Design of Time-constrained Guidance Laws via Virtual Leader Approach

Zhao Shiyu\(^a\), *, Zhou Rui\(^a\), Wei Chen\(^a\), Ding Quanxin\(^b\)

\(^a\)School of Automation Science and Electrical Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

\(^b\)National Defense Key Lab of Fire Control Technique, Luoyang Institute of Electro-optical Equipment of AVIC, Luoyang 471009, China

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Abstract

Guidance problems with flight time constraints are considered in this article. A new virtual leader scheme is used for design of guidance laws with time constraints. The core idea of this scheme is to adopt a virtual leader for real missiles to convert a guidance problem with time constraints to a nonlinear tracking problem, thereby making it possible to settle the problem with a variety of control methods. A novel time-constrained guidance (TCG) law, which can control the flight time of missiles to a prescribed time, is designed by using the virtual leader scheme and stability method. The TCG law is a combination of the well-known proportional navigation guidance (PNG) law and the feedback of flight time error. What's more, this law is free of singularities and hence yields better performances in comparison with optimal guidance laws with time constraints. Nonlinear simulations demonstrate the effectiveness of the proposed law.

Keywords: guidance; missiles; salvo attack; time constraint; virtual leader

1. Introduction

Previous guidance laws have considered a variety of constraints such as minimum time homing\(^[1]\), acceleration saturation\(^[2]\) and terminal impact angle constraints\(^[3-4]\). Guidance problems with flight time constraints have just emerged in recent years as the result of rapid development of cooperative control of multi-vehicle systems, for which, cooperative guidance\(^[5-9]\) of multiple missiles has been devised as an effective countermeasure against the threat of interceptors. As a typical cooperative guidance mission, salvo attack\(^[5]\) requires multiple missiles to hit the target simultaneously, from which, the demand for controlling flight time stems.

Guidance laws with flight time constraints were introduced for the first time in Ref. [5] where an impact-time-control guidance (ITCG) law is proposed and applied to salvo attack missions. J. I. Lee, et al.\(^[6]\) proposed a guidance law called impact-time-and-angle-control guidance (ITACG) which realizes concurrent control of flight time and terminal impact angles. Y. A. Zhang, et al.\(^[7]\) proposed a leader-follower method, with which a time-constrained cooperative guidance scheme was designed.

In fact, path planning problems for unmanned aerial vehicles (UAVs)\(^[10-12]\) have also considered flight time constraints. However, solutions to these time-constrained problems mainly rely on the decoupling of space and time in the problem formulation. Great challenges would be encountered when attempts are made to design missile guidance laws without space and time decoupling.

This article proposes a novel virtual leader approach which can convert a time-constrained guidance problem to a nonlinear tracking problem. This approach not only enables accurate controlled flight time but also obviates the need for time-to-go estimation. Then a new time-constrained guidance law named TCG is designed by using the virtual leader
approach. The proposed law involves the well-known proportional navigation guidance (PNG) command and the feedback of flight time error. As a guidance law with asymptotic stability, TCG is free of singularities and yields better performance compared with existing optimal guidance laws. Nonlinear simulations demonstrate the effectiveness of TCG.

2. Virtual Leader Approach

This section presents a novel virtual leader approach and explains how to use it to convert a time-constrained guidance problem to a nonlinear tracking problem.

Fig.1 shows the homing guidance geometry of the missile $M$ against a stationary target. It is assumed that the missile speed $V$ is constant through the engagement span and the autopilot lag is negligible. The applied acceleration command $a_m$ is normal to the velocity vector $\lambda$, $\sigma$ and $\eta$ denote line-of-sight (LOS) angle, velocity heading angle and the angle between LOS and velocity vector, respectively. Now a virtual leader $M^*$ is introduced against the same target as the missile $M$ homes on. The superscript * for $V^*$, $\lambda^*$, $\sigma^*$, $\eta^*$ and $r^*$ is used to represent parameters of the virtual leader corresponding to those of $M$.

![Fig.1 Homing guidance geometry with virtual leader approach.](image)

The guidance task is that the missile $M$ must hit the target at the pre-designated time $T_d$. The idea of virtual leader approach can be summarized as follows: if the flight time of virtual leader $M^*$ is equal to $T_d$ and the missile $M$ follows the leader effectively to ensure that its flight time is equal to that of the virtual leader, then the missile can hit the target at $T_d$. Subsequently, two critical problems must be solved: 1) How to control the flight time of the virtual leader? 2) How to guide the missile to ensure its flight time to be equal to that of the virtual leader?

