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Probing models of neutrino masses via the flavor structure of the mass matrix

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ABSTRACT

We discuss what kinds of combinations of Yukawa interactions can generate the Majorana neutrino mass matrix. We concentrate on the flavor structure of the neutrino mass matrix because it does not depend on details of the models except for Yukawa interactions while determination of the overall scale of the mass matrix requires to specify also the scalar potential and masses of new particles. Thus, models to generate Majorana neutrino mass matrix can be efficiently classified according to the combination of Yukawa interactions. We first investigate the case where Yukawa interactions with only leptons are utilized. Next, we consider the case with Yukawa interactions between leptons and gauge singlet fermions, which have the odd parity under the unbroken Z_2 symmetry. We show that combinations of Yukawa interactions for these cases can be classified into only three groups. Our classification would be useful for the efficient discrimination of models via experimental tests for not each model but just three groups of models.

1. Introduction

Thanks to the discovery of a Higgs boson h at the CERN Large Hadron Collider (LHC) [1], we have entered the era to explore the origin of particle masses. Coupling constants of $W^{\pm}$, Z, t, b, and τ with h are measured at the LHC [2], and they are consistent with predicted values in the Standard Model (SM). These results strongly suggest that masses of gauge bosons and charged fermions are generated by the vacuum expectation value of the Higgs field, which provides h, as predicted in the SM. Thus, the mechanism to generate their masses in the SM was confirmed. On the other hand, neutrino masses are not included in the SM although neutrino oscillation data uncovered that neutrinos have their masses [3,4]. It is easy to add neutrino mass terms $m_\nu \overline{\nu_L} \nu_R$ to the SM similarly to the other fermion mass terms by introducing right-handed neutrinos ν_R. However, since the neutrino is a neutral fermion in contrast to the other fermions in the SM, another possibility of its mass term exists. That is the Majorana mass term, $(1/2) m_\nu \overline{\nu_L} (\nu_L)^c$. This unique possibility could be the reason why neutrinos are much lighter than the other fermions. New physics models for the Majorana neutrino mass can be found in e.g. Refs. [5–57].

The overall scale of the neutrino mass matrix m_ν generated in new physics models is determined by the structure (tree level, one-loop level, and so on) of the diagram to generate m_ν, masses of new particles in the diagram and coupling constants in the diagram. This means that the determination of the overall scale of m_ν requires to specify many parts of the Lagrangian of each model. On the other hand, the flavor structure (ratios of elements) of m_ν is simply determined by the product of Yukawa coupling matrices and fermion masses. Thus, models to generate m_ν can efficiently be classified according to the combination of Yukawa coupling matrices and fermion masses without the detail of these models. When we construct a new model to generate neutrino masses, it will be noticed indeed that the flavor structure is the key to find an appropriate set of model parameters although the overall scale of m_ν can be easily tuned by using some parameters in the scalar potential.

In this letter, we first classify models for Majorana neutrino masses according to combination of Yukawa interaction between leptons without introducing new fermions. Next, we do the classification for the case where gauge singlet fermions are introduced such that they have the odd parity under the unbroken Z_2 symmetry which can be utilized to stabilize the dark matter. For Yukawa interactions of these new fermions with leptons, Z_2-odd scalars are also introduced. We find that models can be classified into only three groups. The classification could be useful to approach efficiently the origin of Majorana neutrino masses with experimental tests of not each model but each group of models.

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Table 1
Scalar bosons which can have Yukawa interactions with leptons without introducing new fermions. The Yukawa matrix Y_A is antisymmetric, while Y_S^s and Y_S^Δ are symmetric. The lepton number (L#) is assigned to each of scalar fields such that the Yukawa interactions conserve the L# as a convention. Then, the L# is broken in the scalar potential.

Scalar	$SU(2)_L$	$U(1)_Y$	L#	Yukawa	Note
s_1^+	$\underline{\mathbf{1}}$	1	−2	$(Y_A^s)_{\ell\ell'}\left[\overline{L_\ell}\,\epsilon\, L_{\ell'}^c\, s_1^-\right]$	Antisymmetric
s^{++}	$\underline{\mathbf{1}}$	2	−2	$(Y_S^s)_{\ell\ell'}\left[\overline{(\ell_R)^c}\,\ell'_R\, s^{++}\right]$	Symmetric
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	$\underline{\mathbf{2}}$	$\frac{1}{2}$	0	$y_\ell\left[\overline{L_\ell}\,\Phi_2\,\ell_R\right]$	Diagonal
$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$	$\underline{\mathbf{3}}$	1	−2	$(Y_S^\Delta)_{\ell\ell'}\left[\overline{L_\ell}\,\Delta^\dagger \epsilon\, L_{\ell'}^c\right]$	Symmetric

Models of neutrino masses can also be classified according to topologies of diagrams [58] or decompositions of higher mass-dimensional operators [59]. They seem useful to find new models and increase the number of models in order to exhaust all possibilities. In contrast with these classifications, ours would be useful to simplify the situation where many models exist.

