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On annihilator ideals of a polynomial ring over a noncommutative ring

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Abstract

Let R be a ring and let $R[x]$ denote the polynomial ring over R. We study relations between the set of annihilators in R and the set of annihilators in $R[x]$.

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1. Introduction

Let R be a ring. A left (right) annihilator of a subset U of R is defined by $l_R(U) = \{a \in R \mid aU = 0\}(r_R(U) = \{a \in R \mid Ua = 0\})$. Consider the polynomial ring $R[x]$ over R. Let $\Gamma = \{r_R(U) \mid U \subseteq R\}$ and let $\Delta = \{r_{R[x]}(V) \mid V \subseteq R[x]\}$. For a polynomial $f(x) \in R[x]$, C_f denotes the set of coefficients of $f(x)$ and for a subset V of $R[x]$, C_V denotes the set $\bigcup_{f \in V} C_f$. Then $r_{R[x]}(V) \cap R = r_R(V) = r_R(C_V)$. Hence we have a map $\Psi : \Delta \to \Gamma$ defined by $\Psi(I) = I \cap R$ for each $I \in \Delta$. Obviously Ψ is surjective.

McCoy [9] proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$ there exists a nonzero element $c \in R$ such that $cg(x) = 0$. That is; if $r_{R[x]}(g(x)) \neq 0$ then $\Psi(r_{R[x]}(g(x))) \neq 0$. We first generalize this result as follows: Let $f(x)$ be an element of the polynomial ring $R[x]$ over a (not necessarily commutative) ring R. If $r_{R[x]}(f(x)R[x]) \neq 0$, then $\Psi(r_{R[x]}(f(x)R[x])) = r_{R[x]}(f(x)R[x]) \cap R \neq 0$.

If U is a subset of R, then $r_{R[x]}(U) = r_R(U)R[x]$. Hence we also have a map $\Phi : \Gamma \to \Delta$ defined by $\Phi(I) = IR[x]$ for every $I \in \Gamma$. Obviously Φ is injective. We

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consider the case when Φ is bijective. Clearly if Φ is bijective, then its inverse is Ψ. Following [11], a ring R is called an *Armendariz ring* if whenever two polynomials $f(x)=\sum_{i=0}^m a_ix^i$, $g(x)=\sum_{j=0}^n b_jx^j \in R[x]$ satisfy $f(x)g(x)=0$ we have $a_ib_j=0$ for every i and j. We show that Φ is bijective if and only if R is Armendariz. We define a ring R to be *quasi-Armendariz* if whenever two polynomials $f(x)=\sum_{i=0}^m a_ix^i$, $g(x)=\sum_{j=0}^n b_jx^j \in R[x]$ satisfy $f(x)R[x]g(x)=0$ we have $a_iRb_j=0$ for every i and j. Let $\Gamma'=\{r_R(U) \mid U$ is an ideal of $R\}$ and let $\Delta'=\{r_{R[x]}(V) \mid V$ is an ideal of $R[x]\}$. Consider the map $\Psi':\Gamma'\to\Delta'$, the restriction of Ψ to Γ'. We show that Ψ' is bijective if and only if R is quasi-Armendariz. We give a sufficient condition for a ring to be quasi-Armendariz and show that quasi-Baer rings are quasi-Armendariz. We show that some extensions of a quasi-Armendariz ring are quasi-Armendariz. Finally, we consider a ring all of whose homomorphic images are quasi-Armendariz.

2. A generalization of McCoy's theorem

McCoy [9] proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$ there exists a nonzero element $c\in R$ such that $cg(x)=0$. We shall generalize this result. We begin with the following lemma.

