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To understand the rare decay $B_s \to \pi^+\pi^-\ell^+\ell^-$

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ABSTRACT

Motivated by the LHCb measurement, we analyze the $B_s \to \pi^+\pi^-\ell^+\ell^-$ decay in the kinematics region where the pion pairs have invariant masses in the range 0.5–1.3 GeV and muon pairs do not originate from a resonance. The scalar $\pi^+\pi^-$ form factor induced by the strange $\bar{s}s$ current is predicted by the unitarized approach rooted in the chiral perturbation theory. Using the two-hadron light-cone distribution amplitude, we then can derive the $B_s \to \pi^+\pi^-$ transition form factor in the light-cone sum rules approach. Merging these quantities, we present our results for differential decay width which can generally agree with the experimental data. More accurate measurements at the LHC and KEKB in future are helpful to validate our formalism and determine the inputs in this approach.

Very recently, the LHCb has performed an analysis of rare B_s decays into the $\pi^+\pi^-\mu^+\mu^-$ final state [1] and the branching fraction is measured as

$$\mathcal{B}(B_s \to \pi^+\pi^-\mu^+\mu^-) = (8.6 \pm 1.5 \pm 0.7 \pm 0.7) \times 10^{-8}, \qquad (1)$$

where the first two errors are statistical, and systematic respectively. The third error is due to uncertainties on the normalization, i.e. the branching fraction of the $B^0 \to J/\psi(\to \mu^+\mu^-)K^*(\to K^+\pi^-)$. The branching fraction for $B_s \to f_0(980)\mu^+\mu^-$ [1] is determined as:

$$\mathcal{B}(B_s \to f_0(980)(\to \pi^+\pi^-)\mu^+\mu^-) = (8.3 \pm 1.7) \times 10^{-8}, \qquad (2)$$

which lies in the vicinity of the total branching fraction in Eq. (1). Despite the errors, the closeness of the two branching fractions and the differential distribution as shown later in Fig. 4(b) may indicate the dominance of the $f_0(980)$ contributions in the $B_s \to \pi^+\pi^-\mu^+\mu^-$.

The $B_s \to \pi^+\pi^-\mu^+\mu^-$ is a four-body process. Its decay amplitude shows two distinctive features. On the one side, the $\pi^+\pi^-$ final state interaction is constrained by unitarity and analyticity. On the other side, the b mass scale is much higher than the hadronic scale $\Lambda_{\rm QCD}$, which allows an expansion of the hard-scattering kernels in terms of the strong coupling constant α_s and the dimensionless power-scaling parameter $\Lambda_{\rm QCD}/m_b$. In Refs. [2–4], we have developed a formalism that makes use of these two advantages. This approach was also pioneered in Refs. [6,7], and see also Refs. [8–11] for applications to charmless three-body B decays. In doing this, the new formalism can simultaneously merge the perturbation theory at the m_b scale and the low-energy effective theory based on the chiral symmetry to describe the S-wave $\pi\pi$ scattering. The aim of this work is to further examine this formalism by confronting this theoretical framework with the recent data on $B_s \to \pi^+\pi^-\mu^+\mu^-$. An independent analysis that is based on the perturbative QCD approach is also under progress [12].

We start with the differential decay width for $B_s \to \pi^+\pi^-\ell^+\ell^-$. The effective Hamiltonian for the transition $b \to s\ell^+\ell^-$

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

involves various four-quark and the magnetic penguin operators O_i. The $C_i(\mu)$ are the corresponding Wilson coefficients for these local operators O_i. G_F is the Fermi constant, and $V_{tb} = 0.99914 \pm 0.00005$ and $V_{ts} = -0.0405^{+0.011}_{-0.012}$ [13] are the CKM matrix elements. The b and s quark masses are $m_b = (4.66 \pm 0.03)$ GeV and $m_s = (0.095 \pm 0.005)$ GeV [13]. The $b \to sl^+l^-$ transition has the decay amplitude

$$i\mathcal{M}(b \to s\ell^+\ell^-)$$
$$= iN_1 \times \Big\{(C_9 + C_{10})[\bar{s}b]_{V-A}[\bar{\ell}\ell]_{V+A}$$

