



# Emergently thermalized islands in the landscape

Yun-Song Piao

*College of Physical Sciences, Graduate School of Chinese Academy of Sciences, Beijing 100049, China*

Received 24 September 2007; accepted 12 December 2007

Available online 23 December 2007

Editor: T. Yanagida

---

## Abstract

In this Letter, we point out that in the eternal inflation driven by the metastable vacua of the landscape, it might be possible that some large and local quantum fluctuations with the null energy condition violation can stride over the barriers between different vacua and straightly create some islands with radiation and matter in new vacua. Then these thermalized islands will evolve with the standard cosmology. We show that such islands may be consistent with our observable universe, while has some distinctly observable signals, which may be tested in coming observations. © 2008 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/4.0/).

---

Recently, the string landscape with large number of metastable vacua has received increased attentions [1–3], which generally exhibit the cosmological dynamics as eternal inflation [4,5]. The tunnelling between various vacua of landscape may be mediated by the CDL instanton [6], which behaves as a bubble nucleating in new vacuum. However, the bubble universe nucleated is generally empty and negatively curved, which can hardly become our real world. Thus to produce a universe containing the structure, a period of slow roll inflation inside the bubble is needed [7], which reduces the curvature, provides the primordial density perturbation and large number of entropy required by the observable universe. The occurrence of inflation with enough e-folding number requires that the potential above new minimum should have a nearly flat and long plain, which actually means a fine tuning, since the regions with nearly flat potential are generally expected to be quite rare in the string landscape. Thus it will be interesting to check whether there are other possibilities to lead to a universe like ours.

The landscape will be populated during the eternal inflation. Thus in principle there may be many quantum fluctuations with various spatial and temporal scales in each vacua of the landscape, which may violate the NEC [8], see also Ref. [9] for discussions. However, most of these fluctuations are cosmological irrelevant, since generally they will be generated and then return rapidly to their original vacuum. The significant fluctu-

ations are those enough large, as studied in the island universe model [10,11], in which the vacuum background is that with the observed value of the current cosmological constant. Thus with diverse environments of string landscape, we may expect that it is possible that some large and local quantum fluctuations with the NEC violation can stride over the barrier between different vacua in the landscape and straightly create some thermalized regions in new vacua. This might be phenomenally illustrated by applying the HM instanton [12], since the HM instanton may be regarded as a thermal fluctuation whose rate can be given by the difference in the entropies between the fluctuation and the equilibrium state [2]. In a normal landscape in which the barriers between various minima are neither sharp nor broad [13], the CDL instanton and HM instanton will be expected to co-exist [14]. This suggests that in some regions the bubbles with new vacua will be nucleated, while in other regions the jump to the top of the potential barrier will occur, which is mediated by the HM instanton, and then the field will rapidly roll down along the another side of barrier to new vacuum and the same reheating as that after inflation will occur, see Fig. 1. Thus it may be expected that this thermalized region will be followed by a standard evolution of radiation domination. In this Letter we will check whether some of them are able to become our observable universe.

These thermalized regions are generally different from the bubbles nucleated by the CDL instanton. The bubbles are either empty or dominated by the vacuum energy of new vacua, while the thermalized regions are those filled with radiation and mat-

---

*E-mail address:* [yspiao@gucas.ac.cn](mailto:yspiao@gucas.ac.cn).

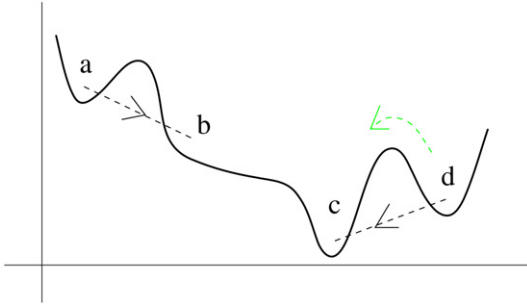


Fig. 1. The landscape of simple potential used to illustrate the islands proposed here. In the eternal inflation, the bubbles with vacuum ‘c’ can be produced by the CDL instanton from either ‘a’ or ‘d’ vacua. The endpoint of tunnelling from ‘a’ would lie on a plain region ‘b’ of potential, and thus the bubble of ‘a’ → ‘c’ will have a period of inflation and then may evolve to our real world, whereas the bubble of ‘d’ → ‘c’ will be empty, as is given by the black dashed lines. However, here we pointed out that ‘d’ → ‘c’ may be also induced by some large and local quantum fluctuation with the NEC violation, which produces some thermalized islands, as is given by the green dashed line. These islands will evolve with the standard FRW cosmology and some of them may like our observable universes. Note that it is also possible that some islands are generated in ‘b’ due to the NEC violating fluctuations in ‘a’. However, in this case the radiation and matter will be diluted quickly and the island universe will then enter an inflation phase dominated by the vacuum energy in ‘b’, which will have the same results as usual slow roll inflation. Here what we concern is the case of the green dashed line, which might provide a different avenue to an observable universe in the landscape. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

