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## Analytic study of properties of holographic superconductors away from the probe limit

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## ABSTRACT

In this Letter, based on the Sturm–Liouville eigenvalue approach, we analytically investigate the properties of holographic superconductors in the background of pure Einstein and Gauss–Bonnet gravity taking into account the backreaction of the spacetime. Higher value of the backreaction parameter results in a harder condensation to form in both cases. The analytical results obtained are found to be in good agreement with the existing numerical results.

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## 1. Introduction

The correspondence between anti-de Sitter and conformal field theories (AdS/CFT) has been a powerful tool to analyze strongly coupled quantum field theories. It provides a correspondence between a gravity theory in a  $(d + 1)$ -dimensional AdS spacetime and a conformal field theory (CFT) living on its  $d$ -dimensional boundary [1–4]. Recently, the correspondence has been used to provide some meaningful theoretical insights to understand the physics of high  $T_c$  superconductors from the gravitational dual.

The central idea behind holographic superconductors comes from the observation that below a critical temperature, electrically charged black holes become unstable to the formation of scalar hair. The mechanism behind this condensation is the breaking of a local  $U(1)$  symmetry near the event horizon of the black hole [5–10]. However, the investigations in most cases have been carried out in the “probe limit” which essentially means that the backreaction of the spacetime has been neglected. Backreaction of the spacetime was considered in [10–12] for a  $2 + 1$ -dimensional holographic superconductor where it was found that even an uncharged scalar field can form a condensate. In [13],  $3 + 1$ -dimensional holographic superconductors in pure Einstein and Gauss–Bonnet gravity have been studied taking backreaction into account numerically. This study was motivated by the fact that earlier such studies have been made on black holes in Gauss–Bonnet gravity in the probe limit [14]. A lot of work has been done thereafter to study the properties of holographic superconductors away from the probe limit [15–21].

In this Letter, we try to substantiate the numerical results of [13] analytically. We apply the Sturm–Liouville (SL) method developed in [22] along with the perturbative technique first employed in [26] (to take into account the effect of the Born–Infeld coupling parameter) to analytically find the relation between the critical temperature and the charge density both in Einstein gravity and Gauss–Bonnet gravity taking backreaction of the spacetime into account. This method has been used earlier in the probe limit with considerable success [23–28]. Our analysis would also help in examining the applicability of the method in the presence of the backreaction.

This Letter is organized as follows. In Section 2, we provide the basic holographic set up for the holographic superconductors, considering the background of a  $4 + 1$ -dimensional electrically charged black hole in anti-de Sitter spacetime. In Section 2.1, taking into account the backreaction of the spacetime in Einstein gravity, we compute the critical temperature in terms of a solution to the SL eigenvalue problem. In Section 2.2, we carry out the same analysis in Gauss–Bonnet gravity. We conclude finally in Section 3.

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## 2. Basic set up

To begin with, we first write down the action for the formation of scalar hair on an electrically charged black hole in 4 + 1-dimensional anti-de Sitter spacetime. This reads

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left( R - 2\Lambda + \frac{\alpha}{4} (R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} - 4R^{\mu\nu} R_{\mu\nu} + R^2) + 16\pi G \mathcal{L}_{matter} \right) \quad (1)$$

where  $\Lambda = -6/L^2$  is the cosmological constant and  $\alpha$  is the Gauss–Bonnet coupling parameter.  $\mathcal{L}_{matter}$  denotes the matter Lagrangian density and takes the form

$$\mathcal{L}_{matter} = -\frac{1}{4} F_{MN} F^{MN} - (D_M \psi)^* D^M \psi - m^2 \psi^* \psi; \quad M, N = 0, 1, 2, 3, 4 \quad (2)$$

where  $F_{MN} = \partial_M A_N - \partial_N A_M$  is the field strength tensor and  $D_M \psi = \partial_M \psi - ie A_M \psi$  is the covariant derivative. The ansatz for the plane-symmetric black hole metric reads

$$ds^2 = -f(r)a^2(r) dt^2 + \frac{1}{f(r)} dr^2 + \frac{r^2}{L^2} (dx^2 + dy^2 + dz^2). \quad (3)$$

We now choose the following ansatz for the gauge field and the scalar field [10]

$$A_M dx^M = \phi(r) dt, \quad \psi = \psi(r) \quad (4)$$

so that the black hole possesses only electric charge.

