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A bound on the scale of spacetime noncommutativity from the reheating phase after inflation

R. Horvat [a,*], J. Trampetic [a,b]

[a] Institute Ruđer Bošković, Bijenička 54, 10000 Zagreb, Croatia
[b] Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany

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ABSTRACT

In an approach to noncommutative gauge theories, where the full noncommutative behavior is delimited by the presence of the UV and IR cutoffs, we consider the possibility of describing a system at a temperature T in a box of size L. Employing a specific form of UV/IR relationship inherent in such an approach of restrictive noncommutativity, we derive, for a given temperature T, an upper bound on the parameter of spacetime noncommutativity $\Lambda_{\rm NC} \sim |\theta|^{-1/2}$. Considering such epochs in the very early universe which are expected to reflect spacetime noncommutativity to a quite degree, like the reheating stage after inflation, or believable pre-inflation radiation-dominated epochs, the best limits on $\Lambda_{\rm NC}$ are obtained. We also demonstrate how the nature and size of the thermal system (for instance, the Hubble distance versus the future event horizon) can affect our bounds.

At the perturbative level noncommutative (NC) quantum field theories (QFT) suffer from the infamous UV/IR mixing problem [1] and therefore lack universality [1–3]. This means that any modification at very high momentum scales does inevitably modify the physics at very small momentum scales in a profound way, rather than involving a soft modification being switched off in the far IR. A treatment of this problem thus necessitate understanding on the UV completion of the theory. The formal reason for such a behavior is a tacit assumption that in four dimensions NC gauge field theories are valid up to arbitrarily large momentum scales.

In two important papers [4,5], it was shown that NC gauge theories can be realized as an effective QFT, underlain by some more fundamental theory such as string theory. In particular, it was shown [4] that at energy scales below the IR cutoff $\Lambda_{\rm IR}$, the NC theory becomes (up to residual effects) an ordinary commutative QFT, thereby diminishing substantially the power of the UV/IR mixing. On the other hand, it was also claimed [4] that the phenomenological effects of the UV completion (for a large class of more general QFTs above the UV cutoff $\Lambda_{\rm UV}$) can be quite successfully modeled by a threshold value $\Lambda_{\rm UV}$. To a good approximation the theory thus becomes an effective QFT with the UV and IR cutoffs obeying the relationship

$$\Lambda_{\rm UV}\Lambda_{\rm IR} \sim \Lambda_{\rm NC}^2, \tag{1}$$

where the scale of noncommutativity is heuristically introduced as $\Lambda_{\rm NC}^{-2} \sim |\theta|$. The full scope of noncommutativity is experienced only in the range delimited by $\Lambda_{\rm IR}$ and $\Lambda_{\rm UV}$, $\Lambda_{\rm IR} < \Lambda_{\rm NC} < \Lambda_{\rm UV}$, while the commutative world (up to residual effects of noncommutativity) resides below $\Lambda_{\rm IR}$. Such a scenario had already been confronted with experimental data [4,6], and in so doing useful information on the scale of noncommutativity $\Lambda_{\rm NC}$ was drawn. Note that with $\Lambda_{\rm NC}$ high enough, the whole standard model can be placed below $\Lambda_{\rm IR}$. In this way, one successfully gets rid of the Lorentz symmetry violating mass term for photons [4], a relic of the theory in which the scope of noncommutativity extends up to $\Lambda_{\rm UV} \to \infty$ and down to $\Lambda_{\rm IR} \to 0$.

Let us first examine how the above scenario affects the UV/IR mixing problem. The phenomenon of UV/IR mixing is best understood by examining the behavior of the (nonplanar) loop graphs with the ordinary product of fields replaced by the Moyal $\star$-product (see, e.g., [7,8]). This results in phase factors [9,10] depending on the virtual momenta of internal loops. In a theory without UV completion ($\Lambda_{\rm UV} \to \infty$), these phase factors although efficient in damping out the high-energy part of the graphs become together inefficient to control the vanishing momenta, i.e., the original UV divergences reappear as IR divergences. On the other hand, in presence of a finite $\Lambda_{\rm UV}$ no one sort of divergence will appear since the said phase factors effectively transform the highest energy scale ($\Lambda_{\rm UV}$) into the lowest one ($\Lambda_{\rm IR}$). Besides the appearance of new infrared divergences in the IR limit

* Corresponding author.
E-mail addresses: horvat@lei3.irb.hr (R. Horvat), josipt@rex.irb.hr (J. Trampetic).