2. Classification of flavor structure

First, we introduce only scalar fields listed in Table 1, which have Yukawa interactions with leptons. We do not always introduce all of them, and we utilize only scalar bosons for required Yukawa interactions. For the Yukawa interaction with the second $SU(2)_L$-doublet scalar field Φ_2, the flavor changing neutral current is forbidden by utilizing a softly-broken Z_2 symmetry as usually done in the two Higgs doublet models. In order to obtain m_ν, we try to connect ν_L to $(\nu_L)^c$ by using these Yukawa interactions and the weak interaction. We do not care how scalar lines are closed because we concentrate on the flavor structure of m_ν. Each charged lepton (ℓ_L, ℓ_R, $(\ell_L)^c$, $(\ell_R)^c$) should appear only once on the fermion line in order to obtain the simplest combinations, which would give the largest contribution to m_ν. In addition, ℓ_L and ℓ_R should appear only in the next to each other on the fermion line. If they do not, the replacement of the structure between them with the mass term of ℓ can give the simpler combination.[1] It is assumed that m_ν is generated via a solo mechanism (a solo kind of fermion lines). Then, we find that only the following five combinations[2] connect ν_L and $(\nu_L)^c$:

$$m_\nu \propto Y_A^s\, y_\ell\, Y_S^s\, y_\ell\, (Y_A^s)^T, \tag{1}$$

$$m_\nu \propto y_\ell\, (Y_S^s)^*\, y_\ell, \tag{2}$$

$$m_\nu \propto g_2\, y_\ell\, (Y_S^s)^*\, y_\ell\, g_2, \tag{3}$$

$$m_\nu \propto Y_S^\Delta, \tag{4}$$

$$m_\nu \propto Y_A^s\, y_\ell^2 + (Y_A^s\, y_\ell^2)^T, \tag{5}$$

where Yukawa matrices Y_A, Y_S^s, y_ℓ, and Y_S^Δ are defined in Table 1. Diagrams of fermion lines for combinations in eqs. (1)–(5) are shown in Figs. 1–5, respectively. The $SU(2)_L$ gauge coupling constant g_2 is shown for clarity although the weak interaction is flavor blind. The combination in eq. (3) gives at least a dimension-9 operator for the Majorana neutrino mass while the others can be a dimension-5 one.

The combination in eq. (5) is the one in the Zee–Wolfenstein model [5,6] of the Majorana neutrino mass at the one-loop level,

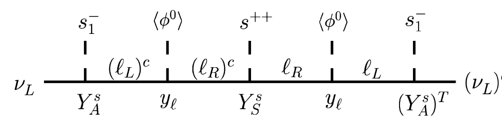

Fig. 1. The diagram of the fermion line for the combination in eq. (1).

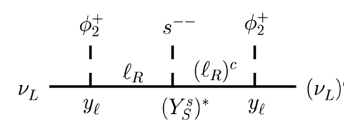

Fig. 2. The diagram of the fermion line for the combination in eq. (2).

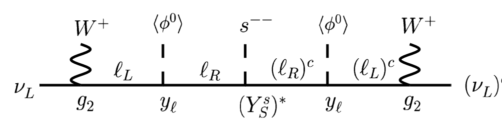

Fig. 3. The diagram of the fermion line for the combination in eq. (3).

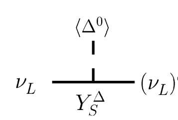

Fig. 4. The diagram of the fermion line for the combination in eq. (4).