The equations of motion for the metric and matter fields computed on this ansatz read

$$f'(r) + 2r \frac{f(r) - 2r^2/L^2}{(r^2 - 2\alpha f(r))} + \gamma \frac{r^3}{2f(r)a^2(r)} \times \left( \frac{2e^2 \phi^2(r) \psi^2(r) + f(r)(2m^2 a^2(r) \psi^2(r) + \phi'^2(r)) + 2f^2(r)a^2(r) \psi'^2(r)}{r^2 - 2\alpha f(r)} \right) = 0, \quad (5)$$

$$a'(r) - \gamma \frac{r^3 (e^2 \phi^2(r) \psi^2(r) + a^2(r) f^2(r) \psi'^2(r))}{a(r) f^2(r) (r^2 - 2\alpha f(r))} = 0, \quad (6)$$

$$\phi''(r) + \left( \frac{3}{r} - \frac{a'(r)}{a(r)} \right) \phi'(r) - 2 \frac{e^2 \psi^2(r)}{f(r)} \phi(r) = 0, \quad (7)$$

$$\psi''(r) + \left( \frac{3}{r} + \frac{f'(r)}{f(r)} + \frac{a'(r)}{a(r)} \right) \psi'(r) + \left( \frac{e^2 \phi^2(r)}{f^2(r) a^2(r)} - \frac{m^2}{f(r)} \right) \psi(r) = 0 \quad (8)$$

where  $\gamma = 16\pi G$  and prime denotes derivative with respect to  $r$ . The fact that  $\gamma \neq 0$  takes into account the backreaction of the spacetime. This limit also allows one to set  $e = 1$  without any loss of generality since the rescalings  $\psi \rightarrow \psi/e$ ,  $\phi \rightarrow \phi/e$  and  $\gamma \rightarrow e^2 \gamma$  can be performed [13].

In order to solve the non-linear equations (5)–(8), we need to fix the boundary conditions for  $\phi(r)$  and  $\psi(r)$  at the black hole horizon  $r = r_+$  (where  $f(r = r_+) = 0$  with  $a(r = r_+)$  finite) and at the spatial infinity ( $r \rightarrow \infty$ ). At the horizon, we require  $\phi(r_+) = 0$  and  $\psi(r_+)$  to be finite for the matter fields to be regular.

Near the boundary of the bulk, we can set  $a(r \rightarrow \infty) \rightarrow 1$ , so that the spacetime becomes a Reissner–Nordström–anti-de Sitter black hole. The matter fields there obey [14]

$$\phi(r) = \mu - \frac{\rho}{r^2}, \quad (9)$$

$$\psi(r) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}} \quad (10)$$

where

$$\lambda_{\pm} = 2 \pm \sqrt{4 - 3(L_{eff}/L)^2}, \quad (11)$$

$$L_{eff}^2 = \frac{2\alpha}{1 - \sqrt{1 - 4\alpha/L^2}} \approx L^2 (1 - \alpha/L^2 + \mathcal{O}(\alpha^2)). \quad (12)$$

The parameters  $\mu$  and  $\rho$  are dual to the chemical potential and charge density of the boundary CFT and choosing  $\psi_- = 0$ ,  $\psi_+$  is dual to the expectation value of the condensation operator  $J$  at the boundary.

