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MULTI-CRITERIA *DE NOVO* PROGRAMMING WITH FUZZY PARAMETERS

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Abstract—A multiple criteria *de Novo* program with fuzzy parameters is developed based on the possibility concept of fuzzy set. This approach is much more flexible than the standard *de Novo* programming and allows the decision maker to choose his appropriate membership grades based on the risk factor he is willing to take. A numerical example is given to illustrate the approach.

INTRODUCTION

de Novo programming, as was formulated by Zeleny in Ref. [1], emphasizes optimal design of the original problem instead of just optimizing a subproblem where the constraints are fixed and given. This approach is much more flexible than the usual multi-objective linear programming (MOLP). However, in real world problems, the technological coefficients and parameters are not precisely known. Due to this uncertainty nature, fuzzy set theory can ideally be used to extend the *de Novo* programming.

In order to introduce our nomenclature, consider the standard MOLP:

$$\max z_k = \sum_{j=1}^{n} C_{kj} X_j, \quad k = 1, \ldots, l,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} X_j \leqslant b_i, \quad i = 1, \ldots, m,$$

$$X_j \geqslant 0, \quad j = 1, \ldots, n,$$

where the values of the parameters b_i represent the given, fixed levels of available resources. The conventional solution concept for a MOLP model is the set of non-dominated solutions [2]. If we change b_i from constants to resource variables with their values to be determined, we obtain the *de Novo* programming formulation [2]:

$$\max z_k = \sum_{j=1}^{n} C_{kj} X_j, \quad k = 1, \ldots, l,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} X_j \leqslant b_i, \quad i = 1, \ldots, m,$$

$$\sum_{i=1}^{m} p_i b_i \leqslant B.$$

$$X_j \geqslant 0, \quad j = 1, \ldots, n,$$

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where X_j. b_i are decision variables for products and resources respectively, p_i are the unit price of resource i, and B is the total available budget. Let $V_j = \Sigma_{i=1}^{m} p_i a_{ij}$ denote the unit cost of producing product j. We can rewrite the *de Novo* programming as follows:

$$\begin{aligned} \max z_k &= \sum_{j=1}^{n} C_{kj} X_j, \quad k = 1, \ldots, l, \\ \text{s.t.} \quad & \sum_{j=1}^{n} V_j X_h \leqslant b_i, \qquad \text{(P1)} \\ & X_j \geqslant 0, \quad j = 1, \ldots, n. \end{aligned}$$

In this paper, we are concerned with fuzziness in the above system design problems in which all parameters, c, p, a and B are expressed by fuzzy subsets.

FUZZY *DE NOVO* PROGRAMMING

Consider the following fuzzy program:

$$\begin{aligned} \max \tilde{z}_k &= \sum_{j=1}^{n} \tilde{C}_{kj} X_j, \quad k = 1, \ldots, l, \\ \text{s.t.} \quad & \sum_{j=1}^{n} \tilde{V}_j X_j \leqslant \tilde{B}, \quad j = 1, \ldots, n, \\ & \tilde{V}_j = \sum_{i=1}^{m} \tilde{p}_i \tilde{a}_{ij}, \quad i = 1, \ldots, m, \qquad \text{(P2)} \\ & X_j \geqslant 0, \end{aligned}$$

where parameters $\tilde{C}_{kj}$, $\tilde{p}_i$, $\tilde{a}_{ij}$, $\tilde{B}$ are fuzzy variables on R characterized by the membership functions $\mu_{\tilde{C}_{kj}}$, $\mu_{\tilde{p}_i}$, $\mu_{\tilde{a}_{ij}}$, $\mu_{\tilde{B}}$, respectively, and $\tilde{V}_j$ are fuzzy functions on R^{2m} defined by

$$\mu_{\tilde{V}_j}(V_j) = \sup \min \left\{ \mu_{\tilde{p}_i}(p_i), \mu_{\tilde{a}_{ij}}(a_{ij}) \mid \forall\, i = 1, \ldots, m, \sum_i p_i a_{ij} = V_j \right\}.$$

Note that the solution to be obtained for problem (P2) should not be a crisp one, but a fuzzy one in nature, with respect to the fuzzy parameters.