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$$+ (C_9 - C_{10})[\bar{s}b]_{V-A}[\bar{\ell}\ell]_{V-A}$$
$$+ 4C_{7L}m_b[\bar{s}i\sigma_{\mu\nu}(1+\gamma_5)b]\frac{q^\mu}{q^2} \times [\bar{\ell}\gamma^\nu\ell]$$
$$+ 4C_{7R}m_b[\bar{s}i\sigma_{\mu\nu}(1-\gamma_5)b]\frac{q^\mu}{q^2} \times [\bar{\ell}\gamma^\nu\ell]\Big\}, \quad (3)$$

where $C_{7L} = C_7$ and $C_{7R} = C_{7L}m_s/m_b$, and

$$N_1 = \frac{G_F}{4\sqrt{2}}\frac{\alpha_{\rm em}}{\pi}V_{tb}V_{ts}^*. \quad (4)$$

The $B \to M_1M_2\ell^+\ell^-$ is a four-body decay mode, whose decay amplitude can be obtained by sandwiching Eq. (3) between the initial and final hadronic states. The spinor product $[\bar{s}b]$ will be replaced by corresponding hadronic matrix elements. A general differential decay width for $B \to M_1M_2\ell^+\ell^-$ with various partial wave contributions has been derived using the helicity amplitude in Ref. [14]. In the $B_s \to \pi^+\pi^-\mu^+\mu^-$ case, the S-wave contribution will dominate and thus the angular distribution is derived as

$$\frac{d^3\Gamma}{dm^2_{\pi\pi}dq^2d\cos\theta_l} = \frac{3}{8}\Big[J_1^c + J_2^c\cos(2\theta_l)\Big], \quad (5)$$

where θ_l is the polar angle between the μ^- and the B_s moving direction in the lepton pair rest frame. The angular coefficients are given by

$$J_1^c = \Big\{|\mathcal{A}^0_{L0}|^2 + |\mathcal{A}^0_{R0}|^2 + 8\hat{m}_l^2|\mathcal{A}^0_{L0}\mathcal{A}^{0*}_{R0}|\cos(\delta^0_{L0} - \delta^0_{R0})$$
$$+ 4\hat{m}_l^2|\mathcal{A}^0_t|^2\Big\}, \quad (6)$$

$$J_2^c = -\beta_l^2\Big\{|\mathcal{A}^0_{L0}|^2 + |\mathcal{A}^0_{R0}|^2\Big\}. \quad (7)$$

In the above equations, $\beta_\ell = \sqrt{1 - 4m^2_\ell/q^2}$, and $\hat{m}_\ell = m_\ell/\sqrt{q^2}$. The helicity amplitude is

$$\mathcal{A}^0_{L/R,0} = \sqrt{N_2}i\frac{1}{m_{\pi\pi}}\Bigg[(C_9 \mp C_{10})\frac{\sqrt{\lambda}}{\sqrt{q^2}}\mathcal{F}_1(q^2)$$
$$+ 2(C_{7L} - C_{7R})\frac{\sqrt{\lambda}m_b}{\sqrt{q^2}(m_B + m_{\pi\pi})}\mathcal{F}_T(q^2)\Bigg],$$
$$\mathcal{A}^0_{L/R,t} = \sqrt{N_2}i\frac{1}{m_{\pi\pi}}\Bigg[(C_9 \mp C_{10})\frac{m_B^2 - m^2_{\pi\pi}}{\sqrt{q^2}}\mathcal{F}_0(q^2)\Bigg], \quad (8)$$

where

$$N_2 = \frac{1}{16\pi^2}N_1N_{\pi\pi}\sqrt{1 - 4m^2_\pi/m^2_{\pi\pi}},$$
$$N_{\pi\pi} = \sqrt{\frac{8}{3}}\frac{\sqrt{\lambda}q^2\beta_\ell}{256\pi^3m_B^3}. \quad (9)$$