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of the external momentum, another two (inseparable) aspects[1] of the UV/IR mixing problem also involve: (i) nonanalytic behavior in the NC parameter when $\theta \to 0$, (ii) pathological behavior when the spatial extension of size $|\theta P|$, for a particle moving with momentum P along the region affected by spacetime noncommutativity, gets reduced to a point $|\theta P| \to 0$. Note that all aspects of the UV/IR mixing problem get disappeared if NC gauge theory can be realized as an effective QFT.

The easiest way to understand the peculiar mixing of UV and IR effects in (1) is to invoke (ii) above, i.e., an interpretation that a quantum in NC gauge theory can be thought of as a straight string connecting two opposite charges [13–15]. Indeed, combining an uncertainty relation for the coordinates

$$\Delta x^\mu \Delta x^\nu > \theta^{\mu\nu}, \tag{2}$$

where $\theta^{\mu\nu}$ has dimensions of $(\text{length})^2$, with the Heisenberg uncertainty principle,[2] $\Delta x^i \Delta p^j \geqslant (1/2)\delta^{ij}$, and switching to the language of effective QFT with $(\Delta p)_{\max} \sim \Lambda_{\text{UV}}$, $(\Delta x)_{\max} \sim \Lambda_{\text{IR}}^{-1}$ (also employing $\Lambda_{\text{NC}}^{-2} \sim |\theta|$), (1) immediately comes about. If we choose (without loss of generality) θ to lie in the $(1,2)$ plane, $\theta^{1,2} = -\theta^{2,1} \equiv \theta$, this means that a particle moving inside the NC plane with momentum P along the one axis, has a spatial extension of size $|\theta P|$ along the other. The string vector is always perpendicular to the direction of motion.

In the present Letter, we require that the effective field-theoretical treatment of NC gauge theories as given by (1) be also capable of describing a system at the temperature T in a box of size L. We note that an ordinary effective QFT is expected to be capable of describing a thermal system provided $\Lambda_{\text{UV}} \gtrsim T$ as long as $T \gg L^{-1}$. We shall focus mainly on radiation-dominated epochs in the very early universe, both post- and pre-inflationary ones, where the temperature is expected to be so high that the field-theoretical treatment of such epochs also requires spacetime noncommutativity. In other words, we require that the above effective description of spacetime noncommutativity be also capable of describing radiation-dominated epochs in the early universe. Such a requirement will bring us a valuable information on the parameter of spacetime noncommutativity Λ_{NC}.

Sticking to a stringy picture for quanta propagating in NC background, one immediately sees that having them in thermal equilibrium in the volume L^3 is precluded if the size of the string exceeds the size of the box L. The maximal size of the string in the field-theoretical treatment, for θ lying in the $(1,2)$ plane and averaging over directions of the momentum of the quantum, can be found to be

$$|\theta p|_{\max} = \frac{1}{\sqrt{2}} \frac{\Lambda_{\text{UV}}}{\Lambda_{\text{NC}}^2}, \tag{3}$$

where p is the total momentum. Thus, the field-theoretical treatment of spacetime noncommutativity as represented by (1) is expected to be capable of describing a thermal system (of size L) if

$$|\theta p|_{\max} \lesssim L \tag{4}$$

together with $\Lambda_{\text{UV}} \gtrsim T$ and $T \gg L^{-1}$. This entails the following upper bound on Λ_{NC}

$$\Lambda_{\text{NC}} \gtrsim (2)^{-1/4} L^{-1/2} T^{1/2}. \tag{5}$$

In the early universe, the reheating stage marks an abrupt transition from a cold, low-entropy phase of the inflatory era and a subsequent high-entropy radiation-dominated epoch [17,18]. In the simplest picture which does not include the preheating stage after inflation [18–20], the vacuum energy of the inflaton field experienced an instantaneous conversion into radiation when the decay rate of the inflaton field had become equal to the expansion rate of the universe H. This event defines the reheating temperature as the maximum temperature of the subsequent radiation-dominated epoch (although it is not necessarily the maximum temperature of the universe after inflation [17]). At the time of reheating the Hubble parameter and the reheating temperature are thus related as

$$H(T_{RH}) = \left(\frac{8\pi^3}{90} g_*(T_{RH}) \right)^{1/2} \frac{T_{RH}^2}{M_{Pl}}, \tag{6}$$

where g_* is the effective number of relativistic degrees of freedom and M_{Pl} is the Planck mass. The reheating temperature in (6) depends, through the inflaton decay width, both on the inflaton mass and on its coupling to matter [17,18].