Lemma 2.1. *Let $f(x)$ and $g(x)$ be two elements of $R[x]$. Then $f(x)Rg(x)=0$ if and only if $f(x)R[x]g(x)=0$.*

Proof. Assume that $f(x)Rg(x)=0$ and take an arbitrary element $\sum_{i=0}^m c_ix^i$ of $R[x]$. Then $f(x)(\sum_{i=0}^m c_ix^i)g(x)=\sum_{i=0}^m f(x)c_ig(x)x^i=0$. This implies $f(x)R[x]g(x)=0$. The "only if part" is clear. □

Theorem 2.2. *Let $f(x)$ be an element of $R[x]$. If $r_{R[x]}(f(x)R[x])\neq 0$, then $r_{R[x]}(f(x)R[x])\cap R\neq 0$.*

Proof. We freely use Lemma 2.1 without mention. Let $f(x)=\sum_{i=0}^m a_ix^i$. If $\deg(f)=0$ or $f=0$, then the assertion is clear. So, let $\deg(f)=m>0$. Assume, to the contrary, that $r_R(f(x)R[x])=0$ and let $g(x)=\sum_{j=0}^n b_jx^j\in R[x]$ be a nonzero element of minimal degree in $r_{R[x]}(f(x)R[x])$. Since $(\sum_{i=0}^m a_ix^i)R[x](\sum_{j=0}^n b_jx^j)=0$, $(\sum_{i=0}^m a_ix^i)R(\sum_{j=0}^n b_jx^j)=0$, and so $a_mRb_n=0$. Hence $a_mR[x]g(x)=a_mR[x](b_{n-1}x^{n-1}+\cdots+b_0)$ and we see $(f(x)R[x]a_m)R[x](b_{n-1}x^{n-1}+\cdots+b_0)=(f(x)R[x]a_m)R[x]g(x)=0$. By hypothesis, we have $a_mR[x](b_{n-1}x^{n-1}+\cdots+b_0)=0$. Therefore $a_m\in l_R(R[x]b_nx^n+R[x](b_{n-1}x^{n-1}+\cdots+b_0))$. Hence $(a_{m-1}x^{m-1}+\cdots+a_0)R[x](b_nx^n+\cdots+b_0)=0$, and so $a_{m-1}Rb_n=0$. Thus we obtain $f(x)R[x](a_{m-1}R[x](b_{n-1}x^{n-1}+\cdots+b_0))=(f(x)(R[x]a_{m-1}R[x])g(x)=0$. Since $g(x)$ is a nonzero element of minimal degree in $r_{R[x]}(f(x)R[x])$, we obtain $a_{m-1}R[x](b_{n-1}x^{n-1}+\cdots+b_0)=0$. Therefore we obtain $a_m,a_{m-1}\in l_R(R[x]b_nx^n+R[x](b_{n-1}x^{n-1}+\cdots+b_0))$. Repeating this process, we obtain

$a_m,\ldots,a_0 \in l_R(R[x]b_n + R[x](b_{n-1}x^{n-1} + \cdots + b_0))$. This implies that $b_0,\ldots,b_n \in r_R(f(x)R[x])$. This is a contradiction. □

A ring R is *semi-commutative* if whenever elements $a,b \in R$ satisfy $ab=0$ then $aRb=0$. We can easily see that reduced rings are semi-commutative.

Corollary 2.3. *Let R be a semi-commutative ring. If $f(x)$ is a zero-divisor in $R[x]$ then there exists a nonzero element $c \in R$ such that $f(x)c=0$.*

3. Armendariz rings and quasi-Armendariz rings

For a ring R, put $rAnn_R(2^R)=\{r_R(U) \mid U \subseteq R\}$ and $lAnn_R(2^R)=\{l_R(U) \mid U \subseteq R\}$. If U is a subset of R, then $r_{R[x]}(U)=r_R(U)R[x]$. Hence we have a map $\Phi : rAnn_R(2^R) \to rAnn_{R[x]}(2^{R[x]})$ defined by $\Phi(I)=IR[x]$ for every $I \in rAnn(R)$. For a polynomial $f(x) \in R[x]$, C_f denotes the set of coefficients of $f(x)$ and for a subset V of $R[x]$, C_V denotes the set $\bigcup_{f\in V} C_f$. Then $r_{R[x]}(V) \cap R = r_R(V) = r_R(C_V)$. Hence we also have a map $\Psi : rAnn_{R[x]}(2^{R[x]}) \to rAnn_R(2^R)$ defined by $\Psi(I)=I \cap R$ for every $I \in \Delta$. Obviously Φ is injective and Ψ is surjective. Clearly Φ is surjective if and only if Ψ is injective, and in this case Φ and Ψ are the inverses of each other.

We consider the case when Φ is surjective.