Here the script t denotes the time-like component of a virtual state decays into a lepton pair. The function λ is related to the magnitude of the $\pi^+\pi^-$ momentum in B_s meson rest frame: $\lambda \equiv \lambda(m^2_{B_s}, m^2_{\pi^+\pi^-}, q^2)$, and $\lambda(a^2, b^2, c^2) = (a^2 - b^2 - c^2)^2 - 4b^2c^2$. The combination of the time-like decay amplitude is introduced in the differential distribution

$$\mathcal{A}^0_t = \mathcal{A}^0_{R,t} - \mathcal{A}^0_{L,t} = 2\sqrt{N_2}C_{10}i\frac{1}{m_{\pi\pi}}\Bigg[\frac{m^2_{B_s} - m^2_{\pi\pi}}{\sqrt{q^2}}\mathcal{F}_0(q^2)\Bigg]. \quad (10)$$

The $B_s \to \pi\pi$ form factors used in Eq. (8) are defined by

$$\langle(\pi^+\pi^-)_S(p_{\pi\pi})|\bar{s}\gamma_\mu\gamma_5 b|\overline{B}_s(p_{B_s})\rangle$$
$$= -i\frac{1}{m_{\pi\pi}}\Bigg\{\Bigg[P_\mu - \frac{m_B^2 - m^2_{\pi\pi}}{q^2}q_\mu\Bigg]\mathcal{F}_1(m^2_{\pi\pi}, q^2)$$
$$+ \frac{m_B^2 - m^2_{\pi\pi}}{q^2}q_\mu\mathcal{F}_0(m^2_{\pi\pi}, q^2)\Bigg\},$$
$$\langle(\pi^+\pi^-)_S(p_{\pi\pi})|\bar{s}\sigma_{\mu\nu}q^\nu\gamma_5 b|\overline{B}_s(p_{B_s})\rangle$$
$$= \frac{\mathcal{F}_T(m^2_{\pi\pi}, q^2)}{m_{\pi\pi}(m_B + m_{\pi\pi})}\Big[(m_B^2 - m^2_{\pi\pi})q_\mu - q^2P_\mu\Big]. \quad (11)$$

As we have shown in Ref. [2], an explicit calculation of the $B_s \to \pi^+\pi^-$ form factors requests the knowledge on generalized light-cone distribution amplitudes [16–20]. The expressions in the light-cone sum rules are given as [2],

$$\mathcal{F}_1(m^2_{\pi\pi}, q^2)$$
$$= N_F\Bigg\{\int_{u_0}^1\frac{du}{u}\exp\Bigg[-\frac{m_b^2 + u\bar{u}m^2_{\pi\pi} - \bar{u}q^2}{uM^2}\Bigg]$$
$$\times\Bigg[-m_b\Phi_{\pi\pi}(u) + um_{\pi\pi}\Phi^s_{\pi\pi}(u) + \frac{1}{3}m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u)$$
$$+ \frac{m_b^2 + q^2 - u^2m^2_{\pi\pi}}{uM^2}\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u)}{6}\Bigg]$$
$$+ \exp[-s_0/M^2]\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u_0)}{6}\frac{m_b^2 - u_0^2m^2_{\pi\pi} + q^2}{m_b^2 + u_0^2m^2_{\pi\pi} - q^2}\Bigg\}, \quad (12)$$

$$\mathcal{F}_-(m^2_{\pi\pi}, q^2) = N_F\Bigg\{\int_{u_0}^1\frac{du}{u}\exp\Bigg[-\frac{m_b^2 + u\bar{u}m^2_{\pi\pi} - \bar{u}q^2}{uM^2}\Bigg]$$
$$\times\Bigg[m_b\Phi_{\pi\pi}(u) + (2-u)m_{\pi\pi}\Phi^s_{\pi\pi}(u)$$
$$+ \frac{1-u}{3u}m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u)$$
$$- \frac{u(m_b^2 + q^2 - u^2m^2_{\pi\pi}) + 2(m_b^2 - q^2 + u^2m^2_{\pi\pi})}{u^2M^2}$$
$$\times\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u)}{6}\Bigg]$$
$$- \frac{u_0(m_b^2 + q^2 - u_0^2m^2_{f_0}) + 2(m_b^2 - q^2 + u_0^2m^2_{\pi\pi})}{u_0(m_b^2 + u_0^2m^2_{\pi\pi} - q^2)}$$
$$\times\exp[-s_0/M^2]\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u_0)}{6}\Bigg\}, \quad (13)$$

