type – III intervals with \(a_C > b_C\) and
\(a_W > b_W\), pessimistic decision cannot be taken. In this case,
the optimistic decision can be taken.
3. Statement of the problem

Let \( F: \mathbb{R}^n \rightarrow \mathbb{I} \) be an interval-valued function where \( \mathbb{R}^n \) be the set of ordered \( n \)-tuples of real numbers and \( \mathbb{I} \) be the set of intervals, \( x = (x_1, x_2, \ldots, x_n) \) be an \( n \)-dimensional decision vector, \( U = (U_1, U_2, \ldots, U_q) \) be a \( q \)-dimensional interval vector whose components are all intervals.

Hence, a general constrained optimization problem with interval-valued objective function can be written as follows:

Maximize \( Z = F(x, U) \)
subject to \( g_j(x) \leq 0, \lambda = 1, 2, \ldots, k \)
and \( h_j(x) = 0, \mu = 1, 2, \ldots, m \)
where \( x \in \mathbb{D} \subseteq \mathbb{R}^n \)

where \( \mathbb{D} \) is the \( n \)-dimensional interval (or box) and is given by \( \mathbb{D} = \{x \in \mathbb{R}^n; l \leq x \leq u\} \). Here \( l, u \in \mathbb{R}^n \) be two vectors given by \( l = (l_1, l_2, \ldots, l_n) \) and \( u = (u_1, u_2, \ldots, u_n) \) such that \( l_j \leq x_j \leq u_j, j = 1, 2, \ldots, n \), \( g_j(x) \leq 0 \) is the \( j \)th inequality constraint and \( h_j(x) = 0 \) is the \( j \)th equality constraint where \( k \) and \( m \) are the number of inequality and equality constraints, respectively.

3.1. Optimal solutions

The interval objective function is defined as \( F: \mathbb{R}^n \rightarrow \mathbb{I} \) and it is expressed as \( F(x, U) = (F^L(x), F^U(x)) \) where \( F^L(x) \) and \( F^U(x) \) are the center and width of the interval function, respectively.

Definition 3.1. A decision vector \( x^* \in \mathbb{D} \) is a minimum point if \( F^L(x^*) \leq F^L(x) \) (maximum if \( F^U(x^*) \geq F^U(x) \) for maximization problem) and \( F^U(x^*) \leq F^U(x) \) for any \( x \in \mathbb{D} \). In this case, the minimum value is denoted by \( F^* \) and the minimizer point by \( x^* \), i.e., \( F^* = \min_{x \in \mathbb{D}} F(x, U) = F(x^*, U) \).

From the above definition it is clear that the problem is a bi-objective optimization problem and the minimizer point \( x^* \) should minimize both criteria simultaneously, which hardly happens in practical problems. So, in our problem, the Pareto optimal solutions or efficient solutions are considered as the optimal solution.

3.2. Interpretation of the solution of the problem with interval-valued objective function

The considered interval-valued objective function is defined as \( F: \mathbb{R}^n \rightarrow \mathbb{I} \). Let us denote the optimizer point as \( x^* \in \mathbb{R}^n \) and the optimized value of the objective function as \( F^* \in \mathbb{I} \), i.e., we want to find the point of the search region for which the interval-valued objective function will be optimum. For this type of problem, the optimum interval means the interval having optimum center (expected value of the interval) with minimum width (uncertainty). Let us consider the following examples to visualize the situation:

Example 3.1. (Function of single variable):

\[
F_1(x, U) = U_1 x + U_2 (x + x \cos x) + U_3 (x^3 + \sin^3 x) + U_4 (x + x^3 + x^5)
\]

where \( U_1 = [2, 4], U_2 = [1.5, 4.5], U_3 = [1, 2], U_4 = [-1, 3]. \]

Now, we shall discuss about the optimizer point (or points) and the optimum value of the interval-valued function for different search regions with the help of graph. To plot the interval-valued function \( F_1(x, U) \) of one real variable we first compute the bounds of the function in the prescribed domain of the variable. Here, the graph consists of two curves, as the corresponding function is a single variable interval-valued function. Among the curves, one represents the graph of upper bound of \( F_1(x, U) \) and the other, the graph of the lower bound.