With the most natural choice $L^{-1} = H$ and using (6), the bound (5) becomes

$$\Lambda_{\text{NC}} \gtrsim \left(\frac{4\pi^3}{90} g_*(T_{RH}) \right)^{1/4} \frac{T_{RH}^{3/2}}{M_{Pl}^{1/2}}. \tag{7}$$

The main reason of why the reheating temperature should not be too high (thus weakening our bounds) is that one inevitably overproduces gravitinos in supergravity theories. The limit from gravitino overproduction is $T_{RH} \lesssim 10^9$–10^{10} GeV [21,22]. Taking the effective number of degrees of freedom at the reheating temperature as for the MSSM ($g_*(T_{RH}) = 915/4$) one obtains for the maximum allowable T_{RH}

$$\Lambda_{\text{NC}} \gtrsim 500 \text{ TeV}. \tag{8}$$

The bound (8) proves to be as powerful as the bound obtained recently from nonobservation of ultrahigh energy neutrino induced events in neutrino observatories [23].

Next we demonstrate how the heuristic arguments leading to (7) can be beefed up by invoking entropic considerations, and, in particular, the holographic bound. For a collection of strings of length $\sim \Lambda_{\text{IR}}^{-1}$ in the volume $\sim \Lambda_{\text{IR}}^{-3}$, the entropy is bounded by the Bekenstein bound S_B [24]. For a macroscopic system in which self-gravitation effects can be disregarded, the Bekenstein bound is given by a product of the energy and the linear size of the system, EL. In the context of effective QFTs, it becomes proportional $\Lambda_{\text{UV}}^4 \Lambda_{\text{IR}}^{-4}$. It should be noted that it is more extensive than the entropy in effective QFTs, $S_{QFT} \sim \Lambda_{\text{UV}}^3 \Lambda_{\text{IR}}^{-3}$. On the other hand, for a weakly gravitating system S_B is bounded by the holographic Bekenstein–Hawking entropy, $S_{BH} \sim \Lambda_{\text{IR}}^{-2} M_{Pl}^2$.[3] Ignoring, for simplicity, the numerical factors, setting[4]

$$S_B \leqslant S_{BH}, \tag{9}$$

[1] For an NC field theory model in which different aspects of the UV/IR mixing problem become disentangled, see [11], with details given in [12].

[2] Since we are not interested in the black-hole regime, we do not invoke the Generalized Uncertainty Principle (GUP) [16] here. Yet the holographic principle is discussed below in a different manner.

[3] It was shown [6] that in the regime obeyed by the present field-theoretical framework, NC thermodynamical laws are an NC deformation of the usual laws [25,26]. Thus the commutative area law, $S_{BH}^{\text{NC}} = AM_{Pl}^2/4$, stays preserved in an NC setting.

[4] Setting, on the other hand, $S_{QFT} \leqslant S_B$ and invoking (1) gives us a consistency condition for the theory, $\Lambda_{\text{UV}} \gtrsim \Lambda_{\text{NC}} \gtrsim \Lambda_{\text{IR}}$. The final option, $S_{QFT} \leqslant S_{BH}$, means that with this bound, at saturation, our effective theory should also be capable of describing systems containing black holes, since it necessarily includes many states with Schwarzschild radius much larger than the box size. There are however compelling arguments for why an ordinary local effective QFT appears unlikely to provide an adequate description of any systems containing black holes.

and invoking (1), one arrives at

$$\Lambda_{\rm NC} \gtrsim \frac{\Lambda_{\rm UV}^{3/2}}{M_{Pl}^{1/2}}. \tag{10}$$

Such a framework is capable of describing a thermal system if $\Lambda_{\rm UV} \gtrsim T$, where $T \gg L^{-1}$. This way, (7) is immediately reproduced. Having employed $\Lambda_{\rm UV} \gtrsim \Lambda_{\rm NC}$ in (10), we obtain that only sub-planckian noncommutativity, $\Lambda_{\rm NC} \lesssim M_{Pl}$, is allowed by the holographic bound. In fact, entropic considerations employed here generalize the bound (5) in such a way that L is being replaced with the largest possible size of the string consistent with the holographic bound at the temperature T. In radiation-dominated cosmologies, such a scale is provided by the Hubble distance, although other choices for L are also possible (see below). Now we are on the much firmer ground with our bounds.

One may object, though, that our bounds include spacetime noncommutativity on the particle theory side only, and do not consider the possibility that noncommutativity can affect particular epochs in the history of the universe or even the whole history as well [27–31]. Since our interest is in radiation dominated epochs in the early universe, the only real concern is a fact that NC spacetime geometry leads to modified dispersion relations [32], affecting in turn various thermodynamical quantities. These become dependent not only on the temperature, but also on the parameter characterizing spacetime noncommutativity. Even so, our effective treatment is expected to provide an adequate description of such systems as long as the hierarchy $\Lambda_{\rm UV} \gtrsim \Lambda_{\rm NC}$, T is respected. In particular, a modification of dispersion relations for the radiation has already been studied [4] in the effective field-theoretical model obeying (1). While at low momentum scales ($k \ll \Lambda_{\rm IR}$) one gets a polarization dependent propagation speed (birefringence), for $\Lambda_{\rm IR} \ll k \ll \Delta M_{\rm SUSY}$ a Lorentz violating mass term $\sim \Delta M_{\rm SUSY}$ emerges, where $\Delta M_{\rm SUSY}$ is the supertrace of the mass matrix. Thus at high temperatures of interest here, $T \gg \Delta M_{\rm SUSY}$, a modification to ordinary dispersion relations turns out to be negligible.