For the first problem, since the virtual leader is introduced artificially, its motion can be designed freely according to our needs. For example, introduce a virtual leader approaching the target along a straight line which means $\lambda^* = \sigma^*$ and $\eta^* = 0$. In this case, the flight time of the virtual leader can be controlled to $T_d$ if the initial virtual leader-target range is set to $r^*(t_0) = V^*T_d$. For the second problem, it is clear that the identical motion parameters that the missile and the virtual leader possess will ensure their identical flight time. For example, if $r(t_0) = r^*(t_0)$ and $\eta(t_0) = \eta^*(t_0)$, the missile and the virtual leader would hit the target at the same time. This means the flight time of the missile will converge to that of the virtual leader when $r \to r^*$ and $\eta \to \eta^*$.

The key idea of the virtual leader approach lies in indirect control of flight time. By this approach, a time-constrained guidance problem can be converted to a nonlinear tracking problem, for solution of which various nonlinear control tools can be applied. By adopting different virtual leaders and different control methods, different guidance laws can be obtained. In addition, an advantage brought by the virtual leader approach is that no information on the time-to-go of the missile is required.

3. Time-constrained Guidance Law

In this section, a new time-constrained guidance law called TCG with asymptotic stability is designed by using virtual leader approach.

3.1. Virtual leader-target kinematics

As mentioned above, the first step is to introduce a virtual leader able to hit the target at $T_d$. Consider a simple case where the virtual leader approaches the target along a straight line. Then the motion equations of the virtual leader are

$$
\begin{align*}
\dot{r}^*(t) &= r^*(t_0) - V^*t \\
\eta^*(t) &= 0
\end{align*}
$$

For the sake of simplicity, choose $V^* = V$. Clearly, by setting

$$
\dot{r}^*(t_0) = VT_d
$$

the flight time of the virtual leader can be controlled to be $T_d$. In addition, the virtual leader approach poses no requirement for the initial position of the virtual leader.

3.2. Missile-target kinematics

This subsection proposes a novel missile-target kinematics in order to derive a time-constrained guidance law free of any singularities.
Consider the missile-target engagement geometry shown in Fig.2. Now establish the Cartesian frame of coordinates with the origin at the initial position of the missile and let \( X \) axis be along with the initial velocity vector. \( y \) is the perpendicular distance from the missile on the \( X \) axis to the target. The independent time variable \( t \) is omitted unless stated otherwise. At the initial time we have

\[
\begin{align*}
\dot{y} &= -a_m \\
\eta &= \lambda
\end{align*}
\]

(3) (4)

With a small LOS angle the missile-target range can be expressed as

\[
r = y/\lambda
\]

(5)

Differentiating Eq. (5) twice gives

\[
\ddot{r} = \frac{\left(\dot{y} \lambda - y \dot{\lambda}\right)\lambda^2 - 2\left(\dot{y} \lambda - y \dot{\lambda}\right)\lambda \ddot{\lambda} - 2\dot{y} \dot{\lambda} \lambda^2}{\lambda^4} = \frac{\dot{y} \lambda - y \dot{\lambda}}{\lambda} - \frac{2\dot{r} \lambda}{\lambda} = \frac{\dot{y} - r \ddot{\lambda} - 2\dot{r} \dot{\lambda}}{\lambda}
\]

(6)

Substituting Eqs.(3)-(4) into Eq.(7) yields a novel missile-target kinematics

\[
\ddot{r} = -a_m - r \ddot{\lambda} - 2\dot{r} \dot{\lambda} \eta
\]

(8)

Note: The missile-target kinematics can also be deduced from the usually used motion equations

\[
\dot{r} = -V \cos \eta
\]

\[
\ddot{r} = \dot{V} \sin \eta = \left(\frac{V^2 \sin \eta}{r} - a_m \right) \sin \eta
\]

(9)

However, Eq. (9) will result in guidance laws with a singularity at \( \sin \eta = 0 \).