Clearly, the difference between the two curves represents the uncertainty of the interval-valued function. Then we can easily find the upper and lower limits of the optimum interval of the given interval-valued function and the optimizer point. The graph has been plotted with the help of MATHEMATICA 7.0 software. Two different search regions have been considered for this discussion.

(i) When the search region is \( \{x: 0 \leq x \leq 2.5\} \)

The optimizer point \( x^* \in [0, 2.5] \) is to be found so that the interval-valued objective function at \( x = x^* \) will be the optimum interval, i.e., \( F^*_1 = F_1(x^*, U) \) be an optimum interval for the search region \( \{x: 0 \leq x \leq 2.5\} \). The solution is obtained by graphical method. The graph has been presented in Fig. 1.

Clearly, the minimizer \( x^* \) of \( F_1(x, U) \) is obtained at \( x = 0 \) as the uncertainty at that point is minimum. However, in case of optimistic decision-making, one can take the minimizer \( x^* \) of \( F_1(x, U) \) as \( x^* = 2.5 \) ignoring the uncertainty. A similar ambiguity arises in case of finding the maximum value of the objective function. Here, we have considered only the lower bound of the function for the search region \( \{x: 0 \leq x \leq 2.5\} \).

\[
F_1(x, U) = \min_{x \in \mathbb{D}} F(x, U) = F(x^*, U) = F_1(x^*, U) = F^*_1 = \min_{x \in \mathbb{D}} F^U(x, U) = F^U(x^*, U).
\]

Figure 1 Graph of \( F_1(x, U) \) for \( \{x: 0 \leq x \leq 2.5\} \).

Figure 2 Graph of \( F_1(x, U) \) for \( \{x: -1 \leq x \leq 1\} \).
decision-making situations – the optimistic and the pessimistic. However, in real life situations, a rational decision-maker has to face different complex situations where he needs to consider some compromise solution.

(ii) When the search region is \( \{x: -1 \leq x \leq 1\} \)

In this case, it is clear that at \( x = 0 \), the uncertainty of the interval function is least and at \( x = 1 \) and \(-1\), the uncertainty is highest. For maximum value of \( F_1(x, U) \), \( x = 1 \) can be taken as the maximizer point ignoring the uncertainty (optimistic decision-making). A similar dilemma will arise in case of finding the minimum value of \( F_1(x, U) \) at \( x = -1 \) in this case. The graph is shown in Fig. 2. In this connection, there arise some questions: what will be the maximum or minimum value of \( F_1(x, U) \)? Whether the maximizer or the minimizer points will be unique? If it is not unique, then what will be the acceptable maximum or minimum value of \( F_1(x, U) \) to a rational decision-maker?

It is clear that the graphical method is highly complicated for two variable problems. In addition, if we consider the constrained optimization problems instead of simple bound constrained problems, the task will be more difficult. On the other hand, for functions with more than two variables, graphical method is not applicable. In this work, we have developed an alternative technique via multi objective programming to solve this type of problems.

4. Solution procedure

The interval-valued objective function \( F(x, U) \) represents the function value with uncertainty. It is already pointed out that the center \( F^c(x) \) and the width \( F^w(x) \) can be considered as the expected value and the possible extent of uncertainty of the given interval-valued function \( F(x, U) \) respectively [18]. The general structure of the proposed optimization technique is comprised by the following steps:

- **Representation of an interval function in its center and width form**: The objective function with interval coefficients is expressed explicitly in terms of center and width and then we apply our technique directly. But, in practice, it is seen that all types of interval-valued functions cannot always be expressed in the above form. Some of those cases can be tackled by this technique under certain restrictions.

- **Construction of multiobjective optimization problem**: In this step, the given problem is reduced to the corresponding noninterval-valued multiobjective optimization problem. The problems for which the interval objective function is explicitly expressible in terms of center and width, the bi-objective optimization problem can be constructed directly. For others, we have to construct the same under certain conditions. The mathematical treatment of the construction of multiobjective optimization problem is discussed in details below.

- **Solution of multiobjective problem**: The Pareto optimal solution for the constructed multiobjective optimization problem is obtained by the GCM. However, any other suitable methods can be applicable for the same, depending on the problem consideration and requirement of the decision-maker.

Now we shall discuss the different forms of interval objective functions.

