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Power Laws for Energy Efficient and Resilient Cities

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Abstract

Urban complexity is a hard-to-grasp concept. This paper firstly aims at investigating this issue through the prism of scale-hierarchic distributions: fractals, power laws, Zipf's laws and Pareto distributions. Authors notably emphasize the mathematical proximity between these distributions. Building on the convergence of Prigogine's dissipative structure theory, industrial ecology and Bejan's constructal law, authors then stress the crucial role that power law distributed complex urban structures have to play to make cities energy efficient and resilient.

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1. Introduction

By launching the "Eco² Cities: Ecological Cities as Economic Cities" initiative, the World Bank is suggesting new approaches based on the synergy and the interdependence of ecological and economic sustainability [HYPERLINK \l "Wor10" 1]. It insists on the necessity to integrate urban forms with urban flows, in order to create a coordinated spatial structure. By analogy with industrial ecology, this new type of urban ecology aim at organising flows within the city in the most efficient way. This optimisation of flows though is not feasible without paying a sufficient attention to the forms within which these flows are taking place. A simultaneous economic and environmental optimum can only be obtained by a cross-optimization of forms and flows, in which the morphology plays a fundamental role to create synergies within urban systems. This paper investigates the issue of urban efficiency and analyses the necessary conditions for the emergence of efficient cities. The first section looks into the

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concepts of complexity, by investigating fractals and Pareto distributions. Authors secondly emphasize the mathematical equivalence between fractals, power laws, Pareto distribution and Zipf's law. But this paper mostly aims at emphasizing and proving the crucial influence that these types of distributions have on cities' energy efficiency and resilience. The demonstration calls on thermodynamics for dissipative systems, industrial ecology and Bejan's constructal law.

2. Fractals, Pareto distributions and urban complexity

Urban complexity can be understood as successive urban scales, revealing hierarchical levels of organization within a city. In these hierarchies, some sets of consecutive levels display a much better determined arrangement than others, which are much looser. The description of a "well structured" set generally introduces the notion of structure: the higher level element is broken down into lower order elements according to a well-defined scheme, that can often be predicted to a great extent beforehand. The hierarchical order linking the frequency of appearance of elements to their size is, as we will see, a fractal order.

The term is a neologism coined by Benoît Mandelbrot 2]} from the Latin *fractus*, itself derived from the verb *frangere*, to break into pieces, to shatter into irregular fragments. Fractal means fragmented, split, irregular, interrupted. Generally speaking, fractal theory is a theory concerning the broken, the fractured, the scattered or yet about the granular, the porous, the tangled. But the strength of the theory is to have identified an order beneath the disorderly appearance of these irregular forms: the complex order of objects folded in multiple ways. The generation of mathematical fractals by iteration of functions creates complexity out of very simple rules, by repeating interminably the action of a generator. More than a theory of disorderly forms, this is a theory of the orderly patterns of chance.

Urban limits, and the size and distribution of land uses and networks obey fractal laws [HYPERLINK \l "PFr94" 3]. The notion of fractal structure accounts for the economic localization of urban activities. On a still higher scale, it makes it possible to synthesize the analysis of urban density with the notion of the hierarchy of central places. Urban geography and in particular the theory of central places underscore the fact that cities exist not in isolation but rather as part of hierarchic systems that Michael Batty and Paul Longley 4]} demonstrate obey in rank and size a fractal distribution, or power law.

Power laws have a tremendous importance in many natural phenomena. They allow describing a wide range of distributions with analogue properties: many small objects and few large objects, many small events, and few large events. This structural law is omnipresent in natural phenomena involving flows: lung and river basin structures, blood system, trees… The inverse power law can mathematically be written as $p(x)=kx^{-\alpha}$, where $p(x)$ is the multiplicity of an object (or the frequency of an event), x the size of the object (or of the event), k and α are constants.

On the other hand, Pareto first described the distribution of wealth by using power laws. It was initially used to describe the allocation of wealth within society, and also known under the "80-20 rule" designation: at the beginning of the 20th century, Pareto observed that 20% of the Italian population owned 80% of the land. Zipf then widened the use of an analogue formula to various types of distribution, such as word frequencies [HYPERLINK \l "Zip49" 5] or city size distributions. Zipf's law is also known as the rank-size distribution.

These types of distribution are omnipresent in man-made phenomena, be they social, economic and cultural: size of cities, wealth within a society, or even the number of visits to internet websites. Interestingly, these types of distributions, the one structuring natural flows, the other unconsciously structuring man-made organisations, are two sides of the same coin. It is easy to prove, after some basic algebra, that Zipf's law and Pareto distribution are synonymous with a power law distribution 6]}.

But the major point resides in the equivalence between power laws are fractal distributions. Fractals obey by definition a scale free distribution: it means that the distribution is the same whatever scale we look at it on. Newman proves that this scale-free property is not just verified by power law distributions, but is even only true of one single type of distribution: power laws [HYPERLINK \l "New05" 6].