Following Rege and Chhawchharia [11] a ring R is called an *Armendariz ring* if whenever two polynomials $f(x)=\sum_{i=0}^{m} a_i x^i$, $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)g(x)=0$ we have $a_ib_j=0$ for every i and j. This name is connected with the work of Armendariz [3]. The following proposition shows that Φ is bijective if and only if R is Armendariz.

Proposition 3.1. *Let R be a ring. The following statements are equivalent*:
(1) *R is Armendariz.*
(2) *$rAnn_R(2^R) \to rAnn_{R[x]}(2^{R[x]}); A \to AR[x]$ is bijective.*
(3) *$lAnn_R(2^R) \to lAnn_{R[x]}(2^{R[x]}); B \to R[x]B$ is bijective.*

Proof. (1) ⇒ (2). For a polynomial $f(x) \in R[x]$, C_f denotes the set of coefficients of $f(x)$ and for a subset S of $R[x]$, C_S denotes the set $\bigcup_{f\in S} C_f$. Let S be a subset of $R[x]$ and let $f(x) \in S$. Since R is Armendariz, $r_{R[x]}(f) = r_{R[x]}(C_f) = r_R(C_f)R[x]$. Hence $r_{R[x]}(S) = \bigcap_{f\in S} r_{R[x]}(f) = \bigcap_{f\in S} r_{R[x]}(C_f) = r_R(C_S)R[x]$.

(2) ⇒ (1). Let $f(x)=\sum_{i=0}^{m} a_i x^i$ be a polynomial in $R[x]$. By hypothesis, $r_{R[x]}(f)=BR[x]$ for some right ideal B of R. If $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ satisfies $f(x)g(x)=0$ then $g(x) \in BR[x]$, and hence $b_0,\ldots,b_n \in B \subseteq r_{R[x]}(f)$. Therefore $a_ib_j=0$ for every i and j.

Similarly we can prove (1) ⇔ (3). □

Following Kaplansky [6], a ring R is called a *Baer ring* if the left annihilator of each subset is generated by an idempotent. We note that the definition of Baer rings is left–right symmetric. A ring R is called a *left* (*resp. right*) *p.p. ring* if the left (resp. right) annihilator of each element of R is generated by an idempotent. A left and right p.p. ring is called a p.p. ring.

We obtain [8, Theorems 9 and 10] as an immediate corollary of Theorem 3.1.

Corollary 3.2. *Let R be an Armendariz ring. Then R is a Baer ring* (*resp. p.p. ring*) *if and only if $R[x]$ is a Baer ring* (*resp. p.p. ring*).

Kerr [7] constructed an example of a commutative Goldie ring R whose polynomial ring $R[x]$ has an infinite ascending chain of annihilator ideals.

Corollary 3.3. *Let R be an Armendariz ring. Then R satisfies the ascending chain condition on right annihilators if and only if so does $R[x]$.*

A ring R is called a *quasi-Armendariz ring* if whenever $f(x)=\sum_{i=0}^{m} a_i x^i$, $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)R[x]g(x)=0$, we have $a_i R b_j = 0$ for every i and j. Put $rAnn_R(id(R))=\{r_R(U) \mid U \text{ is an ideal of } R\}$ and $lAnn_R(id(R))=\{l_R(U) \mid U \text{ is an ideal of } R\}$. In a similar way as in the proof of Proposition 3.1, we can prove the following.

Proposition 3.4. *Let R be a ring. The following statements are equivalent*:
(1) *R is quasi-Armendariz.*
(2) $rAnn_R(id(R)) \to rAnn_{R[x]}(id(R[x])); A \to AR[x]$ *is bijective.*
(3) $lAnn_R(id(R)) \to lAnn_{R[x]}(id(R[x])); B \to R[x]B$ *is bijective.*

For semi-commutative rings, in particular, for reduced rings, we have the following.

Corollary 3.5. *Let R be a semi-commutative ring. Then R is Armendariz if and only if R is quasi-Armendariz.*

Proof. Since R is semi-commutative, $R[x]$ is semi-commutative as well. Hence our assertion is clear. □

We shall give an example of a noncommutative ring which is not quasi-Armendariz.