Let $(X)_\alpha$ be a solution of problem (P2) where $\alpha \in [0, 1]$ represents the degree of possibility to which the solution satisfies the problem. In other words, we define $\alpha \in [0, 1]$ to be safety grade or efficiency level, and 1-α the risk factor. That is

$$\alpha = \min \left\{ \text{poss}\left(\sum_j \tilde{C}_{kj} X_j \right), \text{poss}\left(\sum_j \tilde{V}_j X_j \leqslant \tilde{B} \right) \middle| \forall\, k = 1, \ldots, l, j = 1, \ldots, n \right\},$$

where poss denotes possibility. By means of extension principle we have

$$\text{poss}\left(\sum_j \tilde{C}_{kj} X_j \right) = \sup \min \left\{ \mu_{\tilde{C}_{kj}}(C_{kj}) \middle| \forall\, k = 1, \ldots, l, j = 1, \ldots, n \right\}$$

and

$$\begin{aligned} \text{poss}\left(\sum_j \tilde{V}_j X_j \leqslant \tilde{B} \right) &= \sup \min \left\{ \mu_{\tilde{V}_j}(V_j), \mu_{\tilde{B}}(b) \middle| \forall\, j = 1, \ldots, n, \sum_j V_j X_j \leqslant b \right\}. \\ &= \sup \min \left\{ \sup \min \left\{ \mu_{\tilde{p}_i}(p_i), \mu_{a_{ij}}(a_{ij}) \middle| \forall\, i = 1, \ldots, m, \sum_i p_i a_{ij} = V_j \right\}, \right. \\ &\qquad \left. \mu_{\tilde{B}}(b) \middle| \forall\, j = 1, \ldots, n, \sum_j V_j X_j \leqslant b \right\}. \end{aligned}$$

Therefore, the original formulation of problem (P2) can be, referring to the definition of α, transformed to

$$\begin{aligned}
\max \quad & \tilde{z}_k = \sum_{j=1}^{n} \tilde{C}_{kj} X_j, \quad k = 1, \ldots, l, \\
\text{s.t.} \quad & \operatorname{poss}\Big(\sum_j \tilde{C}_{kj} X_j\Big) \geqslant \alpha, \quad j = 1, \ldots, n, \\
& \operatorname{poss}\Big(\sum_j \tilde{V}_j X_j \leqslant \tilde{B}\Big) \geqslant \alpha, \\
& \alpha \in [0, 1], X_j \geqslant 0
\end{aligned}$$

and further to

$$\begin{aligned}
\max \tilde{z}_k = & \sum_{j=1}^{n} \tilde{C}_{kj} X_j, \quad k = 1, \ldots, l, \\
\text{s.t.} \quad & \mu_{\tilde{C}_{kj}}(C_{kj}) \geqslant \alpha, \quad j = 1, \ldots, n \\
& \mu_{\tilde{p}_i}(p_i) \geqslant \alpha, \quad i = 1, \ldots, m, \\
& \mu_{\tilde{a}_{ij}}(a_{ij}) \geqslant \alpha, \\
& \sum_i p_i a_{ij} = V_j, \\
& \mu_{\tilde{B}}(b) \geqslant \alpha, \\
& \sum_j V_j X_j \leqslant b, \\
& \alpha \in [0, 1], X_j \geqslant 0.
\end{aligned} \tag{P3}$$

THE MEMBERSHIP FUNCTION

There are many ways to construct a membership function [3–6]. The most practical form is a linear form proposed by Zimmerman [6]. Suppose that a decision maker can specify an interval $[P^0, P^1)$ or $(P^1, P^0]$ for the possible values of parameter p (cf. Carlsson and Korhonen [3]), where superscript 0 corresponds to "risk-free" values, i.e. $1 - \alpha = 0$, and superscript 1 to "impossible" values, i.e. $1 - \alpha = 1$. It should be noticed that the type of interval $[P^1, P^0)$ must be for parameters $\tilde{C}_{kj}$ and $\tilde{B}$, and the type of interval $(P^1, P^0]$ for parameters $\tilde{a}_{ij}$ and $\tilde{P}_i$ in order to guarantee the solutions of problem (P3) to be optimal. This means that a system designed on the basis of the possible smallest profit units, the smallest invest budget, the biggest resource prices and the biggest operation costs appear to be "risk-free" design, conversely, a system designed with the possible biggest profits units, the biggest invest budget, the smallest resource prices and the smallest operation costs is most dangerous. In practical solutions, the safety factor should be chosen by considering the tolerance factor of the decision maker, thus fuzzy *de Novo* programming provides the decision maker a chance of comparing different system designs. This comparison would reveal how the system is influenced by the different safety factors.