$$\mathcal{F}_0(m^2_{\pi\pi}, q^2) = \mathcal{F}_1(m^2_{\pi\pi}, q^2) + \frac{q^2}{m^2_{B_s} - m^2_{\pi\pi}}\mathcal{F}_-(m^2_{\pi\pi}, q^2),$$

$$\mathcal{F}_T(m^2_{\pi\pi}, q^2) = 2N_F(m_{B_s} + m_{\pi\pi})$$
$$\times\Bigg\{\int_{u_0}^1\frac{du}{u}\exp\Bigg[-\frac{(m_b^2 - \bar{u}q^2 + u\bar{u}m^2_{\pi\pi})}{uM^2}\Bigg]$$
$$\times\Bigg[-\frac{\Phi_{\pi\pi}(u)}{2} + m_b\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u)}{6uM^2}\Bigg]$$
$$+ m_b\frac{m_{\pi\pi}\Phi^\sigma_{\pi\pi}(u_0)}{6}\frac{\exp[-s_0/M^2]}{m_b^2 - q^2 + u_0^2m^2_{\pi\pi}}\Bigg\}, \quad (14)$$

Table 1
The $B_s \to f_0(980)$ form factors in the light-cone sum rules at LO and NLO in α_s [15].

LO	$F(0)$	a_F	b_F	NLO	$F(0)$	a_F	b_F
F_1	0.185 ± 0.029	$1.44^{+0.13}_{-0.09}$	$0.59^{+0.07}_{-0.05}$	F_1	0.238 ± 0.036	$1.50^{+0.13}_{-0.09}$	$0.58^{+0.09}_{-0.07}$
F_0	0.185 ± 0.029	$0.47^{+0.12}_{-0.09}$	$0.01^{+0.08}_{-0.09}$	F_0	0.238 ± 0.036	$0.53^{+0.14}_{-0.10}$	$-0.36^{+0.09}_{-0.08}$
F_T	0.228 ± 0.036	$1.42^{+0.13}_{-0.10}$	$0.60^{+0.06}_{-0.05}$	F_T	0.308 ± 0.049	$1.46^{+0.14}_{-0.10}$	$0.58^{+0.09}_{-0.07}$

where

$$N_F = B_0 F_{\pi\pi}(m_{\pi\pi}^2)\frac{m_b+m_s}{2m_{B_s}^2 f_B}\exp\left[\frac{m_{B_s}^2}{M^2}\right],$$
$$u_0 = \frac{m_{\pi\pi}^2+q^2-s_0+\sqrt{(m_{\pi\pi}^2+q^2-s_0)^2+4m_{\pi\pi}^2(m_b^2-q^2)}}{2m_{\pi\pi}^2}. \quad (15)$$

In the above the scalar $\pi\pi$ form factor is defined as

$$\langle 0|\bar{s}s|\pi^+\pi^-\rangle = B_0\, F_{\pi\pi}(m_{\pi\pi}^2), \quad (16)$$

and the B_0 is the QCD condensate parameter:

$$\langle 0|\bar{q}q|0\rangle \equiv -f_\pi^2 B_0, \quad (17)$$

with f_π MeV being the pion decay constant at LO. For the numerics, we use $f_\pi = 91.4$ MeV and $\langle 0|\bar{q}q|0\rangle = -(0.24 \pm 0.01)$ GeV^3 (for a review see Ref. [21]), which corresponds to $B_0 = (1.7 \pm 0.2)$ GeV. The M is a Borel parameter introduced to suppress higher twist contributions. Our formulae can be compared to the results for the $B_s \to f_0(980)$ transition [15], with the correspondence