**Form 1**: When the given objective function is linear.

In this form, \( F(U, x) = U_1 x_1 + U_2 x_2 + \cdots + U_n x_n \)

\[
= (U^c_1, U^w_1) x_1 + (U^c_2, U^w_2) x_2 + \cdots + (U^c_n, U^w_n) x_n \\
= (U^c_1 x_1 + U^c_2 x_2 + \cdots + U^c_n x_n) + (U^w_1 |x_1| + U^w_2 |x_2| + \cdots + U^w_n |x_n|) \\
= (F^c, F^w)
\]

where \( F^c = U^c_1 x_1 + U^c_2 x_2 + \cdots + U^c_n x_n, \)

\( F^w = U^w_1 |x_1| + U^w_2 |x_2| + \cdots + U^w_n |x_n| \).

Hence, the problem (3.1) can be reformulated as bi-objective optimization problem as follows:

Maximize \( F^c = U^c_1 x_1 + U^c_2 x_2 + \cdots + U^c_n x_n \)

Minimize \( F^w = U^w_1 |x_1| + U^w_2 |x_2| + \cdots + U^w_n |x_n| \) \hspace{1cm} (4.1)

subject to the given constraints.

To reduce the above noninterval bi-objective optimization problem into single objective constrained optimization problem, we have used the GCM. The reduced problems have been solved by MATHEMATICA 7.0 Software package.

Similarly, to minimize \( F(U, x) \) subject to the same constraints, the given problem can be reformulated as a bi-objective optimization problem as follows:

Minimize \( F^c = U^c_1 x_1 + U^c_2 x_2 + \cdots + U^c_n x_n \)

Minimize \( F^w = U^w_1 |x_1| + U^w_2 |x_2| + \cdots + U^w_n |x_n| \) \hspace{1cm} (4.2)

subject to the same constraints.

The above problem (4.2) can be solved in a similar way as mentioned in the maximization case. For illustration, we shall solve the following example,

**Example 4.1.**

Minimize \( F(U, x) = U_1 x_1 + U_2 x_2 + U_3 x_3 + U_4 x_4 + U_5 x_5 + U_6 x_6 \)

subject to \( x_3 - x_4 - x_9 - x_{11} = 0 \)

\( x_4 - x_7 - x_9 = 0 \)

\( -0.007629 \sin(-x_3 + 1.4847699) x_1 x_2 \\
+ 0.00689543 x_1^2 + 200 = 0 \)

\( 0.007629 \sin(x_3 + 1.4847699) x_1 x_2 + x_6 \)

\( + 0.00689543 x_2^2 = 0 \)

\( 0.007629 \cos(-x_3 + 1.4847699) x_1 x_2 + x_4 \)

\( -0.0006565 x_1^2 - 300 = 0 \)

\( 0.007629 \cos(x_3 + 1.4847699) x_1 x_2 + x_5 \)

\( -0.0006565 x_2^2 = 0 \)

where \( U_1 = [29.5, 31], U_2 = [30, 32.1], U_3 = [27, 29.5], U_4 = [28, 28.5] \) and \( x_1, x_2 \in [340, 420], x_3 \in [0, 0.532599999999995], x_4 \in [0, 400], x_5, x_6 \in [0, 1000], x_7 \in [-1000, 1000], x_8 \in [-300, 300], x_9 \in [-100, 100], x_{10} \in [0, 100], x_{11} \in [-100, 1000]. \)

This is a minimization problem. The objective function of the above problem can be rewritten as follows:
\[ F(U, x) = (30.25, 0.75)x_1 + (31.05, 1.05)x_8 + (28.25, 1.25)x_9 + (28.25, 0.25)x_{10} + (30.25, 0.75)x_{11} = (30.25x_7 + 31.05x_8 + 28.25x_9 + 28.25x_{10} + 30.25x_{11}) + 0.75|x_{11}| = (F^C, F^W) \]

where \( F^C = 30.25x_7 + 31.05x_8 + 28.25x_9 + 28.25x_{10} + 30.25x_{11} \) and \( F^W = 0.75|x_1| + 1.05|x_8| + 1.25|x_9| + 0.25|x_{10}| + 0.75|x_{11}| \).