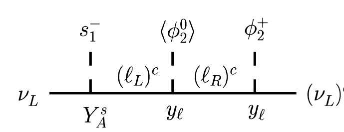

Fig. 5. The diagram of the fermion line for the combination in eq. (5).

which has been excluded already by the neutrino oscillation data [60]. Thus, this combination is ignored below. An example for m_ν in eq. (1) is the Zee–Babu (ZB) model [7,8], which generates m_ν at the two-loop level. The structure in eq. (2) is given in a model in Ref. [9] by Cheng and Li (the CL model), which also generates m_ν at the two-loop level.[3] The Gustafsson–No–Rivera (GNR) model [10] is an example for the combination in eq. (3), in which m_ν is generated at the tree-loop level. Scalar lines of W^+ and s^{--} are connected at the one-loop level by introducing the unbroken Z_2 symmetry and Z_2-odd scalar fields, which provide a dark matter candidate. The structure in eq. (4) is given at the tree level, and an example is the Higgs triplet model (HTM) [9,11]. Since eqs. (2) and (3) have the same flavor structure, that of m_ν is given

[1] Although the electron Yukawa coupling is small, the diagonal matrix y_ℓ would not be negligible because of the tau Yukawa coupling.

[2] Notice that another possible combination $Y_A^s\, g_2 + (Y_A^s\, g_2)^T$ becomes zero.

[3] In Ref. [9], scalar lines of ϕ_2^+ and s^{--} are closed in a little bit complicated way. Instead of that, it seems the simplest to introduce an $SU(2)_L$-doublet scalar field with the hypercharge $Y = 3/2$.

Table 2
Scalar bosons for Yukawa interactions of gauge singlet fermion ψ^0_{iR} with leptons. These scalar bosons and ψ^0_{iR} are Z_2-odd fields. Structures of Yukawa matrices Y^s and Y^η are arbitrary. When ψ^0_R has L# $= x$, lepton numbers $-x-1$ and $x-1$ are assigned to s_2^+ and η, respectively, such that their Yukawa interactions conserve the L# as a convention. The L# is broken in the scalar potential and/or M_ψ.

Scalar	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	Yukawa	Note
s_2^+	$\underline{\mathbf{1}}$	1	$Y^s_{\ell i}\left[\overline{(\ell_R)^c}\,\psi^0_{iR}\,s_2^+\right]$	Arbitrary
$\eta = \begin{pmatrix}\eta^+\\ \eta^0\end{pmatrix}$	$\underline{\mathbf{2}}$	$\frac{1}{2}$	$Y^\eta_{\ell i}\left[\overline{L_\ell}\,\epsilon\,\eta^*\,\psi^0_{iR}\right]$	Arbitrary

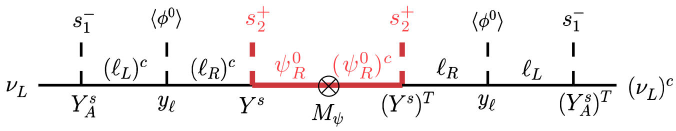

Fig. 6. The diagram of the fermion line for the combination in eq. (6). Bold red lines are for the Z_2-odd particles.

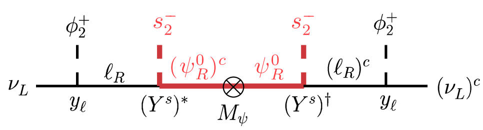

Fig. 7. The diagram of the fermion line for the combination in eq. (7). Bold red lines are for the Z_2-odd particles.

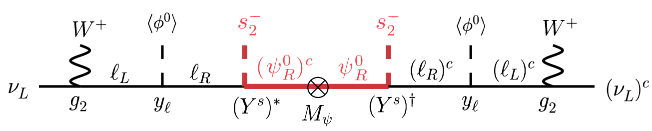

Fig. 8. The diagram of the fermion line for the combination in eq. (8). Bold red lines are for the Z_2-odd particles.

by only three combinations of Yukawa matrices: $Y^s_A\, y_\ell\, Y^s_S\, y_\ell\, (Y^s_A)^T$, $y_\ell\,(Y^s_S)^*\,y_\ell$, and Y^Δ_S.

Next, we impose the unbroken Z_2 symmetry to models and introduce gauge singlet fermions ψ^0_{iR} as the Z_2-odd fields. The fermions have Majorana mass terms, $(1/2)M_{\psi i}\overline{(\psi^0_{iR})^c}\psi^0_{iR}$. We can take the basis where M_ψ is diagonalized without loss of generality. For Yukawa interactions of ψ^0_{iR} with leptons, scalar fields in Table 2 are also introduced as Z_2-odd fields. Scalar fields in Table 1 and the SM fields are Z_2-even ones. Then, the lightest Z_2-odd particle becomes stable. If the lightest Z_2-odd particle is neutral one, it can be a dark matter candidate. We find that the Majorana neutrino mass matrix can be obtained by the following four kinds of combinations of Yukawa matrices and the weak interaction in addition to the five combinations in eqs. (1)–(5):