Example 3.6. Let K be a field of characteristic 2 and let $K[x,y]$ be a polynomial ring over K. Consider the factor ring $R=K[x,y]/(x^2,y^2)$ of $K[x,y]$ by the ideal (x^2,y^2) generated by x^2 and y^2. Then, for any positive integer n, $M_n(R)$ is not a quasi-Armendariz ring.

A ring R is a *subdirect sum* of a family of rings $\{R_i\}_{i\in I}$ if there is an injective homomorphism $f: R \to \prod_{i\in I} R_i$ such that, for each $j \in I$, $\pi_j f: R \to R_j$ is a surjective homomorphism, where $\pi_j: \prod_{i\in I} R_i \to R_j$ is the jth projection. Clearly if R is a subdirect sum of Armendariz rings, then R is an Armendariz ring. Similarly we have the following.

Proposition 3.7. *If R is a subdirect sum of quasi-Armendariz rings, then R is a quasi-Armendariz ring.*

Proof. Let I_k $(k \in K)$ be ideals of R such that each R/I_k is quasi-Armendariz and $\bigcap_{k\in K} I_k = 0$. Suppose that two polynomials $f(x)=\sum_{i=0}^{m} a_i x^i$, $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ satisfy $f(x)R[x]g(x) = 0$. Since R/I_j is quasi-Armendariz for each $j \in J$, we have $a_iRb_j \subseteq I_k$ for every i and j. Hence $a_iRb_j \subseteq \bigcap_{k\in K} I_k = 0$. □

Since a semiprime ring is a subdirect sum of prime rings and since prime rings are quasi-Armendariz rings, we have the following corollary.

Corollary 3.8. *A semiprime ring is a quasi-Armendariz ring.*

A submodule N of a left R-module M is called a *pure submodule* if $L\otimes_R N \to L\otimes_R M$ is a monomorphism for every right R-module L. Following Tominaga [13], an ideal I of R is said to be *right s-unital* if, for each $a \in I$ there is an $x \in I$ such that $ax = a$. By [12, Proposition 11.3.13], for an ideal I, the following conditions are equivalent:
(1) I is pure as a left ideal in R;
(2) R/I is flat as a left R-module;
(3) I is right s-unital.

Theorem 3.9. *The following are equivalent*:
(1) $l_R(Ra)$ *is pure as a left ideal in R for any element* $a \in R$;
(2) $l_{R[x]}(R[x]f)$ *is pure as a left ideal in* $R[x]$ *for any element* $f \in R[x]$;
In this case R is a quasi-Armendariz ring.

Proof. Assume that condition (1) holds. First we shall prove that R is quasi-Armendariz. Suppose $(a_0 + a_1x + \cdots + a_mx^m)R[x](b_0 + b_1x + \cdots + b_nx^n) = 0$ with $a_i, b_j \in R$. We shall prove that $a_iRb_j = 0$ for all i, j.

Let c be an arbitrary element of R. Then we have the following equation:

$$\begin{aligned} 0 &= (a_0 + a_1x + \cdots + a_mx^m)c(b_0 + b_1x + \cdots + b_nx^n) \\ &= a_0cb_0 + \cdots + (a_mcb_{n-3} + a_{m-1}cb_{n-2} + a_{m-2}cb_{n-1} + a_{m-3}cb_n)x^{m+n-3} \\ &\quad + (a_mcb_{n-2} + a_{m-1}cb_{n-1} + a_{m-2}cb_n)x^{m+n-2} \\ &\quad + (a_mcb_{n-1} + a_{m-1}cb_n)x^{m+n-1} + a_mcb_nx^{m+n}. \qquad (\dagger) \end{aligned}$$

Then $a_mcb_n = 0$. Hence $a_m \in l_R(Rb_n)$. By hypothesis, $l_R(Rb_n)$ is right s-unital, and hence there exists $e_n \in l_R(Rb_n)$ such that $a_me_n = a_m$.