Hence, the corresponding bi-objective optimization problem is as follows:

Maximize \( F^C = 30.25x_7 + 31.05x_8 + 28.25x_9 + 28.25x_{10} + 30.25x_{11} \)

Minimize \( F^W = 0.75|x_1| + 1.05|x_8| + 1.25|x_9| + 0.25|x_{10}| + 0.75|x_{11}| \)

subject to \( x_1 - x_9 - x_{10} - x_{11} = 0 \)

\[
\begin{align*}
& x_4 - x_1 - x_9 = 0 \\
& -0.007629 \sin(-x_1 + 1.4847699)x_1x_2 + 0.00689543x_1^2 + 200 = 0 \\
& 0.007629 \sin(x_3 + 1.4847699)x_1x_2 + x_6 + 0.0089543x_2^2 = 0 \\
& 0.007629 \cos(-x_1 + 1.4847699)x_1x_2 + x_4 - 0.00656x_3^2 - 300 = 0 \\
& 0.007629 \cos(x_1 + 1.4847699)x_1x_2 + x_3 - 0.00656x_5^2 = 0 \\
& 0.007629 \cos(x_1 + 1.4847699)x_1x_2 + x_5 \end{align*}
\]

and \( x_1, x_2 \in [340, 420], x_3 \in [0, 0.5235999999999995], x_4 \in [0, 400], x_5, x_6 \in [0, 1000], x_9 \in [-100, 1000], x_7 \in [-300, 300], x_8 \in [-100, 100], x_9 \in [0, 100], x_{11} \in [-100, 1000]. \)

This problem can be solved by GCM. The ideal objective vector is \((9526.92, 289.79)\) and the Pareto optimal solution is \(x^* = (340, 340, 0, 314.497, 0, -1000, 162.092, 152.405, -32.0719, 16.3378, 15.7341)\) with \(F^\text{P} = [9329.3573, 10004.4956] \).

**Form 2:** When the given objective function is nonlinear in \(x\).

In this form,

\[ F(U, x) = U_1f_1(x) + U_2f_2(x) + \cdots + U_nf_n(x) = (U_1^C, U_2^C, \ldots, U_n^C)f_1(x) + (U_1^W, U_2^W, \ldots, U_n^W)f_n(x) \]

subject to the constraints as given in the original problem.

Here, the ideal objective vector is \((1.39204, 0.0)\) and the Pareto optimal solution is \(x^* = (1.13795, 0.435103, 1.0, 0.217551, 0.351424, 0.0)\) with \(F^\text{P} = [0.085043, 2.686498] \).

**Form 3:** When \(F(U, x)\) is a function with interval-valued argument.

In this form, \( F(U, x) = F(U_1f_1(x) + U_2f_2(x) + \cdots + U_nf_n(x)) = F(U_1^Cf_1(x) + U_2^Cf_2(x) + \cdots + U_n^Cf_n(x)) + F(U_1^Wf_1(x) + U_2^Wf_2(x) + \cdots + U_n^Wf_n(x)) \)

subject to the constraints as given in the original problem.
The above objective function can easily be optimized if \( F(u) \) is either an increasing or a decreasing function of single real variable \( u \).

**Case I: When \( F \) is an increasing function.**

\[
F(U, x) = F[F(x), F^u(x)] = \{F^U(x), F^V(x)\} = \{F^x(x), F^y(x)\}.
\]

**Case II: When \( F \) is a decreasing function.**

\[
F(U, x) = F[F(x), F^u(x)] = \{F^U(x), F^V(x)\} = \{F^x(x), F^y(x)\}.
\]

Hence, the problem (3.1) can easily be reduced to a noninterval bi-objective optimization problem in the above two cases. To illustrate the prescribed technique for the optimization problem of Form 3, we shall solve the following example.

**Example 4.3.**

Minimize \( F(U, x) = e^{x_1 x_2 - U_1 x_2} \)

subject to \( \sin(-x_1 + x_2 - 1) = 0 \)

where \( U_1 = [0.98, 1.03] \), \( U_2 = [1.93, 2.09] \)

and \( x_1 \in [-2, 2] \), \( x_2 \in [-1.5, 1.5] \).