Let $\tilde{W}$ be a fuzzy parameter with interval $[W^0, W^1)$ and $\tilde{Q}$ another fuzzy parameter with interval $(Q^1, Q^0]$. We now define two kinds of membership functions corresponding to the two types of intervals. For $\tilde{C}_{kj}$ and $\tilde{B}$, we have

$$\mu_{\tilde{W}}(w) = \begin{cases} 1, & w \leqslant w^0 \\ (w - w^1)/(w^0 - w^1), & w \leqslant w_1 \leqslant w^1, \\ 0, & w \geqslant w^1. \end{cases}$$

For $\tilde{a}_{ij}$ and $\tilde{p}_j$, we have

$$\mu_{\tilde{Q}}(q) = \begin{cases} 0, & Q \leqslant Q^1, \\ (Q - Q^1)/(Q^0 - Q^1), & Q^1 \leqslant Q \leqslant Q^0, \\ 1, & Q \geqslant Q^0. \end{cases}$$

The membership functions are monotonically decreasing for the parameters $\tilde{C}_{kj}$ and $\tilde{B}$, and monotonically increasing for the parameters $\tilde{a}_{ij}$ and $\tilde{P}_i$. Obviously, for any membership function μ_1 its inverse function μ_i^{-1} exists for both cases, and furthermore

for $\tilde{C}_{kj}$ and $\tilde{B}$:

$$\mu_{\tilde{C}_{kj}}(C_{kj}) \geqslant \alpha \Rightarrow C_{kj} \leqslant \mu_{\tilde{C}_{kj}}^{-1}(\alpha), \quad k = 1, \ldots, l, j = 1, \ldots, n,$$

$$\mu_{\tilde{B}}(b) \geqslant \alpha \Rightarrow b \leqslant \mu_{\tilde{B}}^{-1}(\alpha);$$

for $\tilde{a}_{ij}$ and $\tilde{p}_i$:

$$\mu_{\tilde{a}_{ij}}(a_{ij}) \geqslant \alpha \Rightarrow a_{ij} \geqslant \mu_{\tilde{a}_{ij}}^{-1}(\alpha); \quad i = 1, \ldots, m, j = 1, \ldots, n,$$

$$\mu_{\tilde{p}_i}(p_i) \geqslant \alpha \Rightarrow p_i \geqslant \mu_{\tilde{p}_i}^{-1}(\alpha), \quad i = 1, \ldots, m.$$

Therefore, we can rewrite problem (P3) in parametric form (cf. Verdegay [7]):

$$\begin{aligned} \max \tilde{z}_k = \sum_{j=1}^{n} \tilde{C}_{kj} \cdot X_j, \quad & k = 1, \ldots, l, \\ \text{s.t.} \quad C_{kj} \leqslant \mu_{\tilde{C}_{kj}}^{-1}(\alpha), \quad & k = 1, \ldots, l, j = 1, \ldots, n, \\ p_i = \mu_{\tilde{p}_i}(\alpha), \quad & i = 1, \ldots, m, \\ a_{ij} \geqslant \mu_{\tilde{a}_{ij}}^{-1}(\alpha), & \\ b \leqslant \mu_{\tilde{B}}^{-1}(\alpha), & \\ \sum_j \sum_i p_i a_{ij} X_j \leqslant b, & \\ \alpha \in [0, 1], X_j \geqslant 0, & \end{aligned}$$