Under the change of coordinates  $z = \frac{r_+}{r}$ , the field equations (5)–(8) become

$$f'(z) + \frac{2r_+^2}{z^3} \frac{(2r_+^2 - z^2 f(z))}{(r_+^2 - 2\alpha z^2 f(z))} - \gamma \frac{r_+^2}{2z^3 a^2(z) f(z)} \times \frac{\{2r_+^2 \phi^2(z) \psi^2(z) + f(z)(z^4 \phi'^2(z) - 6r_+^2 a^2(z) \psi^2(z)) + 2a^2(z) f^2(z) z^4 \psi'^2(z)\}}{(r_+^2 - 2\alpha z^2 f(z))} = 0, \quad (13)$$

$$a'(z) + \gamma \frac{r_+^2}{z^3 a(z) f^2(z)} \frac{(r_+^2 \phi^2(z) \psi^2(z) + a^2(z) f^2(z) z^4 \psi'^2(z))}{(r_+^2 - 2\alpha z^2 f(z))} = 0, \quad (14)$$

$$\phi''(z) - \left( \frac{1}{z} + \frac{a'(z)}{a(z)} \right) \phi'(z) - \frac{2r_+^2 \psi^2(z)}{z^4 f(z)} \phi(z) = 0, \quad (15)$$

$$\psi''(z) - \left( \frac{1}{z} - \frac{a'(z)}{a(z)} - \frac{f'(z)}{f(z)} \right) \psi'(z) + \frac{r_+^2}{z^4} \left( \frac{\phi^2(z)}{f^2(z) a^2(z)} + \frac{3}{f(z)} \right) \psi(z) = 0 \quad (16)$$

where prime now denotes derivative with respect to  $z$ . These equations are to be solved in the interval  $(0, 1)$ , where  $z = 1$  is the horizon and  $z = 0$  is the boundary. The boundary condition  $\phi(r_+) = 0$  now becomes  $\phi(z = 1) = 0$ .

### 2.1. Effect of backreaction in Einstein gravity

With the above set up in place, we now move on to investigate the relation between the critical temperature and the charge density. At the critical temperature  $T_c$ ,  $\psi = 0$ , so Eq. (14) reduces to

$$a'(z) = 0. \quad (17)$$

Hence Eq. (13) (with  $\alpha = 0$ ) and the field equation (15) reduces to

$$f'(z) + \frac{2}{z^3} (2r_{+(c)}^2 - z^2 f(z)) - \gamma \frac{z \phi'^2(z)}{2} = 0, \quad (18)$$

$$\phi''(z) - \frac{1}{z} \phi'(z) = 0. \quad (19)$$

With the boundary condition (9), the solution of Eq. (19) reads

$$\phi(z) = \lambda r_{+(c)} (1 - z^2) \quad (20)$$

where

$$\lambda = \frac{\rho}{r_{+(c)}^3}. \quad (21)$$

This leads to the following solution for the metric from Eq. (18) consistent with the condition  $f(z = 1) = 0$

$$f(z) = r_{+(c)}^2 \left\{ \frac{1}{z^2} - (1 + \gamma \lambda^2) z^2 + \gamma \lambda^2 z^4 \right\} = \frac{r_{+(c)}^2}{z^2} g_0(z) \quad (22)$$

where

$$g_0(z) = 1 - (1 + \gamma \lambda^2) z^4 + \gamma \lambda^2 z^6. \quad (23)$$

Now using the solution (20), we find that as  $T \rightarrow T_c$ , the equation for the field  $\psi$  approaches the limit

$$-\psi''(z) + \left( \frac{1}{z} - \frac{f'(z)}{f(z)} \right) \psi'(z) - \frac{3}{z^2 g_0(z)} \psi(z) = \lambda^2 \frac{(1 - z^2)^2}{g_0^2(z)} \psi(z). \quad (24)$$

Near the boundary, we define [22]

$$\psi(z) \sim z^3 F(z) \quad (25)$$

where  $F(0) = 1$ . Substituting this form of  $\psi(z)$  in Eq. (24), we obtain

$$-F''(z) + \left( \frac{1}{z} - \frac{f'(z)}{f(z)} - \frac{6}{z} \right) F'(z) + \frac{3}{z} \left\{ \left( \frac{1}{z} - \frac{f'(z)}{f(z)} \right) - \frac{2}{z} - \frac{1}{z g_0(z)} \right\} F(z) = \lambda^2 \frac{(1 - z^2)^2}{g_0^2(z)} F(z) \quad (26)$$

to be solved subject to the boundary condition  $F'(0) = 0$ .