Replacing c by e_mc in Eq. (†), we obtain

$$a_0e_ncb_0 + \cdots + (a_me_ncb_{n-2} + a_{m-1}e_ncb_{n-1})x^{m+n-2} + a_me_ncb_{n-1}x^{m+n-1} = 0.$$

Then we obtain $a_mcb_{n-1} = a_ne_ncb_{n-1} = 0$. Hence $a_m \in l_R(Rb_n + Rb_{n-1})$. Since $l_R(Rb_{n-1})$ is right s-unital, there exists $f \in l_R(Rb_{n-1})$ such that $a_mf = a_m$. If we put $e_{n-1} = e_nf$, then $a_m e_{n-1} = a_m$ and $e_{n-1} \in l_R(Rb_n + Rb_{n-1})$. Next, replacing c by $e_{n-1}c$ in Eq. (†), we obtain $a_m c b_{n-2} = 0$ in the same way as above. Hence we have $a_m \in l_R(Rb_n + Rb_{n-1} + Rb_{n-2})$. Continuing this process, we obtain $a_m R b_k = 0$ for all $k = 0, 1, \ldots, n$. Thus we get $(a_0 + \cdots + a_{m-1}x^{m-1})R[x](b_0 + \cdots + b_n x^n) = 0$. Using induction on $m+n$, we obtain $a_i R b_j = 0$ for all i, j. Thus we proved that R is quasi-Armendariz. Using [13, Theorem 1] we can see that condition (2) holds.

Conversely, suppose that condition (2) holds. Let a be an element of R. Then $l_{R[x]}(R[x]a)$ is right s-unital. Hence, for any $b \in l_R(Ra)$, there exists a polynomial $f \in R[x]$ such that $bf = b$. Let a_0 be the constant term of f. Then $a_0 \in l_R(Ra)$ and $ba_0 = b$. This implies that $l_R(Ra)$ is right s-unital. Therefore condition (1) holds. □

Corollary 3.10. *Let R be a commutative ring. Then each principal ideal of R is flat if and only if each principal ideal of $R[x]$ is flat. In this case R is an Armendariz ring.*

Proof. For each $a \in R$, $R/l_R(a) \cong Ra$ holds. Hence this corollary follows from Theorem 3.9. □

A ring R is called *quasi-Baer* if the left annihilator of every left ideal of R is generated by an idempotent. Note that this definition is left–right symmetric. Some results of a quasi-Baer ring can be found in [5] and [10]. Let R be a quasi-Baer ring and let $a \in R$. Then $l_R(Ra) = Re$ for some idempotent $e \in R$, and so $R/l_R(Ra) \cong R(1-e)$ is projective. Therefore a quasi-Baer ring satisfies the hypothesis of Theorem 3.9. The first statement of the following corollary is a special case of [4, Theorem 1.8].

Corollary 3.11. *A ring R is a quasi-Baer ring if and only if $R[x]$ is a quasi-Baer ring. In this case R is a quasi-Armendariz ring.*

Now we consider some extensions of quasi-Armendariz rings. Let R be a ring and let n be a positive integer. Let $M_n(R)$ denote the ring of $n \times n$ matrices over R and e_{ij} denote the (i,j)-matrix unit.

Theorem 3.12. *If R is a quasi-Armendariz ring and let S be a subring of $M_n(R)$ such that $e_{ii} S e_{jj} \subseteq S$ for all $i, j \in \{1, \ldots, n\}$. Then S is also a quasi-Armendariz ring.*

Proof. Let $\alpha(x) = \sum_{k=0}^{m} a^k x^k$ and $\beta(x) = \sum_{k=0}^{n} b^k x^k$ be two polynomials in x over S and suppose $\alpha(x) S[x] \beta(x) = 0$. We can consider $S[x]$ as a subset of $M_n(R[x])$. Then, for any $ce_{pq} \in e_{pp} S e_{qq}$ where $c \in R$, $\alpha(x) ce_{pq} \beta(x) = 0$ in $M_n(R[x])$. Considering the (i,j)-components of both sides of this equation, we have $\sum_{t=0}^{m+n} (\sum_{r+s=t} a_{ip}^r c b_{qj}^s) x^t = (\sum_{r=0}^{m} a_{ip}^r x^r) c (\sum_{s=0}^{n} b_{qj}^s x^s) = 0$. Since $\{c \in R \mid ce_{pq} \in e_{pp} S e_{qq}\}$ forms an ideal of R, $(\sum_{r=0}^{m} a_{ip}^r x^r) Rc (\sum_{s=0}^{n} b_{qj}^s x^s) = 0$. Since R is quasi-Armendariz, $a_{ip}^r c b_{qj}^s = 0$ for all r, s. By hypothesis on S, every element of S is a sum of such ce_{pq}, we conclude that $a^r S b^s = 0$ for all r, s. □

To prove that the class of quasi-Armendariz rings is Morita stable, we need the following.