ter, which thus more like some islands emerging from the dS background sea. Note that in this Letter what is referred as the “island” is such an emergently thermalized region which is generated by a large fluctuation in original vacuum but emerges in a different vacuum, see the green dashed line in Fig. 1, which is actually slightly not in its original meaning [10]. Thus here the emergence of island will be inevitably related to the tunneling in the landscape, especially the HM tunneling, as has been mentioned. In this sense, the emerging probability of islands in new vacua may be given by that of the HM instanton between different vacua, while in Refs. [10,11] with the cosmological constant background, it is not clear in what case we can calculate the emerging probability of island.

The “thermalized” here means that the resulting state is a thermal state with radiation and matter, in which all components are assumed to be in thermal equilibrium. Thus this part of region is similar to that after the reheating following a slow roll inflation. This can be distinguished from the discussion in Ref. [15], in which the state after the fluctuation is assumed to be an observable universe with structures, and also from the recycling universe proposed in Ref. [16], see also Ref. [17], in which the state after the fluctuation is another dS spacetime.

Though the spawning of island in the landscape is actually a quantum process, it may be regarded phenomenally or semi-classically as an NEC violating evolution to study, as was done in Ref. [11], see also Ref. [18]. Thus in this sense the island actually shares some remarkable successes of inflation model. The reason is that the inflation can be generally regarded as an accelerated stage, and so may defined as an epoch when the comoving Hubble length decreases, which actually occurs equally

during an NEC violating expansion. When the island emerges, the change of local background may be depicted by the drastic evolution of local Hubble parameter ‘h’, where the “local” means that the quantities, such as the scale factor ‘a’ and ‘h’, only character the values of the NEC violating region. We begin with introducing ‘ε’ defined  $-\dot{h}/h^2$ , which can be regarded as  $\epsilon \simeq \frac{1}{h\Delta t} \frac{\Delta h}{h}$ , and thus actually describes the change of h in unit of Hubble time. During the NEC violating fluctuation,  $\dot{h} > 0$ , thus  $\epsilon < 0$  can be deduced. We assume here that ε is constant for simplicity. Thus after making the integral for the definition of ε, we have  $a \sim h^{1/|\epsilon|}$ .

The more rapid the fluctuation is, in principle the stronger it can be, which in some sense is also a reflection of the uncertainty relation between the energy and time in quantum dynamics. Therefore to make the NEC violating fluctuate be so strong as to be able to create the islands of our observable universe, we should take the time scale of the NEC violation

$$\int dt = \int_i^e \frac{dh}{|\epsilon|h^2} \simeq \frac{1}{|\epsilon|h_i} \quad (1)$$

be vanishingly small, where the subscript ‘i’ and ‘e’ denote the initial and end value of the NEC violating fluctuation respectively, and thus  $h_i$  is determined by the energy scale of original vacuum. To make  $\int dt \rightarrow 0$ , we need  $h_i$  or  $|\epsilon|$  is quite large. The larger  $h_i$  is the more violent the fluctuation is anticipative. However, as will be showed here,  $h_i$  is required to be so small as to solve the horizon problem of standard cosmology. Thus Eq. (1) suggests  $|\epsilon| \gg 1$ , which means that though during the fluctuation the change of h is drastic, the expansion of the scale factor is extremely slow, since  $a \sim h^{1/|\epsilon|}$ .