$$m_\nu \propto Y^s_A\, y_\ell\, Y^s\, M_\psi^{-1}\, (Y^s)^T\, y_\ell\, (Y^s_A)^T, \tag{6}$$

$$m_\nu \propto y_\ell\, (Y^s)^*\, M_\psi^{-1}\, (Y^s)^\dagger\, y_\ell, \tag{7}$$

$$m_\nu \propto g_2\, y_\ell\, (Y^s)^*\, M_\psi^{-1}\, (Y^s)^\dagger\, y_\ell\, g_2, \tag{8}$$

$$m_\nu \propto Y^\eta\, M_\psi^{-1}\, (Y^\eta)^T, \tag{9}$$

where Yukawa matrices Y^s and Y^η are defined in Table 2. Figs. 6–9 correspond to diagrams of fermion lines for combinations in eqs. (6)–(9), respectively. The part M_ψ^{-1} is given by assuming ψ^0_{iR} are heavier than the other particles. If it is not the case, M_ψ^{-1} can be replaced with M_ψ.

The Krauss–Nasri–Trodden (KNT) model [12] of m_ν at the three-loop level is an example for the combination in eq. (6). The structure in eq. (7) is realized, for example, in the Aoki–Kanemura–Seto (AKS) model [13] at the three-loop level by introducing the Z_2-odd real singlet scalar boson. Since the three-loop diagram utilizes the scalar interaction with two Higgs doublet fields, the AKS model can explain not only m_ν and the dark matter but also the baryon asymmetry of the universe via the electroweak baryogenesis scenario. An example of the combination in eq. (9) is the Ma model [14], where m_ν is generated at the one-loop level. No model is known for m_ν in eq. (8).[4] Flavor structures of combinations in eqs. (7) and (8) are the same because the weak interaction does not change the flavor. Therefore, the flavor structure of m_ν is determined by three combinations when we use the Yukawa interactions in Table 2: $Y^s_A\, y_\ell\, Y^s\, M_\psi^{-1}\, (Y^s)^T\, y_\ell\, (Y^s_A)^T$, $y_\ell\, (Y^s)^*\, M_\psi^{-1}\, (Y^s)^\dagger\, y_\ell$, and $Y^\eta\, M_\psi^{-1}\, (Y^\eta)^T$.

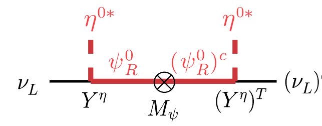

Fig. 9. The diagram of the fermion line for the combination in eq. (9). Bold red lines are for the Z_2-odd particles.

It is clear that combinations in eqs. (1)–(4) and eqs. (6)–(9) can be classified further to only the following three groups:

$$\text{Group-I}: \quad m_\nu \propto Y^s_A\, y_\ell\, X_{SR}\, y_\ell\, (Y^s_A)^T, \tag{10}$$

$$\text{Group-II}: \quad m_\nu \propto y_\ell\, X^*_{SR}\, y_\ell, \tag{11}$$

$$\text{Group-III}: \quad m_\nu \propto X_{SL}, \tag{12}$$

where symmetric matrices X_{SR} and X_{SL} are given by

$$X_{SR} = Y^s_S, \quad Y^s M_\psi^{-1} (Y^s)^T, \quad Y^s M_\psi (Y^s)^T, \tag{13}$$

$$X_{SL} = Y^\Delta_S, \quad Y^\eta M_\psi^{-1} (Y^\eta)^T, \quad Y^\eta M_\psi (Y^\eta)^T. \tag{14}$$

The matrix X_{SR} is for the effective interactions of right-handed charged leptons while the matrix X_{SL} is for the ones of left-handed leptons. As long as we concentrate on the flavor structure, it seems difficult to discriminate the origin of X_{SR} (X_{SL}) in eq. (13) (eq. (14)).