The above equation can be put in the Sturm–Liouville form

$$\frac{d}{dz} \{ p(z) F'(z) \} - q(z) F(z) + \lambda r(z) F(z) = 0 \quad (27)$$

with

$$\begin{aligned} p(z) &= z^3 g_0(z), \\ q(z) &= 3z^5 \{ 3(1 + \gamma \lambda^2) - 5\gamma \lambda^2 z^2 \}, \\ r(z) &= \frac{z^3 (1 - z^2)^2}{g_0(z)}. \end{aligned} \quad (28)$$

With the above identification, we can once again write down the eigenvalue  $\lambda^2$  which minimizes the expression

**Table 1**

A comparison of the analytical and numerical results for the critical temperature and the charge density with backreaction in Einstein gravity.

$\gamma$	$\lambda_{SL}^2$	$(T_c/\rho^{1/3})_{SL}$	$(T_c/\rho^{1/3})_{numerical}$
0	18.23	0.196	0.197
0.025	16.3817	0.159	0.161
0.05	14.8114	0.128	0.128
0.075	13.4269	0.103	0.089
0.1	12.1816	0.082	0.079
0.125	11.0604	0.066	0.053
0.15	10.0647	0.053	0.045
0.175	9.1912	0.043	0.031
0.2	8.4294	0.035	0.026

$$\lambda^2 = \frac{\int_0^1 dz \{p(z)[F'(z)]^2 + q(z)[F(z)]^2\}}{\int_0^1 dz r(z)[F(z)]^2} = \frac{\int_0^1 dz z^3 [g_0(z)[F'(z)]^2 + 3z^2 \{3(1 + \gamma\lambda^2) - 5\gamma\lambda^2 z^2\} [F(z)]^2]}{\int_0^1 dz \frac{z^3(1-z^2)^2}{g_0(z)} [F(z)]^2} \tag{29}$$

To estimate it, we use the following trial function

$$F = F_{\tilde{\alpha}}(z) \equiv 1 - \tilde{\alpha}z^2 \tag{30}$$

which satisfies the conditions  $F(0) = 1$  and  $F'(0) = 0$ .

Hence, we obtain (with the backreaction parameter  $\gamma = 0$ )

$$\lambda_{\tilde{\alpha}}^2 = \frac{2(18 - 27\tilde{\alpha} + 14\tilde{\alpha}^2)}{6(3 - 4 \ln 2) + 16(2 - 3 \ln 2)\tilde{\alpha} + (17 - 24 \ln 2)\tilde{\alpha}^2} \tag{31}$$

which attains its minimum at  $\tilde{\alpha} \approx 0.7218$ . The critical temperature therefore reads

$$T_c = \frac{1}{4\pi} f'(r_{+(c)}) = \frac{1}{\pi \lambda_{\tilde{\alpha}=0.7218}^{1/3}} \rho^{1/3} \approx 0.196 \rho^{1/3} \tag{32}$$

which is in very good agreement with the exact  $T_c = 0.197 \rho^{1/3}$  [12].

Now in order to include the effect of backreaction, we set  $\gamma = 0.025$  and put the value of  $\lambda^2$  obtained for the corresponding value of  $\gamma$  (which in this case is  $\gamma = 0$ ) in the right hand side of Eq. (29) to get the value of  $\lambda^2$  for  $\gamma = 0.025$

$$\lambda_{\tilde{\alpha}}^2 = \frac{1.32909 - 1.90819\tilde{\alpha} + 0.97677\tilde{\alpha}^2}{0.06168 - 0.05909\tilde{\alpha} + 0.017301\tilde{\alpha}^2} \tag{33}$$

which attains its minimum at  $\tilde{\alpha} \approx 0.6780$ . The critical temperature therefore reads

$$T_c = \frac{1}{4\pi} f'(r_{+(c)}) = \frac{1}{\pi} \left(1 - \frac{1}{2} \gamma \lambda_{\tilde{\alpha}=0.7218}^2\right) r_{+(c)} = \frac{1}{\pi} \frac{(1 - \frac{1}{2} \gamma \lambda_{\tilde{\alpha}=0.7218}^2)}{\lambda_{\tilde{\alpha}=0.6780}^{1/3}} \rho^{1/3} \approx 0.1588 \rho^{1/3} \tag{34}$$

which is in very good agreement with the exact  $T_c = 0.161 \rho^{1/3}$  [13]. Increasing the value of  $\gamma$  in steps of 0.025 and repeating the above process, we can obtain the values of  $\lambda^2$  for various values of the backreaction parameter.