The scale of the NEC violating region is generally required to be larger than the Hubble scale of original vacuum [10], see also Refs. [19–21]. This is also consistent with application of the HM instanton action in which the region tunnelling to the top of the barrier corresponds to the Hubble scale of the original vacuum, which may be understood by using the stochastic approach to inflation [22–25]. This result sets the initial value of local evolution of a, and since it is nearly unchanged during the fluctuation,

$$a_e \simeq a_i \simeq 1/h_i, \quad (2)$$

may be deduced, which means that the smaller  $h_i$  is, the larger the scale of local thermalized region after the fluctuation is. To obtain enough e-folding number for solving the horizon problem of standard cosmology, we need  $a_e \gg 1/h_e$ , thus  $h_i \ll h_e$  is required, which may be also seen as follows:

$$\mathcal{N} \equiv \ln \left( \frac{a_e h_e}{a h} \right) \quad (3)$$

is the e-folding number of mode with some scale  $\sim 1/k$ , where  $k = ah$ , which leaves the horizon before the end of the NEC violating fluctuation, and thus  $k_e$  is the last mode to be generated. When taking  $ah = a_0 h_0$ , where the subscript ‘0’ denotes the present time, we generally have  $\mathcal{N} \sim 50$ , which is required by observable cosmology, see Ref. [26] for a discussion on the

value of  $\mathcal{N}$ . By using Eqs. (2) and (3), we may obtain approximately

$$\mathcal{N} \simeq \ln\left(\frac{h_e}{h_i}\right) \simeq \ln\left(\frac{T_e}{\Lambda_i}\right)^2, \quad (4)$$

where  $\Lambda_i \simeq h_i^{1/2}$  is the energy scale of original vacuum and  $T_e \simeq h_e^{1/2}$  is the thermalized temperature after the NEC violating fluctuation, which may be the same as the reheating temperature after inflation, and  $m_p^2 = 1$  has been taken. When taking  $\mathcal{N} \simeq 50$  and  $T_e \sim 10^{15}$  GeV, we have  $\Lambda_i \sim$  TeV. For a lower  $T_e$ ,  $\Lambda_i$  is required to be smaller. Thus unlike the case in Refs. [10,11], it seems that here the efolding number required to solve the horizon problem of standard cosmology cannot be always obtained. To have an enough efolding number, an enough low original vacuum should be selected. The reason is that the smaller  $\Lambda_i$  is, the larger the Hubble scale of corresponding dS vacuum is, and thus the size of local universe after the fluctuation and the efolding number. This result also suggests that if our universe is actually such an island originated from last vacuum, then the observations made in our universe might have recorded some information on last vacuum. For example, for the efolding number  $\mathcal{N} > 50$ , we may know that its energy scale should be low, and further if the thermalization temperature  $T_e$  is about  $10^{15}$  GeV, our universe must not be originate from a fluctuation of those vacua with the energy density larger than TeV scale. In principle, the  $T_e$  should be required to be lower than that the monopoles production needs, while higher than TeV. This cannot only avoid the monopole problem afflicting the standard cosmology, but helps to provide a solution to the matter genesis.

The calculations of primordial scalar perturbation during the evolution with the NEC violation have been done in Refs. [11, 18,27,28]. The emergence of island in the landscape corresponds to the limit case with  $\epsilon \ll -1$ . In Ref. [11], which firstly calculates the curvature perturbation of island universe, in which the background vacuum is taken as the observed value of cosmological constant, it has been shown that the spectrum of Bardeen potential  $\Phi$  before the thermalization is dominated by an increasing mode and is nearly scale invariant, which under some conditions may induce scale invariant curvature perturbation. Whether the resulting spectrum is scale invariant is determined by the physics at the epoch of thermalization. Thus in this case there is generally an uncertainty, see the discussions in Ref. [11]. However, later it was noted that the curvature perturbation may be also induced by the entropy perturbation [27], or by the perturbation of test scalar field [28] before the thermalization, which under certain conditions may have a nearly scale invariant spectrum and proper amplitude required by the observations. The curvature perturbation induced by the entropy perturbation has same form as that by the Bardeen potential [27], only up to a numerical factor with unite order, which, more importantly, is not dependent of the physical detail of thermalized surface. Thus unlike the case in Refs. [10,11], when with more freedom degrees provided by the landscape, since in low energy the landscape may be approximated as the space of a set of fields with a complicated and rugged potential, it may be

more natural to consider the curvature perturbation induced by the entropy perturbation, which will definitely give the scale invariant spectrum with proper amplitude. In Refs. [11,27], the amplitude of curvature perturbation is given by