Since the exponential function \( e^x \) is an increasing function of single real variable \( u \), so here Case I will be applicable. Hence, the given problem can be reduced as follows:

Minimize \( F(U, x) = e^{(\sin(-x_1 + x_2 - 1))} = e^{0.098.01.03} \)

subject to \( \sin(-x_1 + x_2 - 1) = 0 \)

and \( x_1 \in [-2, 2] \), \( x_2 \in [-1.5, 1.5] \).

It can easily be solved by our proposed method. For this problem, ideal objective vector is \( (0.163556, 0.018304) \) and the Pareto optimal solution is \( x^* = (0.5, 1.5)^T \) with \( F^{min} = [0.071005, 0.092551] \).

**Form 4:** When \( F(U, x) \) is a sum of several functions with interval-value argument.

In this form

\[
F(U, x) = F_1(U_1 f_1(x) + \ldots + U_user(x)) \pm F_2(U_2 f_2(x) + \ldots + U_user(x)) \pm \ldots \pm F_n(U_n f_n(x) + \ldots + U_user(x))
\]

\[
= F_1\left(\sum_{i=1}^{n} U_i f_i(x)\right) \pm F_2\left(\sum_{i=1}^{n} U_i f_i(x)\right) \pm \ldots \pm F_n\left(\sum_{i=1}^{n} U_i f_i(x)\right)
\]

subject to \( \sin(-x_1 + x_2 - 1) = 0 \)

and \( x_1 \in [-2, 2] \), \( x_2 \in [-1.5, 1.5] \).

Maximize \( F(U, x) = \left(x_2 - 2.75 x_1^2 + 4.9 x_1 - 0.4 x_1^3\right)^3 \)

subject to \( \pi x_1 + x_2 \geq 0 \)

\[
-\pi x_1 + x_2 \leq 0
\]

\( U_1 = [1.2, 1.35], \ U_2 = [4.5, 5.3], \ U_3 = [2.7, 3], \ U_4 = [0.75, 1.1], \ U_5 = [9.5, 9.8] \)

and \( x_1 \in [-1.5, 3.5], \ x_2 \in [0, 15] \).

Here, the objective function has three term functions, these are cubic, square root and exponential, respectively. All these functions are increasing functions of single real variable \( x \). Rewriting the problem, we have

Maximize \( F(U, x) = \left(x_2 - 1.275 x_1^2 + 4.9 x_1 - 0.4 x_1^3\right)^3 \)

subject to \( \pi x_1 + x_2 \geq 0 \)

\[
-\pi x_1 + x_2 \leq 0
\]

\( U_1 = [1.2, 1.35], \ U_2 = [4.5, 5.3], \ U_3 = [2.7, 3], \ U_4 = [0.75, 1.1], \ U_5 = [9.5, 9.8] \)

and \( x_1 \in [-1.5, 3.5], \ x_2 \in [0, 15] \).

The solution will be as follows: Ideal objective vector is \( (7346.51, 28.933) \) and the Pareto optimal solution is \( x^* = (0.318517, 18.1628)^T \) with \( F^{min} = [7142.780957, 7515.29596] \).

**Form 5:** When \( F(U, x) \) is the ratio of two interval-value functions.

In this form,

\[
F(U, x) = \frac{U_1 f_{11}(x) + \ldots + U_{user}(x) + \ldots + \frac{U_2 f_{22}(x) + \ldots + U_{user}(x)}{U_1 f_{11}(x) + \ldots + U_{user}(x)}}{U_1 f_{12}(x) + \ldots + U_{user}(x) + \ldots + \frac{U_2 f_{22}(x) + \ldots + U_{user}(x)}{U_1 f_{12}(x) + \ldots + U_{user}(x)}}
\]

where

\[
F^U(x) = F^U_1(x) \pm F^U_2(x) \pm \ldots \pm F^U_n(x)
\]

\[
F^V(x) = F^V_1(x) \pm F^V_2(x) \pm \ldots \pm F^V_n(x)
\]
where \( f^1_i(x) = U^1_i f_1(x) + \cdots + U^1_m f_m(x) \); \( f^m_i(x) = U^m_i f_1(x) + \cdots + U^m_m f_m(x) \); and \( f^2_i(x) = U^2_i f_1(x) + \cdots + U^2_m f_m(x) \).