which is equivalent to

$$\begin{aligned} \max \tilde{z}_k = \sum_{j=1}^{n} \tilde{C}_{kj} X_j, \quad & k = 1, \ldots, l, \\ \text{s.t.} \quad C_{kj} = \mu_{\tilde{C}_{kj}}^{-1}(\alpha), \quad & j = 1, \ldots, n, \\ P_i = \mu_{\tilde{p}_i}^{-1}(\alpha), \quad & i = 1, \ldots, m, \\ a_{ij} = \mu_{\tilde{a}_{ij}}^{-1}(\alpha), & \\ b = \mu_{\tilde{B}}^{-1}(\alpha), & \\ \sum_j \sum_i p_i a_{ij} X_j, X_j \leqslant b, & \\ \alpha \in [-0, 1], X_j \geqslant 0, & \end{aligned}$$

and further leads to

$$\max \quad (\tilde{z}_k)_\alpha = \sum_{j=1}^{n} \mu_{\tilde{C}_{kj}}^{-1}(\alpha) X_j, \quad k = 1, \ldots, l, \tag{P4}$$

$$\text{s.t.} \quad \sum_j \sum_i \mu_{\tilde{p}_i}^{-1}(\alpha) \mu_{\tilde{a}_{ij}}^{-1}(\alpha) X_j \leqslant \mu_{\tilde{B}}^{-1}(\alpha), \quad i = 1, \ldots, m, j = 1, \ldots, n,$$

$$\alpha \in [0, 1], X_j \geqslant 0,$$

where

$$\mu_{\tilde{C}_{kj}}^{-1}(\alpha) = C_{kj}^0 + \alpha(C_{kj}^1 - C_{kj}^0), \quad k = 1, \ldots, l, j = 1, \ldots, n,$$

$$\mu_{\tilde{p}_i}^{-1}(\alpha) = p_i^1 + \alpha(p_i^0 - p_i^1), \quad i = 1, \ldots, m,$$

$$\mu_{\tilde{a}_{ij}}^{-1}(\alpha) = a_{ij}^1 + \alpha(a_{ij}^0 - a_{ij}^1), \quad i = 1, \ldots, m, j = 1, \ldots, n,$$

$$\mu_{\tilde{B}}^{-1}(\alpha) = b^0 + \alpha(b^1 - b^0).$$

For a given $\alpha \in [0, 1]$ we can now design the optimal system with respect to each objective separately. Since only one budgetary constraint is involved, the optimal solution for each objective can be easily obtained for any number of variables and any number of resources by finding

$$\forall\, k \in K, \max_j \left\{\mu_{\tilde{C}_{kj}}^{-1}(\alpha) / \sum_i [\mu_{\tilde{p}_i}^{-1}(\alpha)\mu_{\tilde{a}_{ij}}^{-1}(\alpha)]\right\},$$

say

$$\mu_{\tilde{C}_{kT}}^{-1}(\alpha) / \sum_i [\mu_{\tilde{p}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{iT}}^{-1}(\alpha)],$$

then

$$X_{kj}^* = \begin{cases} \mu_{\tilde{B}_0}^{-1}(\alpha) / \sum_j [\mu_{\tilde{p}_i}^{-1}(\alpha)\, \mu_{\tilde{a}_{ij}}^{-1}(\alpha)], & \text{for } j = T, \\ 0, & \text{otherwise.} \end{cases}$$

Let us use the foursome $(z^*, X^*, b^*, B^*)_\alpha$ to denote the "ideal system design" for a given safety factor α, where z^*, X^*, b^* and B^* are l-vector, n-vector, m-vector and unit-vector, respectively. The "ideal system design" is not a solution for the given budget, but it can serve as a reference point for judging the multi-criteria desirability of alternative system designs. If the number of decision variables is equal to the number of decision criteria, the ideal $(z^*, X^*, b^*, B^*)_\alpha$ can be obtained by solving the following systems of linear algebraic equations:

$$\mu_{\tilde{C}_{1j}^*}^{-1}(\alpha) \cdot X_{1j}^* = \mu_{\tilde{C}_{11}}^{-1}(\alpha) X_1 + \mu_{\tilde{C}_{12}}^{-1}(\alpha) X_2 + \cdots + \mu_{\tilde{C}_m}^{-1}(\alpha),$$