$$\mathcal{P}_s \cong |\epsilon| h_e^2, \quad (5)$$

which only depends on  $|\epsilon|$  and  $h_e$ . When  $|\epsilon| \rightarrow \infty$ , the perturbation amplitude will be divergent. Thus for our purpose, the value of  $|\epsilon|$  seems to require a litter fine tuning. For example, having taken  $T_e \sim 10^{15}$  GeV as the thermalization temperature and  $|\epsilon| \sim 10^2$ , we have  $\mathcal{P}_s \sim 10^{-10}$  for (5), which is just the observed amplitude of CMB [29]. In principle,  $\epsilon$  may be taken as a larger value, however, in this case to have a proper amplitude, the thermalization temperature  $T_e$  should be smaller. Here the fine tuning for  $\epsilon$  is actually similar to that appearing in the inflation model. They are related to each other by a dual transformation  $|\epsilon| \leftrightarrow 1/|\epsilon|$  [27], which also corresponds to a duality between their background evolutions, i.e., between the nearly exponent expansion with  $|\epsilon| \simeq 0$  and the slow expansion  $|\epsilon| \rightarrow \infty$ .

The calculations of primordial tensor perturbation during the evolution with the NEC violation have been done in Ref. [30]. The case corresponding to the island universe is given in Eq. (13) of Ref. [11]. The spectrum is

$$\mathcal{P}_T \cong k^3 \left| \frac{v_k^{(e)}}{a_e} \right|^2 \simeq \frac{k^2}{a_e^2}, \quad (6)$$

where the gauge invariant variable  $v_k$  is related to the tensor perturbation  $h_k$  by  $v_k = ah_k$ . It can be seen that the tensor spectrum is quite blue, which is actually a reflection of the rapid increase of background energy density during the NEC violating fluctuation, see Ref. [30] for details. This result indicates that the tensor amplitude in the island will be intensely suppressed on large scale. To calculate the tensor amplitude, we may replace  $a_e$  by using  $k_e = a_e h_e$  in Eq. (6) and have

$$\mathcal{P}_T \simeq h_e^2 \left( \frac{k}{k_e} \right)^2, \quad (7)$$

which gives the value of tensor amplitude in various scales. Thus the value on large scale may be obtained by combining Eqs. (3) and (4), and then substituting the result into Eq. (7), which is  $\mathcal{P}_T \cong h_i^2$ . This is quite low, for example, taking  $h_i^2 \simeq \Lambda_i \sim$  TeV, we have  $\mathcal{P}_T \sim 10^{-60}$ . Note that this result on large scale is the same as that in Ref. [10], in which the dS sea phase is suddenly matched to a radiation dominated phase and the NEC violating mediated phase is neglected. However, in their paper they argued that the tensor spectrum is scale invariant and thus has same amplitude in all scales, while in our calculation the spectrum is actually strong blue, which only is same as that in Ref. [10] on large scale. The discrepancy between Refs. [10] and [11] lies in the NEC violating phase neglected by Ref. [10], which it is that tilts the tensor spectrum. In small scale, which corresponds to take  $k = k_e$ , we have  $\mathcal{P}_T \simeq h_e^2$ , which is actually quite large for a high thermalization scale.

In summary, we point out that in the eternal inflation driven by the metastable vacua of the landscape, it may be possible

that some large and local quantum fluctuations with the NEC violation can stride over the barriers between different vacua in the landscape and straightly create some thermalized islands in new vacua. We show that these islands may be consistent with our observable universe. This result suggests that with the landscape the observable universe may be some of many thermalized regions, which appear either by a slow roll inflation after the nucleation of bubbles, followed by the reheating, or by a straightly thermalization in new vacua without the slow roll inflation, spawned within the eternally inflating background.

The observations in principle can determined whether we live in an emergently thermalized island or in a reheating region after inflation inside bubble. It has been shown that in the island the tensor amplitude is negligible on large scale, while there exists a large class inflation model, such as large field inflation model, with moderate amplitude of tensor perturbation, see, e.g., Ref. [31] for the various inflation models. Thus it seems that the detection of a stochastic tensor perturbation will be consistent with the inflation model, while rule out the possibility that a straightly thermalized region is regarded as our real world. The low tensor amplitude on large scale is also not conflicted with the inflation model, e.g., some small field inflation models. Thus in this case other distinguishabilities need to be considered. The bubble after the nucleation described by the CDL instanton is generally negatively curved, and thus the corresponding universe is an open universe, while the island led to by the NEC violating fluctuation may be closed. Thus in principle if the cosmological dynamics is actually controlled by a landscape with many metastable vacua, the curvature measurement of our universe will be significant to make clear where we live in.