Additionally, according to Salingaros and West 7]}, a fractal distribution obeys a power law, with α the fractal dimension of the structure considered. The equivalence between Pareto distribution, Zipf's law and scale-free distribution though is not only a mathematical curiosity, but has a much more profound meaning, which is closely related to the emergence of forms within complex systems. The following section aims at investigating the relationships between order emergence and energy efficiency.

3. Power laws for efficient cities

Cities are extremely complex structures that can be conceptualised and analysed through various prisms. Thermodynamics is the most obvious theoretical framework that can be used when it comes to consider cities as energy systems. However, using thermodynamics to assess cities energy efficiency appears to be everything but easy. Classical thermodynamics, that is widely based on the second law of thermodynamics (entropy maximization principles), fails to properly assess cities [HYPERLINK \l "Sal11" 8]. The main reason for this is that the second law can only be applied to closed systems. Cities are mainly driven by external flows: energy flows, material flows, information flows, etc. As such, applying the second law of thermodynamics for closed systems to cities is a physical nonsense. Fortunately, recent developments in thermodynamics provide interesting insights for open flow-driven systems such as cities.

The major breakthrough made by Prigogine 9,10]} opened the new era of non-linear thermodynamics applied to open flow-driven systems. Prigogine's work and what followed shed a new light on complex open systems, notably concerning the emergence of order within systems. Flow-driven systems are pushed away from the thermodynamic equilibrium (the one predicted by the second law) towards steady-states, defined by entropy production optima. For open systems such as cities, there is strictly speaking no thermodynamics equilibrium as for simple closed systems, but rather a highly organized and complex steady-state, far from the thermodynamic equilibrium.

Following these entropy production considerations, industrial ecology offers another sound framework to analyse cities as complex and organised open systems. Without going into too much detailed considerations, industrial ecology considers the relationships between energy flows and order emergence from another angle. In a nutshell, Kay [HYPERLINK \l "JJK02" 11] analyses order emergence as a response from the system to make a more effective use of the available energy flows. The more complex and organised the system, the more effectively it uses energy. For the exact same amount of available energy, a complex self-organised system will produce more than a less organised one.

The last insight concerning the relationships between energy aspects and the emergence of order in complex systems can be found in the 'constructal theory', first coined by Bejan in 1996. The empirical law he defines provides a tool to predict the emergence of structures, given the initial external conditions. In other words, given the flows to which the system is exposed, the constructal law predicts the type of structure the most likely to emerge in the system 11]}. This law has been verified for a wide range of natural phenomena, both in inanimate and living systems: river deltas, blood vessels, pulmonary highways, trees, etc.

Fortunately and interestingly, all these approaches are converging into the very same idea: A power law distribution for subsets within a complex system is the most efficient organization for a flow driven system. Salat and Bourdic [HYPERLINK \l "Sal11" 8] show that these three approaches lead to the same idea of power law distributions. Kay's most effective structure is in fact in the continuity of Prigogine's work, investigating order emergence within complex flow driven systems. On the other hand, Heitor Reis 13]} shows that Bejan's constructal law can be proved for "point-to-volume" and "point-to-area" flows, starting from Prigogine's entropy production optimum theory. In the end, these three approaches are just different ways to tackle the same issue, and lead to the exact same conclusion concerning order and energy efficiency. The most energy efficient structure for a complex flow-driven system is a highly organized state, based on power law distributions. This very fundamental result highlights the crucial influence that power law distributions – be they called fractal, Pareto or Zipf distributions – have on urban energy efficiency.

4. Historical cities are complex and resilient: lessons for the future urban world

Power laws and urban complexity influence is not limited to urban energy efficiency. Urban resilience is a hard-to-grasp concept, but will be one of the crucial issues in the century to come regarding climate change. Understood as the ability to overcome crisis and shocks, the resilience of cities is an issue that is worth being investigated in the context of climate change adaptation and fossil-fuel scarcity.

Once again, the use of fractal geometry as a tool of classification of urban phenomena constitutes a big step forward in furthering our understanding of their complexity. Fractal geometry reveals a “hidden order” in the form of urban fabrics, uncovering internal spatial structures and organizations in levels of complexity on different scales that cannot be seen using the analytical grids of ordinary geometry. Fractal geometry also allows to build explanatory models for the morphogenesis of urban fabrics into fractal objects.

Historical cities, from Sienna to San Gimignano, from Suzhou to Beijing, from Tunis to Jerusalem, are a vast laboratory for examining the relations between people, the climate and the urban environment. Faced the forces of nature – the soil, the sun, and the wind – these fractal cities were the outcome of generations of patient efforts. Historical cities the crucible in which the conception of cities has to be forged. Conversely, the planners of the modernist city set out to raze real cities in the name of abstract principles and unreal theories about the primacy of right angles and simplicity, when all historical cities are multi-scale systems complexified by their irregular topography and hydrography, and by the curving paths marked out by human beings focalized on such centres of attraction as marketplaces or mosques.