We mention here the type-I [15] and the type-III seesaw [16] models, where gauge singlet fermions (for the type-I) or $\mathrm{SU}(2)_L$-triplet Majorana fermions (for the type-III) are introduced. The structure of m_ν in these models can be included in the Group-III because Yukawa matrices Y_A and y_ℓ are not used to generate m_ν. However, they are exceptions because new scalar fields are not introduced. Discussion in the next section (namely, $\tau \to \overline{\ell_1}\ell_2\ell_3$ ($\ell_1, \ell_2, \ell_3 = e, \mu$) for the Group-III) is not applicable for these models.[5]

3. Discussion

The neutrino mass matrix m_ν is expressed as $U^*_{\mathrm{MNS}}\mathrm{diag}(m_1 e^{i\alpha_{12}}, m_2, m_3 e^{i\alpha_{32}})U^\dagger_{\mathrm{MNS}}$, where m_i ($i = 1$–3) are the neutrino mass eigenvalues, α_{12} and α_{32} are the Majorana phases [61], and U_{MNS} is the Maki–Nakagawa–Sakata (MNS) matrix [62] of the lepton flavor mixing. The Group-I gives $m_1 = 0$ or $m_3 = 0$

[4] The combination in eq. (8) gives at the least a dimension-9 operator for m_ν, and it might be four-loop realization at the least. Then, too small neutrino masses might be generated.

[5] There is the box diagram with the W boson and neutral fermions from $\mathrm{SU}(2)_L$-singlet or triplet, but the interaction of the neutral fermions with W is suppressed by $\sqrt{m_\nu/M_R}$ (the mixing between ν_L and the fermions), where M_R denotes the fermion mass.

because of $\mathrm{Det}(m_\nu) \propto \mathrm{Det}(Y_A) = 0$. Although this has been known for the Zee–Babu model [8] (an example of models in the Group-I), our statement is more model-independent. The Group-I is excluded if the absolute neutrino mass is directly measured at the KATRIN experiment [63] whose estimated sensitivity is 0.35 eV at 5σ confidence level. The indirect bound on the sum of neutrino masses, $\sum_i m_i < 0.23$ eV (90% confidence level), was obtained by cosmological observations [64], and sensitivity to $\sum_i m_i = \mathcal{O}(0.01)$ eV is expected in future experiments [65].

The flavor structure of m_ν is constrained by the neutrino oscillation data, and the constrained structure can be translated into constraints on the flavor structure (ratios of elements) of X_{SR} of the Group-II and X_{SL} of the Group-III. Hereafter, we denote X_{SR} of the Group-II and X_{SL} of the Group-III as X for simplicity. These interactions can cause the lepton flavor violating (LFV) decays $\tau \to \overline{\ell}_1 \ell_2 \ell_3$ ($\ell_1, \ell_2, \ell_3 = e, \mu$). Ratios of the decay branching ratios (BR) of these LFV decays can be determined by the flavor structure of X independently on the overall scale of m_ν. In order to evade the strong constraint $\mathrm{BR}(\mu \to \overline{e}ee) < 1.0 \times 10^{-12}$ [66], LFV decays $\tau \to \overline{\ell}_1 \ell_2 \ell_3$ can be observed at the Belle II experiment [67] only for $X_{ee} = 0$ or $X_{e\mu} = 0$, which constrains ratios of $\mathrm{BR}(\tau \to \overline{\ell}_1 \ell_2 \ell_3)$ as discussed in the HTM (included in the Group-III) [68]. For $X_{ee} = 0$ ($X_{e\mu} = 0$), LFV decays $\tau \to \overline{\ell} ee$ ($\tau \to \overline{\ell} e\mu$) do not occur. Since $X_{e\ell}$ elements for the Group-II are enhanced by $1/m_e$ for a given m_ν, it is likely that $\mathrm{BR}(\tau \to \overline{e} e\mu)$ for $X_{ee} = 0$ or $\mathrm{BR}(\tau \to \overline{e} ee)$ for $X_{e\mu} = 0$ is larger than the others. For $X_{ee} = X_{e\mu} = 0$, only $\tau \to \overline{e}\mu\mu$ can be observed for the Group-II as shown in the GNR model [10], while $\tau \to \overline{\mu}\mu\mu$ is also possible for the Group-III. Notice that $X_{ee} = 0$ for the Group-II and III results in $(m_\nu)_{ee} = 0$, which is excluded if the neutrinoless double beta decay (see e.g. Ref. [69]) is observed or $m_3 < m_1$ (the inverted mass ordering of neutrinos) is determined by neutrino oscillation experiments (see e.g. Ref. [70]). Notice also that $(X_{SR})_{ee} = 0$ for the Group-I does not mean $(m_\nu)_{ee} = 0$. Therefore, if $(m_\nu)_{ee} = 0$ is excluded by these neutrino experiments, the observation of $\tau \to \overline{\ell} ee$ indicates the Group-I because the situation is inconsistent for the Group-II and III.