It should be fairly said that the discussions here is inevitably slightly speculative, since a full description for the phenomena with the NEC violation is still lacked for the moment. However, the results showed here might have captured some essentials of emergently thermalized island in the landscape, which might be interesting and significant to phenomenological study of landscape cosmology.

### Acknowledgements

This work is supported in part by NNSFC under Grant Nos. 10405029, 10775180, in part by the Scientific Research

Fund of GUCAS (No. 055101BM03), in part by CAS under Grant No. KJCX3-SYW-N2.

### References

- [1] R. Bousso, J. Polchinski, *JHEP* 0006 (2000) 006.
- [2] S. Kachru, R. Kallosh, A. Linde, S.P. Trivedi, *Phys. Rev. D* 68 (2003) 046005.
- [3] L. Susskind, hep-th/0302219.
- [4] A. Vilenkin, *Phys. Rev. D* 27 (1983) 2848.
- [5] A.D. Linde, *Phys. Lett. B* 175 (1986) 395.
- [6] S.R. Coleman, F. De Luccia, *Phys. Rev. D* 21 (1980) 3305.
- [7] B. Freivogel, M. Kleban, M.R. Martinez, L. Susskind, *JHEP* 0603 (2006) 039.
- [8] S. Winitzki, gr-qc/0111109; T. Vachaspati, astro-ph/0305439.
- [9] S. Winitzki, gr-qc/0612164.
- [10] S. Dutta, T. Vachaspati, *Phys. Rev. D* 71 (2005) 083507; S. Dutta, *Phys. Rev. D* 73 (2006) 063524.
- [11] Y.S. Piao, *Phys. Rev. D* 72 (2005) 103513.
- [12] S.W. Hawking, I.G. Moss, *Nucl. Phys. B* 224 (1983) 180.
- [13] When the barrier is broad enough, the regions of space which tunnel to the top of the barrier will undergo the slow roll eternal inflation. In this case below the slow roll eternally inflating regime a period of usual slow roll inflation is still needed. The relevant cosmology has been discussed in P. Batra, M. Kleban, hep-th/0612083.
- [14] A. Linde, hep-th/0611043.
- [15] L. Dyson, M. Kleban, L. Susskind, *JHEP* 0210 (2002) 011.
- [16] J. Garrige, A. Vilenkin, *Phys. Rev. D* 57 (1998) 2230.
- [17] K.M. Lee, E.J. Weinberg, *Phys. Rev. D* 36 (1987) 1088.
- [18] Y.S. Piao, E. Zhou, *Phys. Rev. D* 68 (2003) 083515.
- [19] E. Farhi, A.H. Guth, *Phys. Lett. B* 183 (1987) 149.
- [20] T. Vachaspati, M. Trodden, *Phys. Rev. D* 61 (2000) 023502.
- [21] A. Borde, M. Trodden, T. Vachaspati, *Phys. Rev. D* 59 (1999) 043513.
- [22] A.A. Starobinsky, in: H.J. de Vega, N. Sanchez (Eds.), *Current Topics in Field Theory, Quantum Gravity and Strings*, in: *Lecture Notes in Physics*, vol. 26, Springer, Heidelberg, 1986, p. 107.
- [23] A.S. Goncharov, A.D. Linde, V.F. Mukhanov, *Int. J. Mod. Phys. A* 2 (1987) 561.
- [24] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood, Chur, Switzerland, 1990, in: *Contemp. Concepts Phys.* 5 (2005) 1, hep-th/0503203.
- [25] A.D. Linde, *Nucl. Phys. B* 372 (1992) 421.
- [26] A.R. Liddle, S.M. Leach, *Phys. Rev. D* 68 (2003) 103503.
- [27] Y.S. Piao, *Phys. Rev. D* 76 (2007) 083505.
- [28] Y.S. Piao, *Phys. Rev. D* 74 (2006) 043509.
- [29] D.N. Spergel, et al., WMAP Collaboration, astro-ph/0603449.
- [30] Y.S. Piao, *Phys. Rev. D* 73 (2006) 047302.
- [31] D.H. Lyth, A. Riotto, *Phys. Rep.* 314 (1999) 1.