Historical cities have proved their capacity to absorb successive transformations without losing their essential structure. In Paris, no more than half of the buildings predating 1900 subsist within its historical boundaries and yet the city has managed to maintain its character thanks to the tenacious hold of the structure created by Baron Haussmann. Far from destroying the historical urban complexity, Haussmann added one more scale to the Parisian network by cutting wide boulevards through the old urban fabric. Unconsciously increasing the scale hierarchy, he gave a new coherence to the city, opening at the same time a new era for motorised transports. In the historical European city, the extremely complex substrate, the subdivisions and the street grid can be traced back to the Middle Ages and sometimes even to the Roman Empire. The capacity of the city to retain its identity despite changes has vanished from the modernist city, since it has lost its distinctive character and its transformative power.

The capacity to survive disasters and even to rise out of its ashes, like Lisbon after the 1755 earthquake, London after the Great Fire in 1666, Kyoto after the fires in the Middle Ages, Tokyo after the 1923 earthquake, is what authors call urban resilience – a complex concept related to the permanence of a memory at once social, symbolic and material. The vast majority of historical cities is resilient and has managed to survive the centuries, often outlasting the civilisations that built them. Cities worldwide will be confronted to various types of perturbations and chocks in the century to come. Will modernist cities manage to survive the century and hold out against the growing risks linked to climate change? How will their structure evolve and behave if confronted to a rise in prices due to natural resources scarcity? Unfortunately, this adaptation ability, or resilience, is rarely –if not never – taken into account in urban policy processes.

Urban tissues resilience is an indicator for cities’ stability and has therefore a strong influence on long term economic value. Resilience and efficiency of urban systems is heavily dependent on its level of complexity. In a highly dense and connected city with high levels of complexity, functional mix allows sparing significant amounts of inputs (materials, energy…). Furthermore, high levels of complexity and density make it easier to manage residual needs in a circular economy optimized by smart grids. The role of complexity becomes thus even more important when cities are confronted to exogenous or endogenous stresses (energy and natural resources scarcity, climate change, or economic crisis).

The major problem of the contemporary city is the disconnection between scales. The 20th-century technicist urban planners who ignored the fractal structure of historical cities divided the city into two spatial scales dedicated to two types of relations and behaviours: the greater metropolitan region traversed and structured by large transit infrastructures dedicated to speed and summarily zoned; and the neighbourhood, celebrated as the building block of the sustainable city, when its concept, boundaries and limits remained blurry and ill-defined. Two stances were adopted as a result. The first involved razing the old fabric and inordinately enlarging the urban grid to bring it in line with the major regional throughways. This was the position taken by Le Corbusier [HYPERLINK \l "LeC42" 14], modernism and the new towns in France. We know today that this approach is a failure, that it engenders inhuman cities, entirely given over to speed and to the ever-growing intensification of transports and energy consumption. This floating city, drifting in a territory that is too big for it, loses all urbanity, all identity, and all definition. It stops being a city. In this sense, the 20th century will have been the century of the demise of cities.

5. Conclusions

Exploring the concept of complexity applied to urban organization, this paper shows the crucial role of scale hierarchy to make cities both efficient and resilient. Scale hierarchy is omnipresent in natural and man-made phenomena, be it described by fractals, power laws, Pareto distributions or Zipf's laws, which are closely related theoretical concepts. When applied to urban systems, the thermodynamics of complex flow-driven systems, industrial ecology and Bejan's constructal approaches converge toward the same conclusion: power laws are a fundamental distribution, allowing an effective use of the available energy flows.

The central point is that an urban system's structural efficiency is the highest when it is configured according to a fractal structure. The spatial distribution of elements in such a structure obeys a Pareto distribution – that is, an inverse power law found throughout the organization of living organisms and economic systems. The scale relationships between the different hierarchic levels of an arborescence, a leaf, and the blood and oxygen circulation systems in our bodies obey such a mathematical law. It states the frequency of an element's appearance and the span of a connection based on its hierarchic level: the smaller an element is, the more often it will be encountered in the system; the bigger an element is the rarer it will be. This fundamental law defines in itself the manner in which living organisms and things should be organized to optimize their access to energy, the use that they make of it, and their resilience.

The tremendous spreading and widening or urban areas around the world urges us to pay attention to the structural parameters underpinning urban efficiency, be it environmental, energy, economic, and even social efficiency. Historical cities have survived and prospered thanks to a process of growing complexity, absorbing successive transformations without losing their essential structure. Their slow and organic growth has naturally led to dense, interconnected, complex, scale hierarchic, resilient and efficient tissues. Because of the incredible growth of cities around the world throughout the last century, these former mechanisms have unfortunately not been operating anymore in modern urban development. Modern urban tissues are extremely inefficient because of their lack of complexity, due to simplistic approaches. Authorities, urban planners, and fund raisers should be provided with tools that allow them to generate complex, resilient and efficient urban tissues.

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