$$\mu_{\tilde{C}_{2j}^*}^{-1}(\alpha) \cdot X_{2j}^* = \mu_{\tilde{C}_{21}}^{-1}(\alpha) X_1 + \mu_{\tilde{C}_{22}}^{-1}(\alpha) X_2 + \cdots + \mu_{\tilde{C}_{2n}}^{-1}(\alpha),$$

$$\vdots$$

$$\mu_{\tilde{C}_{nj}^*}^{-1}(\alpha) \cdot X_{nj}^* = \mu_{\tilde{C}_{n1}}^{-1}(\alpha) X_1 + \mu_{\tilde{C}_{n2}}^{-1}(\alpha) X_2 + \cdots + \mu_{\tilde{C}_{nn}}^{-1}(\alpha),$$

and

$$(z_k^*)_\alpha = \sum_{j=1}^{n} \mu_{\tilde{C}_{kj}}^{-1}(\alpha) \cdot X_j^*, \quad k = 1, \ldots, l, j = 1, \ldots, n,$$

$$(b_i^*)_\alpha = \sum_{j=1}^{n} \mu_{\tilde{a}_{ij}}^{-1}(\alpha) \cdot X_j^*, \quad i = 1, \ldots, m, j = 1, \ldots, n,$$

$$(B^*)_\alpha = \sum_{j=1}^{n} \sum_{i=1}^{m} \mu_{\tilde{p}_i}^{-1}(\alpha) \mu_{\tilde{a}_{ij}}^{-1}(\alpha) X_j^*, \quad i = 1, \ldots, m, j = 1, \ldots, n.$$

Furthermore, based on the ideal $(z^*, X^*, b^*, B^*)_\alpha$ we can obtain the optimal solution $(z', x', b', B')_\alpha$ for any other budget level, such that $\forall\, \alpha \in [0, 1]$, $X' = X^* \cdot B'/B^*$, $z' = \mu_{\tilde{C}}^{-1}(\alpha) \cdot X'$ and $b' = \mu_{\tilde{a}}^{-1}(\alpha) \cdot X'$.

A NUMERICAL EXAMPLE

Consider the following multi-criteria design problem:

$$\max \tilde{z}_1 = [2^0, 5^1)X_1 + 12X_2,$$

$$\tilde{z}_2 = 4x_1 + [1^0, 3^1)X_2,$$

$$\text{s.t.} \quad X_1 + (1^1, 4^0)X_2 \leqslant b_1,$$

$$2X_1 + (2^1, 3^0]X_2 \leqslant b_2,$$

$$((0.5^1, 2^0] + 2)X_1 + ((0.5^1, 2^0] \cdot (1^1, 4^0] + (2^1, 3^0])X_2 \leqslant [200^0, 250^1),$$

$$X_1, X_2, X_3 \geqslant 0,$$

where the membership functions are defined by

$$\mu_{\tilde{p}}(p) = \begin{cases} 1, & p \leqslant p^0, \\ (p - p^0)/(p^1 - p^0), & p^0 \leqslant p \leqslant p^1, \\ 0, & p \geqslant p^1, \end{cases}$$

for the fuzzy parameters with the type of interval $[p^0, p^1)$, and

$$\mu_{\tilde{p}}(p) = \begin{cases} 0, & p \leqslant p^1, \\ (p - p^1)/(p^0 - p^1), & p^1 \leqslant p \leqslant p^0, \\ 1, & p \geqslant p^0, \end{cases}$$

for the fuzzy parameters with the type of interval $(p^1, p^0]$. These parameter values are pictured in Fig. 1.