In this form, we have to assume that \( 0 \not\in [f^2_i(x), f^2_i(x)] \) \( \forall x \in [l, u] \).

Now, the following cases will arise:

**Case I:** \( f^1_i(x) \geq 0 \) and \( f^2_i(x) > 0 \) \( \forall x \in [l, u] \)

\[
F(U, x) = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = [F^i(x), F^R(x)]
\]

**Case II:** \( f^1_i(x) < 0 < f^2_i(x) \) and \( f^2_i(x) > 0 \) \( \forall x \in [l, u] \)

\[
F(U, x) = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = [F^i(x), F^R(x)]
\]

**Case III:** \( f^1_i(x) \leq 0 \) and \( f^2_i(x) > 0 \) \( \forall x \in [l, u] \)

\[
F(U, x) = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = [F^i(x), F^R(x)]
\]

The other cases, when \( f^2_i(x) < 0 \) \( \forall x \in [l, u] \), can be tackled by changing the maximization problem to minimization problem.

Hence, in this case also, the problem (3.1) can easily be reduced to a noninterval bi-objective optimization problem. To illustrate the method for **Form 5**, let us consider the following example.

**Example 4.5.**

Maximize \( F(U, x) = \frac{U_1 \cos x_1 \cos x_2 + U_2 \log(x_1 + x_2 + 1) + U_3 x_2}{U_4 x_1 + U_5 x_2 + U_6 e^{-x_1 + x_2}} \)

subject to \( x_1 + x_2 \geq 20 \)

\( x_1^2 + x_2^2 \leq 90^2 \)

where \( U_i = [-60, -50], U_2 = [-35, -25], U_3 = [1.14], U_4 = [7.11], U_5 = [5.7], U_6 = [0.5, 1.5] \)

and \( x_1, x_2 \in [10, 100] \).

The given objective function can be rewritten as

\[
F(U, x) = \frac{(-55.5 \cos x_1 \cos x_2 + (-30.5 \log(x_1 + x_2 + 1) + (7.5, 6.5) x_2}{(9.2) x_1 + (6.1) x_2 + (10.5) e^{-(x_1 + x_2)}}
\]

\[
= \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]}
\]

where

\( f^1_i(x) = -55 \cos x_1 \cos x_2 - 30 \log(x_1 + x_2 + 1) + 7.5 x_2 \)

\( f^2_i(x) = [\cos x_1 \cos x_2 + 5 \log(x_1 + x_2 + 1)] + 6.5 x_2 \)

\( f^2_i(x) = 9 x_1 + 6 x_2^2 + e^{-x_1 + x_2} f^2_i(x) = 2 x_1 + x_2^2 + 0.5 e^{-x_1 + x_2} \)

It is very easy to observe that \( f^1_i(x) < 0 < f^2_i(x) \) and \( f^2_i(x) > 0 \) \( \forall x_1, x_2 \in [10, 100] \). So **Case II** formulation will be applicable here.

\[
F(U, x) = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = \frac{[f^1_i(x), f^2_i(x)]}{[f^2_i(x), f^2_i(x)]} = [F^i(x), F^R(x)]
\]

where

\[
F^R(x) = \frac{5 x_1 + 3 x_2^2 + 0.5 e^{x_1 + x_2}}{7 x_1 + 5 x_2^2 + 0.5 e^{x_1 + x_2}}
\]

Solving the above problem by GCM, we get the solution as follows: Ideal objective vector is \((-0.0310945, 0.0150939)\) and the Pareto optimal solution is \( x^* = (28.266558, 62.814655) \) with \( F^{\min} = [-0.002283, 0.041469] \).

5. Numerical examples and comparative study

To test the performance of the proposed method, three numerical examples, taken from Ishibuchi and Tanaka [18], Chanas and Kuchta [20] and Inuiuchi and Sakawa [36] have been solved and the obtained results are compared with the previous results.