In Table 1, we compare our analytical values obtained by the SL approach with the existing numerical results in the literature [13].

### 2.2. Effect of backreaction in Gauss–Bonnet gravity

In this section, we study the relation between the critical temperature and the charge density taking into account the effect of the Gauss–Bonnet coupling parameter  $\alpha$ .

In this case, using Eq. (17), Eq. (13) (with  $\alpha \neq 0$ ) reduces to

$$f'(z) + \frac{2r_{+(c)}^2}{z^3} \frac{(2r_{+(c)}^2 - z^2 f(z))}{(r_{+(c)}^2 - 2\alpha z^2 f(z))} - \gamma \frac{r_{+(c)}^2}{2} \frac{z\phi'^2(z)}{(r_{+(c)}^2 - 2\alpha z^2 f(z))} = 0. \tag{35}$$

The solution of the above equation upto first order in the Gauss–Bonnet coupling parameter  $\alpha$  reads

$$f(z) = \frac{r_{+(c)}^2}{z^2} \{g_0(z) + \alpha g_1(z)\} \tag{36}$$

where

$$g_1(z) = 1 - 2(1 + \gamma\lambda^2)z^4 + 2\gamma\lambda^2 z^6 + (1 + \gamma\lambda^2)^2 z^8 - 2(1 + \gamma\lambda^2)\gamma\lambda^2 z^{10} + \gamma^2 \lambda^4 z^{12}. \tag{37}$$

**Table 2**

A comparison of the analytical and numerical results for the critical temperature and the charge density with backreaction in Gauss–Bonnet gravity ( $\alpha = 0.1$ ).

$\gamma$	$(T_c/\rho^{1/3}) _{SL}$	$(T_c/\rho^{1/3}) _{numerical}$
0	0.1867	0.185
0.1	0.066	0.051
0.2	0.0114	0.008

For  $\alpha \neq 0$ , we define near the boundary

$$\psi(z) \sim z^{\Delta_+} F(z). \quad (38)$$

Substituting this form of  $\psi(z)$  in Eq. (24), we obtain

$$-F''(z) + \left( \frac{1}{z} - \frac{f'(z)}{f(z)} - \frac{2\Delta_+}{z} \right) F'(z) + \frac{\Delta_+}{z} \left\{ \left( \frac{1}{z} - \frac{f'(z)}{f(z)} \right) - \frac{\Delta_+(\Delta_+ - 1)}{z^2} - \frac{3}{z^2(g_0 + \alpha g_1)} \right\} F(z) = \lambda^2 \frac{(1-z^2)^2}{(g_0 + \alpha g_1)^2} F(z) \quad (39)$$

to be solved subject to the boundary condition  $F'(0) = 0$ .

The above equation can once again be put in the Sturm–Liouville form with

$$\begin{aligned} p(z) &= z^{2\Delta_+ - 3} (g_0 + \alpha g_1), \\ q(z) &= z^{2\Delta_+ - 5} \left\{ \Delta_+ (2(g_0 + \alpha g_1) - z(g'_0 + \alpha g'_1)) - \Delta_+(\Delta_+ - 2)(g_0 + \alpha g_1) - 3 \right\}, \\ r(z) &= \frac{z^3(1-z^2)^2}{g_0(z)}. \end{aligned} \quad (40)$$

With the above identification, we can once again proceed to find the minimum value of the eigenvalue  $\lambda^2$  as in the earlier section.