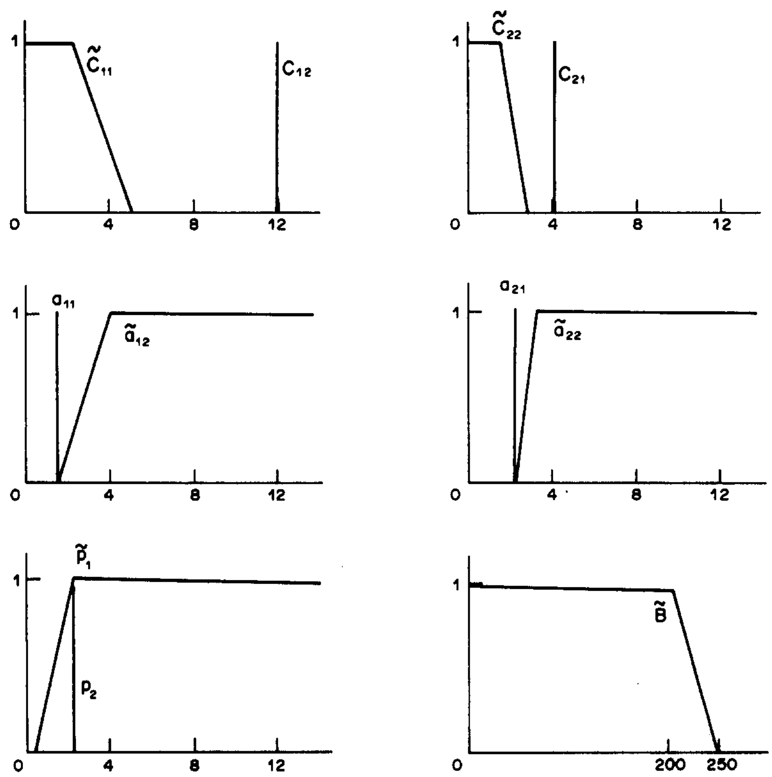

Fig. 1. The membership functions of the parameters.

For a given safety factor α the fuzzy parameters can be expressed by means of their membership functions as:

$$(\tilde{C}_{11})_\alpha = \mu_{\tilde{C}_{11}}^{-1}(\alpha) = 5 - 3\alpha, (\tilde{C}_{22})_\alpha = \mu_{\tilde{C}_{22}}^{-1}(\alpha) = 3 - 2\alpha,$$

$$(\tilde{a}_{12})_\alpha = \mu_{\tilde{a}_{12}}^{-1}(\alpha) = 1 + 3\alpha, (\tilde{a}_{22})_\alpha = \mu_{\tilde{a}_{22}}^{-1}(\alpha) = 2 + \alpha,$$

$$(\tilde{p}_1)_\alpha = \mu_{\tilde{p}_1}^{-1}(\alpha) = 0.5 + 1.5\alpha, (\tilde{B})_\alpha = \mu_{\tilde{B}}^{-1}(\alpha) = 250 - 50\alpha,$$

$$(\tilde{V}_1)_\alpha = \mu_{\tilde{V}_1}^{-1}(\alpha) = 2.5 + 1.5\alpha, (\tilde{V}_2)_\alpha = \mu_{\tilde{V}_2}^{-1}(\alpha) = 2.5 + 4a + 4.5a^2.$$

Thus the original formulation can be rewritten as:

$$\max (\tilde{z}_1)_\alpha = (5 - 3\alpha)X_1 + 12X_2,$$

$$\max (\tilde{z}_2)_\alpha = 4X_1 + (3 - 2\alpha)X_2,$$

$$\text{s.t.} \quad (2.5 + 1.5\alpha)X_1, + (2.5 + 4\alpha + 4.5\alpha^2)X_2 \leqslant 250 - 50\alpha,$$