3.3 Time-constrained guidance law with asymptotic stability

As mentioned in Section 2, the flight time of the missile \( M \) will converge to that of the virtual leader \( M^* \) if

\[
r \to r^*, \eta \to \eta^* \text{ as } t \to \infty
\]

(10)

Note that \( \eta \to \eta^* = 0 \) drives the missile velocity in the LOS direction and subsequently results in small miss distance and small acceleration command in the terminal phase.

Define the distance error as

\[
e_r = r^* - r
\]

(11)

From Eq.(1) and Eq.(11) the velocity error can be obtained as

\[
\dot{e}_r = -V - \dot{r}
\]

(12)

Note that \( \dot{r} \) can also be expressed by \( \dot{r} = -V \cos \eta \).

Then Eq.(12) can be written as \( \dot{e}_r = -V + V \cos \eta \).

Hence \( \dot{e}_r = 0 \Leftrightarrow \eta = 0 \), \( \eta \in [0, 2\pi) \). Subsequently, Eq.(10) can be rewritten as

\[
e_r \to 0, \dot{e}_r \to 0 \text{ as } t \to \infty
\]

(13)

Define state variables as

\[
z_1 = e_r, z_2 = \dot{e}_r
\]

(14)

Then the state equations are

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
-1
\end{bmatrix}
\]

(15)

The objective is to obtain control laws to ensure the convergence of the system Eq. (15). Substituting Eq. (8) into Eq. (15) yields

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} \bullet
f(r, \dot{r}, \lambda, \dot{\lambda}, \eta, a_m)
\]

(16)

where

\[
\begin{align*}
z_1(t_0) &= VT_d - V_{t_0} - r(t_0) \\
z_2(t_0) &= -V + V \cos \eta(t_0)
\end{align*}
\]

(17)

\[
f(r, \dot{r}, \lambda, \dot{\lambda}, \eta, a_m) = \frac{a_m + r \dot{\lambda} + 2 \dot{r} \dot{\lambda}}{\eta}
\]

(18)

Choose the acceleration command as

\[
a_m = -r \ddot{\lambda} - 2 \dot{r} \dot{\lambda} + \eta u
\]

(19)

Then the system Eq. (16) can be transformed into

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

(20)

where \( u \) is a new control command to be designed to ensure the convergence of system Eq. (20). Of course, there are many other control methods that can be used to design \( u \). Here design \( u \) as a state-feedback law as follows:

\[
u = k_1 z_1 + k_2 z_2 \quad (k_1, k_2 \in \mathbb{R})
\]

(21)

Eq. (20) can be rewritten as

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix}
\]

(22)

It is easy to prove that \((z_1, z_2)\) asymptotically converges to \((0, 0)\) if and only if \( k_1 < 0 \) and
From Eq.(19) and Eq.(21), an error feedback time-constrained guidance law with asymptotic stability can be derived as

$$a_m = Nv_e \dot{\lambda} + \Delta_1 + \Delta_2$$

where $N = 2$ is the navigation ratio and $v_e = -\dot{r}$ is the closing velocity.

Guidance law in Eq. (23) consists of three terms which are, respectively, the well-known PNG command, an error feedback command for reducing the flight time error and driving missile velocity into the LOS direction and thirdly $\Delta_2$. The third term has a second order derivation of the LOS angle, which might induce difficulties in implementation. Numerous simulations demonstrate that $\Delta_2$ has little influence on the general performances of the law. Therefore, with $\Delta_2$ omitted, a practical time-constrained guidance law can be obtained as

$$a_m = Nv_e \dot{\lambda} + \eta (k_e \dot{e} + k_e \dot{e})$$

where $\eta = 0$.

Note: Since the feedback gain contains $\eta$, the error feedback loop would be cut off and $a_m$ will remain zero through the engagement if

$$\eta = 0$$

As a result, the proposed law is not able to control the flight time in the case of $\eta(t_0) = 0$. However, zero miss distance is still guaranteed in this case since $\eta(t_0) = 0$ just means the missile velocity aims at the target directly.

### 4. Nonlinear Numerical Simulations

Nonlinear numerical simulations are performed in this section to investigate the characteristics of the proposed law. The guidance law in Eq. (25) is called TCG and the one in Eq. (23) is denoted as TCG+$\Delta_2$.