$$m_{f_0} \leftrightarrow m_{\pi\pi}, \quad \Phi^i_{f_0}(u) \leftrightarrow \Phi^i_{\pi\pi}(u), \quad f_{f_0} \leftrightarrow B_0 F_{\pi\pi}(m_{\pi\pi}^2), \quad (18)$$

where f_{f_0} is the decay constant of $f_0(980)$ defined by the scalar current. The twist-3 distribution amplitudes, $\Phi^s_{\pi\pi}(u)$ and $\Phi^\sigma_{\pi\pi}(u)$, for the scalar $\pi\pi$ state have the same asymptotic forms with the ones for a scalar resonance [22], while the twist ones can be similarly expanded in terms of the Gegenbauer moments. Inspired by this similarity, we can plausibly introduce an intuitive matching:

$$\mathcal{F}_i^{B_s\to\pi\pi}(m_{\pi\pi}^2, q^2) = \frac{1}{f_{f_0}} B_0 F_{\pi\pi}(m_{\pi\pi}^2) F_i^{B_s\to f_0}(q^2). \quad (19)$$

Here we have assumed the dominance of the $f_0(980)$ which is justified in the $B_s \to \pi^+\pi^-\mu^+\mu^-$ as shown in the data in Eq. (2) and Eq. (1).

The $B_s \to f_0(980)$ form factors have been calculated in the light-cone sum rules at leading order (LO) and next-to-leading order (NLO) in α_s [15,23–25], and in the perturbative QCD approach [26–31] in Ref. [32]. The momentum distribution in the form factors has been parametrized in the form:

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i q^2/m_{B_s^2} + b_i(q^2/m_{B_s}^2)}. \quad (20)$$

Numerical results for these quantities where $f_{f_0} = (0.18 \pm 0.015)$ GeV [33] are taken from Ref. [15] and are collected in Table 1. Using a different value for f_{f_0} for instance in Refs. [22,34] will not induce any difference to the generalized form factor, since such effects will cancel as demonstrated in Eq. (19). In the following calculation, we will use the NLO results for the $B_s \to f_0$ transition. Using the LO results can reduce the differential decay width by about 40%.

The scalar $\pi\pi$ form factor $F_{\pi\pi}(m_{\pi\pi}^2)$ has been calculated within a variety of approaches using (unitarized) chiral perturbation theory (CHPT) [35–42] and dispersion relations [43]. In terms of the isoscalar S-wave states

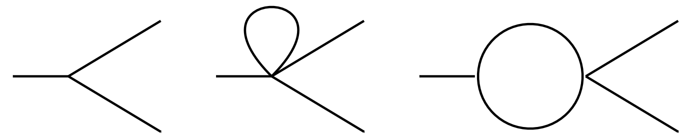

Fig. 1. Feynman diagrams for the scalar form factor at tree-level and one-loop level in CHPT. The wave function renormalization diagrams are not shown here.

$$|\pi\pi\rangle_{I=0} = \frac{1}{\sqrt{3}}\left|\pi^+\pi^-\right\rangle + \frac{1}{\sqrt{6}}\left|\pi^0\pi^0\right\rangle, \quad (21)$$
$$|K\bar{K}\rangle_{I=0} = \frac{1}{\sqrt{2}}\left|K^+K^-\right\rangle + \frac{1}{\sqrt{2}}\left|K^0\bar{K}^0\right\rangle, \quad (22)$$

the scalar form factors are defined as

$$\sqrt{2}B_0\, F_1^s(s) = \langle 0|\bar{s}s|\pi\pi\rangle_{I=0}, \quad (23)$$
$$\sqrt{2}B_0\, F_2^s(s) = \langle 0|\bar{s}s|K\bar{K}\rangle_{I=0}, \quad (24)$$