$$\alpha \in [0, 1], X_1, X_2 \geqslant 0.$$

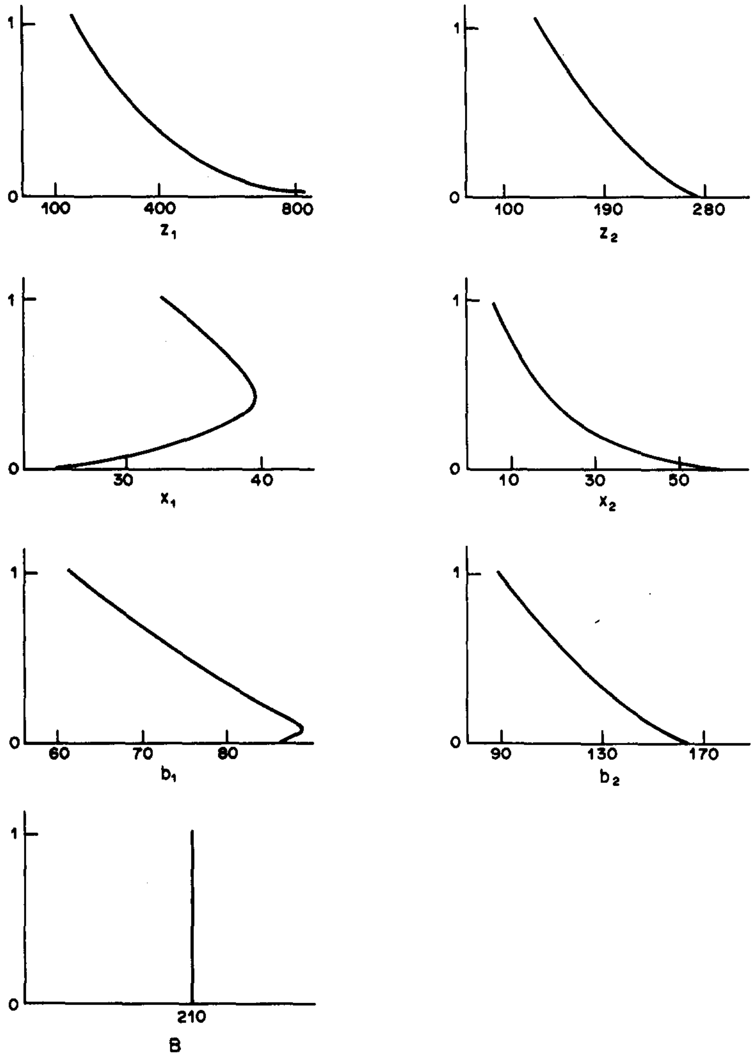

Fig. 2. System designs for different values of *a*.

By choosing different values of α one can get a set of different solutions which show the relationship of the ideal system design with the required successful possibility. For instance if we let $\alpha = 0.8$, then we obtain the optimal solutions with respect to each objective separately as follows:

$$(z_1^*)_{0.8} = 293.7063, \quad \text{for } X_1 = 0, X_2 = 24.4755,$$

$$(z_2^*)_{0.8} = 227.0270, \quad \text{for } X_1 = 56.7568, X_2 = 0.$$

The other components of the solution can be obtained by solving the following system of linear equations:

$$293.7063 = 2.6X_1^* + 12X_2^*,$$

$$227.0270 = 4X_1^* + 1.4X_2^*,$$

$$b^* = 3.7X_1^* + 8.58X_2^*,$$

$$b_1^* = X_1^* + 3.4X_2^*,$$

$$b_2^* = 2x_1^* + 2.8X_2^*,$$

which yields $X_1^* = 52.1447, X_2^* = 13.17775, b^* = 305.9785, b_1^* = 96.9482, b_2^* = 141.1964$. Therefore the system ideal for the safety factor $\alpha = 0.8$ is $(\tilde{z}^*)_{\alpha=0.8} = (z_1^* = 293.7063,\ z_2^* = 227.0270,\ X_1^* = 52.1447,\ X_2^* = 13.1775,\ b_1^* = 96.9482,\ b_2^* = 141.1864,\ B^* = 305.9985)_{\alpha=0.8}$. On the other hand, the optional design for the original budget $(\tilde{B})_{\alpha=0.8} = 210$ will be proportionally as: $(\tilde{z}')_{\alpha=0.8} = (z_1' = 201.5641,\ z_2' = 155.8036,\ X_1' = 35.7858,\ x_2' = 9.0434,\ b_1' = 66.5334,\ b_2' = 96.8934,\ B' = 210)_{\alpha=0.8}$. Furthermore, the optional system designs for $\alpha = 0$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1.0 are also obtained and the solutions are pictured in Fig. 2.

CONCLUSION

Fuzzy *de Novo* programming with multi-criteria extends the flexibility of the standard *de Novo* programming. The most promising advantage is that fuzzy *de Novo* programming allows the decision maker to deal with an uncertainty situation realistically. Furthermore, fuzzy *de Novo* programming can be solved easily based on fuzzy set and possibility theory. The final optimal solutions presents a set of solutions based on different safety factors chosen by the decision maker.

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