To estimate it, we first set  $\alpha = 0.1$ ,  $\gamma = 0$  and again use the trial function (30) to obtain

$$\lambda_{\tilde{\alpha}}^2 = \frac{1.74 - 2.5\tilde{\alpha} + 1.26194\tilde{\alpha}^2}{0.0527 - 0.0496\tilde{\alpha} + 0.0144\tilde{\alpha}^2} \quad (41)$$

which attains its minimum at  $\tilde{\alpha} \approx 0.7078$ . The critical temperature therefore reads

$$T_c = \frac{1}{4\pi} f'(r_{+(c)}) = \frac{1}{\pi \lambda_{\tilde{\alpha}=0.7078}^{1/3}} \rho^{1/3} \approx 0.1867 \rho^{1/3} \quad (42)$$

which is in very good agreement with the exact  $T_c = 0.185 \rho^{1/3}$  [14].

For  $\alpha = 0.1$ ,  $\gamma = 0.1$ , we get

$$\lambda_{\tilde{\alpha}}^2 = \frac{1.1838 - 1.4036\tilde{\alpha} + 0.6594\tilde{\alpha}^2}{0.0726 - 0.0763\tilde{\alpha} + 0.0239\tilde{\alpha}^2} \quad (43)$$

which attains its minimum at  $\tilde{\alpha} \approx 0.3495$ . In computing this result, we have used the value of  $\lambda^2$  corresponding to  $\gamma = 0.075$  (with  $\alpha = 0$ ) from Table 1 in Eq. (40) to calculate the expression which minimizes  $\lambda^2$  and obtained the value of  $\lambda^2$  corresponding to  $\alpha = 0.1$ ,  $\gamma = 0.1$ . The critical temperature therefore reads

$$\begin{aligned} T_c &= \frac{1}{4\pi} f'(r_{+(c)}) = \frac{1}{\pi} \left( 1 - \frac{1}{2} \gamma \lambda_{\gamma=0.075, \alpha=0}^2 \right) r_{+(c)} \\ &= \frac{1}{\pi} \frac{(1 - \frac{1}{2} \gamma \lambda_{\gamma=0.075, \alpha=0}^2)}{\lambda_{\tilde{\alpha}=0.3495, \alpha=0.1}^{1/3}} \rho^{1/3} \approx 0.066 \rho^{1/3} \end{aligned} \quad (44)$$

which is in very good agreement with the exact  $T_c = 0.051 \rho^{1/3}$  [13].

In Table 2, we compare our analytical values obtained by the SL approach with the existing numerical results in the literature [13].

We would like to mention that there is a slight discrepancy between the analytical values obtained by the SL method and the numerical values for the Gauss–Bonnet case. This is because the analytical calculations have been carried out to first order in the Gauss–Bonnet parameter  $\alpha$ . In order to get analytical results closer to the numerical values, one must carry out higher order calculations involving the Gauss–Bonnet parameter  $\alpha$ .

### 3. Conclusions

In this Letter, we perform analytic computation of 3 + 1-dimensional holographic superconductors in the background of pure Einstein and Gauss–Bonnet gravity taking into account the backreaction of the spacetime. We apply the Sturm–Liouville eigenvalue problem to obtain the relation between the critical temperature and the charge density in both Einstein and Gauss–Bonnet gravity. It is observed that higher value of the backreaction parameter results in a harder condensation to form in both cases. Further, the condensation is even harder to form in the presence of the Gauss–Bonnet parameter. Our results are in very good agreement with the existing numerical results [13].

As final remarks, we would like to mention the novelty of our analytic results. The consistency between the analytical and numerical results indicates that the Sturm–Liouville method is a powerful analytic approach to investigate holographic superconductors even in the presence of backreaction. Further, the analytical approach is always more desirable than the numerical approach since the numerical results become less reliable when the temperature  $T$  approaches to zero [7,22]. In this temperature limit, the numerical solutions to the non-linear field equations become extremely difficult and therefore the determination of the nature of the condensate becomes very problematic unless analytic techniques are adopted. Hence, the analytic method is always more reliable while performing computations as  $T \rightarrow 0$ . It should be also noted that the validity of the analytical results obtained by the Sturm–Liouville method can be confirmed (without referring to the numerical results) by comparing them with the results obtained from an alternative analytic technique known as the matching method [14]. Our results would also serve as a first step to analyze holographic superconductors immersed in an external magnetic field [29] taking into account the backreaction of the spacetime.

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