where the notation ($\pi = 1$, $K = 2$) has been introduced for simplicity, and the convention $F_{\pi\pi}(m_{\pi\pi}^2) = 2/\sqrt{3}F_1^s(m_{\pi\pi}^2)$. In the CHPT, expressions have already been derived by calculating the diagrams in Fig. 1 up to NLO [36,40–42]:

$$F_1^{\rm CHPT}(s) = \frac{\sqrt{3}}{2}\left[\frac{16m_\pi^2}{f^2}\left(2L_6^r - L_4^r\right) + \frac{8s}{f^2}L_4^r + \frac{s}{2f^2}J_{KK}^r(s) + \frac{2}{9}\frac{m_\pi^2}{f^2}J_{\eta\eta}^r(s)\right], \quad (25)$$
$$F_2^{\rm CHPT}(s) = 1 + \frac{8L_4^r}{f^2}\left(s - m_\pi^2 - 4m_K^2\right) + \frac{4L_5^r}{f^2}\left(s - 4m_K^2\right) + \frac{16L_6^r}{f^2}\left(4m_K^2 + m_\pi^2\right) + \frac{32L_8^r}{f^2}m_K^2 + \frac{2}{3}\mu_\eta + \left(\frac{9s - 8m_K^2}{18f^2}\right)J_{\eta\eta}^r(s) + \frac{3s}{4f^2}J_{KK}^r(s). \quad (26)$$

With the increase of the invariant mass of the $\pi\pi$ system, higher order contributions become more important. It has been proposed that the unitarized approach can sum higher order corrections and extend the applicability to the scale around 1 GeV [44]. A sketch of the resummation scheme is shown in Fig. 2. In this figure, the $K(s)$ is the S-wave projected kernel of meson–meson scattering amplitudes [40,41]:

$$K(s) = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}, \quad (27)$$

$$K_{11} = \frac{2s - m_\pi^2}{2f^2}, \quad K_{12} = K_{21} = \frac{\sqrt{3}s}{4f^2}, \quad K_{22} = \frac{3s}{4f^2}, \quad (28)$$

where the subscripts 1, 2 denote the $\pi\pi$ and $K\bar{K}$ state, respectively. The function $g(s)$ is the loop integral which can be calculated in the cutoff-regularization scheme with $q_{\rm max} \sim 1$ GeV being the cutoff [cf. Erratum of Ref. [44]] or in dimensional regularization. In the latter scheme, the meson loop function $g_{ii}(s)$ is given by

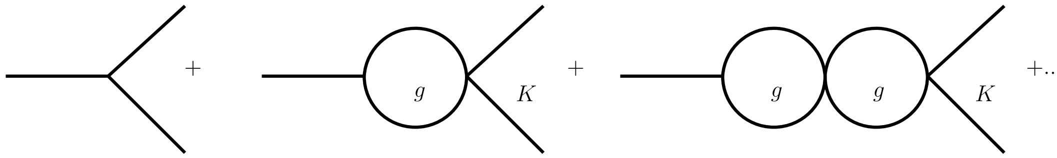

Fig. 2. The s-channel diagrams to the scalar $\pi\pi$ form factors in CHPT. With the increase of the $\pi\pi$ invariant mass, higher order contributions may become important. In the unitarized approach [44], these diagrams can be summed.

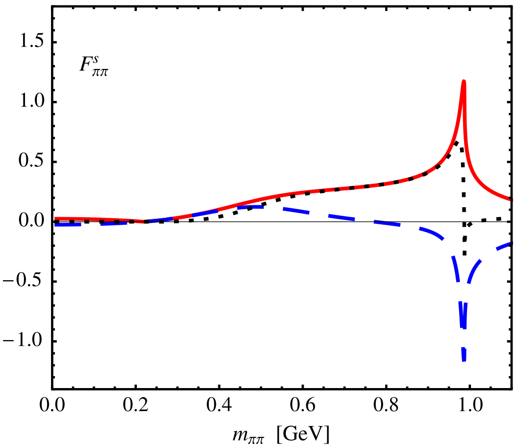

Fig. 3. The $\pi\pi$ scalar form factor obtained in the unitarized chiral perturbation theory. The modulus, real part and imaginary part are shown in solid, dashed and dotted curves, respectively.