**Example 5.1.**

Maximize \( F(U, x) = U_1 x_1 + U_2 x_2 + U_3 x_3 \)

subject to \( 4.6 x_1 + 7.6 x_2 + 3.6 x_3 \leq 21 \)

\( 5.8 x_1 + 3.6 x_2 + 7.8 x_3 \leq 31 \)

\( 7.5 x_1 + 6.5 x_2 + 6.8 x_3 \leq 41 \)

where \( U_1 = [15, 17], U_2 = [15, 20], U_3 = [10, 30] \)

and \( x_1, x_2, x_3 \geq 0 \)

This example has been taken from Ishibuchi and Tanaka [18]. The objective function can be expressed as

\[
F(U, x) = \frac{(-16, 1) x_1 + (17.5, 2.5) x_2 + (20, 10) x_3}{(16 x_1 + 17.5 x_2 + 20 x_3, |x_1| + 2.5 |x_2| + 10 |x_3|)} = (F^C, F^W)
\]

where \( F^C = 16 x_1 + 17.5 x_2 + 20 x_3 \)

and \( F^W = |x_1| + 2.5 |x_2| + 10 |x_3| \)

Hence, the given problem is reduced to the following bi-objective optimization problem:

Maximize \( F^C = 16 x_1 + 17.5 x_2 + 20 x_3 \)

Minimize \( F^W = |x_1| + 2.5 |x_2| + 10 |x_3| = x_1 + 2.5 x_2 + 10 x_3 \)

subject to \( 4.6 x_1 + 7.6 x_2 + 3.6 x_3 \leq 21 \)

\( 5.8 x_1 + 3.6 x_2 + 7.8 x_3 \leq 31 \)

\( 7.5 x_1 + 6.5 x_2 + 6.8 x_3 \leq 41 \)

and \( x_1, x_2, x_3 \geq 0 \)

For this problem, the ideal objective vector is \((88.8061, 0.0)\).

Now solving the above problem by GCM, we get the Pareto optimal solution as \( x^* = (4.14417, 0, 0.538009) \) with \( F^{max} = [67.54264, 86.59116] \) where \( F^C = 17.5 x_2 + 20 x_3 \)

and \( F^W = |x_1| + 2.5 |x_2| + 10 |x_3| \)

Ishibuchi and Tanaka [18] applied the weighted method to solve the reduced multiobjective optimization problem. They obtained a set of three Pareto optimal solutions as \( x^d = (0, 1.13, 3.45), x^d = (3.48, 0, 1.39), x^d = (4.57, 0, 0) \) after choosing the suitable weights varying from 0 to 1. The corresponding objective function values are as follows:
According to the definition of interval order relations of Mahato and Bhunia [22], it is clear that the Pareto optimal solution of our method is better than the same of Ishibuchi and Tanaka [18] as the objective function values of our method is better than \( F(x^r), F(x^u), \) and \( F(x^p).\)

Now, consider the following example taken from Chanas and Kuchta [20].

**Example 5.2.**

Maximize \( F(U, x) = U_1 x_1 + U_2 x_2 \)
subject to \( 10 x_1 + 60 x_2 \leq 1080 \)
\( 10x_1 + 20x_2 \leq 400 \)
\( 10 x_1 + 10 x_2 \leq 240 \)
\( 30 x_1 + 10 x_2 \leq 420 \)
\( 40 x_1 + 5 x_2 \leq 520 \)
where \( U_1 = [-20, 50], \quad U_2 = [0, 10] \)
and \( x_1, x_2 \geq 0 \)

The objective function of the above problem can be expressed as

\[
F(U, x) = (15, 35) x_1 + (5, 5) x_2 = (15 x_1 + 5 x_2, 35 |x_1| + 5 |x_2|) = (15 x_1 + 5 x_2, 35 x_1 + 5 x_2) = \langle F^C, F^W \rangle
\]

where \( F^C = 15 x_1 + 5 x_2 \) and \( F^W = 35 x_1 + 5 x_2 \)

Hence, the given problem reduces to

Maximize \( F^C = 15 x_1 + 5 x_2, \)
Minimize \( F^W = 35 x_1 + 5 x_2 \)
subject to the same constraints.

For this problem, the ideal objective vector is \((210, 0)\). Now solving the same problem with the help of GCM, we get the Pareto optimal solution as \( x^* = (0, 18) \) with \( F_{max} = [0, 180] = (90, 90) \).