The discussion above did not require the discovery of new particles. If a charged scalar boson is discovered and dominantly decays into leptons, the branching ratios are expected to be given by Y_A (y_ℓ) when the Group-I (II) is assumed. The flavor structure of y_ℓ is known, and decays via the y_ℓ are dominated by the decay into τ. The flavor structure of Y_A is determined by the neutrino oscillation data as $(Y_A)_{e\mu}/(Y_A)_{e\tau} = -(U_{\mathrm{MNS}})^*_{\tau 1}/(U_{\mathrm{MNS}})^*_{\mu 1}$ and $(Y_A)_{\mu\tau}/(Y_A)_{e\tau} = -(U_{\mathrm{MNS}})_{e1}/(U_{\mathrm{MNS}})^*_{\mu 1}$ for $m_1 < m_3$. For $m_1 > m_3$, they are given by $(Y_A)_{e\mu}/(Y_A)_{e\tau} = -(U_{\mathrm{MNS}})_{\tau 3}/(U_{\mathrm{MNS}})_{\mu 3}$ and $(Y_A)_{\mu\tau}/(Y_A)_{e\tau} = -(U_{\mathrm{MNS}})^*_{e3}/(U_{\mathrm{MNS}})^*_{\mu 3}$. Ratios of decay branching ratios $\mathrm{BR}(s_1^- \to e\nu) : \mathrm{BR}(s_1^- \to \mu\nu) : \mathrm{BR}(s_1^- \to \tau\nu)$ are roughly given by $2:5:5$ for $m_1 < m_3$ and $2:1:1$ for $m_1 > m_3$ [71]. Therefore, Group-I and II can be tested by measuring leptonic decays of the charged scalar boson at the collider experiments.

When a group of models is favored by the experiments discussed above, we will try to discriminate models in the group by using details of each model. For example, the doubly-charged scalar boson is introduced in the ZB model in the Group-I while it does not exist in the KNT model of the Group-I. Thus, if the doubly-charged scalar boson is discovered at the collider experiments, the ZB model would be favored among models in the Group-I. This is the same for the CL model and the GNR model in the Group-II and the HTM in the Group-III. Even if groups of models have not been discriminated, collider experiments can test each models by measuring properties (e.g. decay patterns) of new particles as usually studied for model by model.

4. Conclusion

In this letter, we have studied the systematic classification of models for generating Majorana neutrino masses m_ν according to combinations of Yukawa interactions. If we use Yukawa interactions for leptons by introducing new scalar fields relevant for these Yukawa interactions, the flavor structure of m_ν is given by three combinations: $Y_A^s\, y_\ell\, Y_S^s\, y_\ell\, (Y_A^s)^T$, $y_\ell\, (Y_S^s)^*\, y_\ell$, and Y_S^Δ. The Yukawa matrix Y_A is antisymmetric while Y_S^s and Y_S^Δ are symmetric. The Yukawa couplings y_ℓ are proportional to charged lepton masses. For the case where gauge singlet Z_2-odd fermions ψ_{iR}^0 and Z_2-odd scalar fields are additionally introduced, the flavor structure of m_ν is determined also by $Y_A^s\, y_\ell\, Y^s\, M_\psi^{-1}\, (Y^s)^T\, y_\ell\, (Y_A^s)^T$, $y_\ell\, (Y^s)^*\, M_\psi^{-1}\, (Y^s)^\dagger\, y_\ell$, and $Y^\eta\, M_\psi^{-1}\, (Y^\eta)^T$. The Yukawa matrices Y_S^s and Y_S^η are symmetric, and M_ψ is the Majorana mass matrix for ψ_{iR}^0. Combining these results, we have found that models can be classified into only three groups: $m_\nu \propto Y_A^s\, y_\ell\, X_{SR}\, y_\ell\, (Y_A^s)^T$, $y_\ell\, X_{SR}^*\, y_\ell$, and X_{SL}. Here, X_{SR} and X_{SL} are some symmetric matrices. Although the structure of m_ν in the type-I seesaw and the type-III seesaw models can be classified in the Group-III, these models are exceptions to the discussion in this letter. Our classification enable us to approach efficiently to the origin of Majorana neutrino masses by testing not each model but each groups of models.

We concentrated on Majorana neutrino masses in this letter. The similar classification of models for Dirac neutrino masses is also desired because the nature may respect the lepton number conservation. This will be presented elsewhere [72].

Acknowledgements

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