$$J^r_{ii}(s) \equiv \frac{1}{16\pi^2}\left[1-\log\left(\frac{m_i^2}{\mu^2}\right)-\sigma_i(s)\log\left(\frac{\sigma_i(s)+1}{\sigma_i(s)-1}\right)\right]$$
$$= -g_{ii}(s), \tag{29}$$

with $\sigma_i(s)=\sqrt{1-4m_i^2/s}$. Imposing the unitarity constraints, the scalar form factor can be expressed in terms of the algebraic coupled-channel equation [36,38]

$$F(s) = R(s)[I+g(s)K(s)]^{-1}$$
$$= R(s)[I-g(s)K(s)]+\mathcal{O}(p^6), \tag{30}$$

where the above equation has been expanded up to NLO in the chiral expansion. The $R(s)=(R_1(s),R_2(s))$ includes both tree-level contributions, and other higher order corrections that have not been summed. Thus this function has no right-hand cut, and can be obtained by matching onto the CHPT results in Eqs. (25)–(26) [38,45]:

$$R_1(s) = \frac{\sqrt{3}}{2}\left\{\frac{16m_\pi^2}{f^2}\left(2L_6^r-L_4^r\right)+\frac{8s}{f^2}L_4^r\right.$$
$$\left.-\frac{m_\pi^2}{72\pi^2f^2}\left[1+\log\left(\frac{m_\eta^2}{\mu^2}\right)\right]\right\}, \tag{31}$$

$$R_2(s) = 1+\frac{8L_4^r}{f^2}\left(s-4m_K^2-m_\pi^2\right)+\frac{4L_5^r}{f^2}\left(s-4m_K^2\right)$$
$$+\frac{16L_6^r}{f^2}\left(4m_K^2+m_\pi^2\right)+\frac{32L_8^r}{f^2}m_K^2+\frac{2}{3}\mu_\eta$$
$$+\frac{m_K^2}{36\pi^2f^2}\left[1+\log\left(\frac{m_\eta^2}{\mu^2}\right)\right]. \tag{32}$$

With the above formulae and the fitted results for the low-energy constants L_i^r in Ref. [38] (evolved from m_ρ to the scale $\mu=2q_{\rm max}/\sqrt{e}$), we show the strange $\pi\pi$ form factor in Fig. 3. The modulus, real part and imaginary part are shown as solid, dashed and dotted curves.

Equipped with the results for scalar form factor and heavy to light transition, we can explore the differential branching fraction for the $B_s\to\pi^+\pi^-\mu^+\mu^-$. Our theoretical results for $d\mathcal{B}/dm_{\pi\pi}$ are given in the left panel of Fig. 4. This clearly shows the peak corresponding to the $f_0(980)$. In order to compare with the experimental data, we also give the binned results on the right panel in Fig. 4 from 0.5 GeV to 1.3 GeV. Theoretical errors shown in this panel arise from the ones in the form factors. The experimental data (with triangle markers) has been normalized to the central value given in Eq. (1). The comparison in this panel shows a general agreement between our theoretical prediction and the experimental data except in a few bins. This agreement is very encouraging.

In spite of the agreement, there exist some differences in our results and data. For instance our theoretical result does not show the enhancement at $m_{\pi\pi}\simeq(800, 1100, 1250)$ MeV as given in the data. The excess at 800 MeV may come from the tail of the $B_s\to\eta(\to\pi^+\pi^-\pi^0,\pi^+\pi^-\gamma)\mu^+\mu^-$, while in the range above 1 GeV, the contribution from the $f_0(1370)$ may not be negligible.