Fig.3 shows the block diagram of the homing loop in which the acceleration saturation is 5g and the flight control dynamics is chosen as

$$W(s) = \frac{1}{s^2 + \frac{2\zeta\omega}{\omega} + 1}$$

where $\omega = 20$ rad/s and $\zeta = 0.707$. Assume accurate $\eta, \dot{\lambda}, r$ and $\dot{r}$ can be obtained. $\dot{\lambda}$ is given by

$$\dot{\lambda} = \frac{s - \tau}{s + \tau}$$

where $\tau = 0.1$ s.

Fig.4 shows the trajectories by TCG and TCG+$\Delta_2$ with different values of $\tau_d$. It is observed that the disparities between real flight times and designated ones are within 1 s in all cases. The trajectories and flight times by TCG are quite close to those by TCG+$\Delta_2$. It can be seen from Fig.5 that TCG+$\Delta_2$ results in relatively larger acceleration commands than TCG, especially, in the terminal phase. $e_e$ and $\dot{e}$, for the case of TCG are, respectively, shown by Fig.6 and Fig.7, where the errors converge to zero asymptotically. That means the flight time error is reduced to zero and the velocity vector is driven in the LOS direction gradually.

Consider the case where the target is moving slowly. Here is investigated a scenario where the target moves with a 45° heading angle at a speed of 20 m/s. It is noticeable that all flight time errors by TCG are less than 1 s in the cases of $\tau_d = 40, 50, 60$ s.

Fig.8 shows the trajectories by TCG with various initial heading angles as 0°, 45°, 90°, 135° and 180°. Different initial heading angles do not lead to significant distinctions in trajectories and flight times,
which indicates the small sensitivity of TCG to the initial heading angles. It can be seen that the highest or the lowest launch angle of 180° or 0° may result in relatively large flight time errors, but note that zero miss distance is still guaranteed in these cases.

Time-constrained guidance laws have found an important application in salvo attacks where the target is demanded to be hit by multiple missiles simultaneously. Fig. 9 illustrates a salvo attack scenario. The simulations parameters are assumed the same as those in Ref. [5]. It can be seen that the TCG causes effective decline in the flight time derivation.

![Fig.4 Trajectories with different $T_d$.](image)

![Fig.5 Missile acceleration with different $\theta_d$.](image)

![Fig.6 Distance error by TCG with different $T_d$.](image)

![Fig.7 Closing velocity error by TCG with different $T_d$.](image)

![Fig.8 Trajectories by TCG with different initial heading angles($T_d=50$ s).](image)

![Fig.9 Salvo attack by TCG.](image)

4. 2. Comparisons with ITCG

Regarded as the first guidance law dealing with flight time constraints, ITCG [5] can be expressed as

$$a_m = NV \dot{\lambda} - \frac{60V^4}{NAR_{go}} (T_d - \hat{T}_{go})$$  \hspace{1cm} (30)$$

where $R_{go}$ is the range between the missile and the target, $\hat{T}_{go} = (1 + \eta^2/10)R_{go}/V$ the estimation of...
As an optimal guidance law, ITCG suffers from singularities at $\dot{\theta}$ and $R_{go}=0$, around which the acceleration commands become prohibitively large. That means ITCG results in increased demands for acceleration towards the end of the pursuit. In contrast, as a guidance law with asymptotic stability, TCG needs relatively large acceleration command at the beginning and settles to smaller amplitudes at the end of the pursuit and, consequently, leads to smaller miss distances. Moreover, initial launch angles such as $0^\circ$ or $180^\circ$ can also lead to the failure of ITCG. However, as shown in Fig.8, TCG can still guarantee the zero miss distance regardless of a relatively large flight time error it has in these cases.

5. Conclusions

This article proposes a novel virtual leader approach to design guidance laws with flight time constraints. By using this approach, a new time-constrained guidance law is derived which controls flight time precisely. Nonlinear simulations demonstrate the high effectiveness of the proposed law. Naturally, the new law can be applied to salvo attacks and other time-critical guidance missions. The future work is expected to focus on applying the virtual leader approach to guidance problems with terminal impact angle constraints.

References


Biography:

Zhao Shiyu  Born in 1984, he received B. S. and M. S. degrees from Beijing University of Aeronautics and Astronautics in 2006 and 2009, respectively. His research interests include optimal control and cooperative control. E-mail: zsybeijing@gmail.com

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