To solve the problem, Chanas and Kuchta [20] used a generalized approach with the help of \( t_0, t_1 \)-cut of intervals. According to them, the set of Pareto optimal solutions is \( \{ x^{(1)} = (0, 18), x^{(2)} = (6, 17), x^{(3)} = (8, 16), x^{(4)} = (9, 15), x^{(5)} = (10, 12), x^{(6)} = (13, 0) \} \) depending on the values of the parameters \( t_0 \) and \( t_1 \) lying in the interval \( [0, 1] \). This problem has also been solved by Suprajitno and Bin Mohd [42] using the modified simplex method for interval linear programming problems. In this case, the cost coefficients as well as the decision variables are considered as intervals. The solutions obtained by them are given by \( x_1 = [9.999999999999997, 10.000000000000001], x_2 = [11.999999999999999, 12.000000000000001] \) and \( x_1 = [12.999999999999999, 13.000000000000001], x_2 = [0.000000000000000, 0.000000000000000] \). Clearly, the values of the decision variables are intervals but negligible width.

Now, consider the following example of Inuiguchi and Sakawa [36].

**Example 5.3.**

Maximize \( F(U, x) = U_1 x_1 + x_2 + U_3 x_3 + U_4 x_4 + U_5 x_5 \)
subject to
\( x_1 + 3 x_2 - 4 x_3 + x_4 + x_5 + 2 x_7 + 4 x_8 \leq 10 \)
\( 5 x_1 + 2 x_2 + x_3 - 4 x_4 + 3 x_5 + 7 x_6 + 2 x_7 + 3 x_8 \leq 84 \)
\( 3 x_1 - x_3 - 4 x_5 + x_6 \leq 18 \)
\( -3 x_1 - 4 x_2 + 8 x_3 + 2 x_4 + 3 x_5 - 4 x_6 + 5 x_7 - x_8 \leq 10 \)
\( 12 x_1 + 8 x_2 - 3 x_3 + 4 x_4 + x_5 + x_6 \leq 40 \)
\( x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 12 \)
\( 8 x_1 - 12 x_2 - 3 x_3 + 4 x_4 - x_5 \leq 30 \)
\( -5 x_1 - 6 x_2 + 12 x_3 + x_4 - x_5 \leq 100 \)
where \( U_1 = [0, 1], \quad U_2 = [-1, 1], \quad U_3 = [-1, 1], \quad U_4 = [-3, -1], \quad U_5 = [0, 1], \quad U_6 = [0, 1], \)
and \( x_i \geq 0, \quad j = 1, 2, ..., 8. \)

For this problem, the ideal objective vector obtained is \((22,128,0.692308)\). Now solving the above problem by reducing it in bi-objective optimization problem, we get the solution as \( F_{max} = [10.05588, 15.37842] = (12.71715, 2.66127) \) with the maximizer \( x^* = (0, 3,58715, 3,06422, 2,92659, 0, 0, 0, 9,6422) \). The solution obtained Inuiguchi and Sakawa [36] is \((0, 3.9548, 3.5372, 1.4008, 0, 0, 1837, 6,1122, 7,1189)\) with \( F_{max} = [6.1357, 22.3076] \).

According to Mahato and Bhunia’s [22] order relation (in pessimistic point of view), the above two solutions are incomparable. The expected value of the new solution is worse than the previous one, but the uncertainty of our solution is far less than Inuiguchi and Sakawa’s [36] solution. In this context, we can conclude that the new solution is quite compatible with our goal.

6. Concluding remarks

In this paper, an alternative technique for solving interval objective constrained optimization problems has been proposed by converting the problem to crisp multiobjective optimization problem. Our goal is to find the optimum value of the considered problems in the form of interval with minimum uncertainty. The center and width of the interval objective functions are considered here as the expected value and the extent of uncertainty of the objective function, respectively. However, sometimes, it becomes very difficult to express an interval-valued function in its center and width form. A number of functional forms have been investigated here for which the proposed technique can be applied to construct the corresponding multiobjective optimization problem. Then, GCM has been applied to obtain the Pareto optimal solution of the multiobjective problems. To investigate the effectiveness and efficiency of the proposed technique, an adequate number of examples have been solved. As a result, we can conclude that this technique will be helpful to tackle the uncertainty in different branches of Operational Research and Management Science. For future research, the proposed technique can also be extended for the interval optimization problems with interval-valued constraints.
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