Integrating out the $m_{\pi\pi}$, we have the branching fraction:

$$\mathcal{B}(B_s\to f_0(980)(\to\pi^+\pi^-)\mu^+\mu^-) = (4.1\pm1.6)\times10^{-8}, \tag{33}$$

which deviates from the data by about 2σ. However, one expects the experimental result in Eq. (2) would get somewhat reduced. This can be witnessed by the $B^-\to J/\psi K^-$ and $B^-\to K^-\mu^+\mu^-$ [13]

$$\frac{\mathcal{B}(B^-\to K^-\mu^+\mu^-)}{\mathcal{B}(B^-\to J/\psi K^-)}$$
$$= \frac{(4.49\pm0.23)\times10^{-7}}{(1.027\pm0.031)\times10^{-3}}\sim 4.4\times10^{-4}. \tag{34}$$

If this ratio were not sensitive the light meson in the final state which is true in most cases, the branching fraction for the $B_s\to J/\psi f_0(980)$ [13]

$$\mathcal{B}(B_s\to J/\psi f_0) = (1.39\pm0.14)\times10^{-4},$$

would indicate

$$\mathcal{B}(B_s\to f_0(980)\mu^+\mu^-)\sim 6.1\times10^{-8}. \tag{35}$$

This value is smaller by about 30% than the central value given in Eq. (1), and is more consistent with our theoretical result. The future measurement with more data at the experimental facilities like LHC and KEKB will be able to clarify this point, and thus to examine our theoretical formalism more precisely. We strongly encourage our experimental colleagues to conduct such measurements.

In summary, in this work we have analyzed the $B_s\to\pi^+\pi^-\ell^+\ell^-$ that has focused on the region where the pion pairs have invariant masses in the range 0.5–1.3 GeV and muon pairs do not originate from a resonance. We have adopted the approach proposed in our previous work [2–4] (see also Ref. [5] for an overview) which makes uses of the two-hadron light-cone

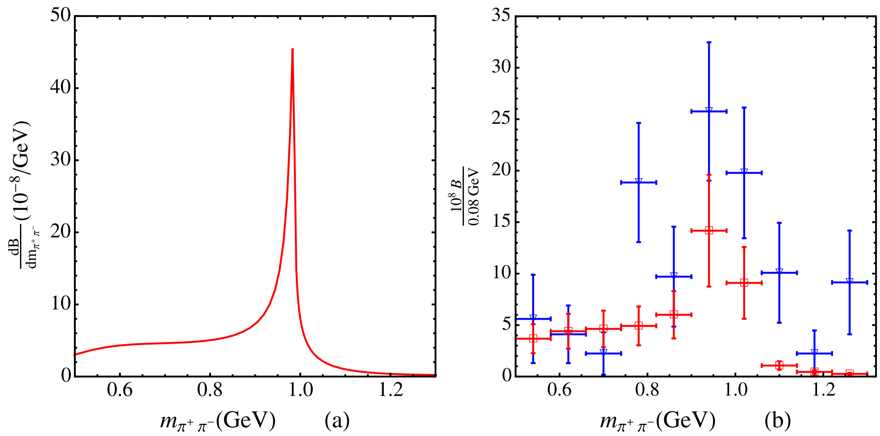

Fig. 4. The differential branching ratio for the $B_s \to \pi^+\pi^-\ell^+\ell^-$. The experimental data (with triangle markers) has been normalized to the central value of the branching fraction: $\mathcal{B}(B_s^0 \to \pi^+\pi^-\mu^+\mu^-) = (8.6 \pm 1.5 \pm 0.7 \pm 0.7) \times 10^{-8}$. Theoretical predictions (with square markers) are based on the result for the time-like scalar form factors derived in the unitarized CHPT.

distribution amplitude. The scalar $\pi^+\pi^-$ form factor induced by the strange $\bar{s}s$ current is predicted by the unitarized chiral perturbation theory. The heavy to light transition can then be handled by the light-cone sum rules approach. Merging these quantities, we have presented our theoretical results for differential decay width and compared with the experimental data. Except in a few bins, our theoretical results are in alignment with the data. We have also discussed the disagreement and given our expectation. More accurate measurements at the LHC and KEKB in future are helpful to validate/falsify our formalism and determine the inputs in this approach.

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