Proposition 3.13. *If R is a quasi-Armendariz ring, then, for any nonzero idempotent $e \in R$, eRe is a quasi-Armendariz ring.*

Proof. Let $f(x) = \sum_{i=0}^{m} a_i x^i$, $g(x) = \sum_{j=0}^{n} b_j x^j \in eRe[x]$ be polynomials satisfying $f(x)eRe[x]g(x) = 0$. Since $f(x)e = f(x)$ and $eg(x) = g(x)$, we obtain $f(x)R[x]g(x) = 0$, and hence $a_i R b_j = 0$ for each i and j. Also since $a_i e = a_i$ and $eb_j = b_j$ for each i and j, we conclude that $a_i eReb_j = 0$ for each i and j. □

Corollary 3.14. *If R is a quasi-Armendariz ring and if R is Morita equivalent to a ring S, then S is a quasi-Armendariz ring.*

For any ring R and any positive integer n, $T_n(R)$ denotes the ring of all $n \times n$ upper triangular matrices over R.

Corollary 3.15. *If R is a quasi-Armendariz ring, then, for any positive integer n, $T_n(R)$ is also a quasi-Armendariz ring.*

In the same way as in [2, Theorem 2] we can prove the following.

Theorem 3.16. *If R is a quasi-Armendariz ring, then the polynomial ring $R[X]$ is a quasi-Armendariz ring for any set X of commutative indeterminates.*

4. Quasi-Gaussian rings

For $f \in R[x]$, the content A_f of f is the ideal of R generated by the coefficients of f. For any subset S of $R[x]$, A_S denotes the ideal $\sum_{f \in S} A_f$. A commutative ring R is Gaussian if $A_{fg} = A_f A_g$ for all $f, g \in R[x]$. We extend this notion to noncommutative rings as follows. A ring R is said to be *quasi-Gaussian* if $A_{fRg} = A_f A_g$ for all $f, g \in R[x]$.

Theorem 4.1. *A ring R is quasi-Gaussian if and only if every homomorphic image of R is quasi-Armendariz.*

Proof. Obviously if R is a quasi-Gaussian ring then every homomorphic image of R is quasi-Armendariz. □

Suppose that every homomorphic image of R is quasi-Armendariz. Let $f, g \in R[x]$. Then $\bar{f}\bar{R}\bar{g} = 0$ in $(R/A_{fRg})[x]$. Since R/A_{fRg} is quasi-Armendariz, $A_{\bar{f}}A_{\bar{g}} = 0$ in R/A_{fRg}. This implies that $A_f A_g = A_{fRg}$.

Corollary 4.2. *If R is a quasi-Gaussian ring and if R is Morita equivalent to a ring S, then S is a quasi-Gaussian ring.*

Proof. Clearly a homomorphic image of R is Morita equivalent to a homomorphic image of the ring S. Hence this corollary follows from Theorem 4.1 and Corollary 3.14. □

Example 4.3. An ideal I of a commutative ring is said to be locally principal if $IR_M = I_M$ is a principal ideal for each maximal ideal M of R. A commutative ring R is said to be arithmetical if its lattice of ideals is distributive, or equivalently, if every finitely generated ideal of R is locally principal. It is well-known that an arithmetical ring is Gaussian, and so, in particular a principal ideal ring is Gaussian (see [1, p. 83] and [2, p. 2269]). Therefore, a ring R which is Morita equivalent to an arithmetical ring, is quasi-Gaussian.

Example 4.4. A ring R is *fully idempotent* if $I^2 = I$ for every two-sided ideal I of R. Obviously a ring R is fully idempotent if and only if every homomorphic image of R is semiprime. Von Neumann regular rings are fully idempotent. By Theorem 4.1 and Corollary 3.8, a fully idempotent ring is a quasi-Gaussian ring.

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