The possibility of having a radiation-dominated epoch taken place before inflation is quite generic in many extensions of the ΛCDM model, especially in those with a symmetry-breaking phase transition characterized by a high-energy scale (e.g., GUT symmetry breaking) [17]. The solution to such a transition has been obtained long ago [33]. Recently, the effect of such a pre-inflation radiation-dominated epoch to CMB anisotropy has been studied [34,35]. Even the inflation itself may occur in a state with thermal relativistic matter, when the equation of state turns into that of inflationary matter under influence of the NC structure of spacetime [32]. All such thermal epochs are expected to be describable altogether by our NC effective QFT approach if the hierarchy $\Lambda_{\rm UV} \gtrsim \Lambda_{\rm NC}$, T is respected, supplemented also by a holographic bound $\Lambda_{\rm UV} \lesssim M_{Pl}$. Having the temperature in these epochs much higher than in the phase of reheating, the bound on the scale of noncommutativity (8) is upraised by a tremendous amount. For instance, for a GUT scale of about 10^{15} GeV one would have $\Lambda_{\rm NC} \gtrsim 10^{10}$ TeV. For a radiation-dominated epoch near the Planck epoch, the bound is obviously saturated by the holographic constraint near the Planck mass. It is important to note that our bounds do not require any prior specification of the cutoffs as long as the aforementioned hierarchy between the scales stays respected.

Finally, we would like to see to what extent choices different than $L^{-1} = H$ could affect our bound (8). For that purpose we resort to a particular cosmological model, which, as a bonus, is also proven to be successful in describing the present accelerated phase of the universe. We choose the model for holographic dark energy [36–38], a stuff prevailing at present times but suppressed at earlier cosmological epochs. For the sake of illustration, we consider the popular Li's model [37]. This model belongs to a class of non-interacting and saturated HDE models, with a choice for L in the form of the future event horizon,

$$d_E = a \int_a^\infty \frac{da}{a^2 H}, \tag{11}$$

with a being a scale factor. The vacuum energy density, assumed not to be responsible for the early-time inflation, is parametrized as $\rho_\Lambda = (3/8\pi) M_{Pl}^2 d_E^{-2}$. Extracting d_E amounts to knowing ρ_Λ during the radiation-domination epoch, in which ρ_Λ occupies only a tiny fraction of the total energy density. In a two-component universe ρ_Λ evolution is governed by [37,39][5]

$$\Omega'_\Lambda = \Omega_\Lambda^2 (1 - \Omega_\Lambda)\left[\frac{1}{\Omega_\Lambda} + \frac{2}{\sqrt{\Omega_\Lambda}}\right], \tag{12}$$

where the prime denotes the derivative with respect to $\ln(a)$. In (12) $\Omega_\Lambda = \rho_\Lambda/\rho_{crit}$, where ρ_{crit} is the critical density. With $\Omega_\Lambda \ll 1$ and $\rho_{crit} \simeq \rho_{rad}$, we obtain

$$\rho_\Lambda(a) \simeq g_*(a)\ \rho_{rad_0} a^{-3}, \tag{13}$$

where ρ_{rad_0} denotes the radiation energy density at the present time. In turn this, together with a solution of (12) for the matter-dominated epoch,

$$\rho_\Lambda(a) \simeq \rho_{m_0} a^{-2}, \tag{14}$$

determines the ratio $L^{-1/2}/H^{1/2}$ at the time of reheating as

$$\frac{L^{-1/2}(T_{RH})}{H^{1/2}(T_{RH})} \sim \frac{T_0}{T_{RH}^{1/4} T_{MR}^{3/4}}, \tag{15}$$

where T_0 is the temperature of the universe at present and T_{RM} is the temperature at the moment when the matter density becomes equal to that of the radiation. Plugging some relevant numbers and $T_{RH} = 10^{10}$ GeV, one finds the ratio (14) to be $\sim 10^{-10}$. This leads, via (5), to a insignificant bound on $\Lambda_{\rm NC}$. As has been already made clear with (7), the holographic bound on $\Lambda_{\rm NC}$ virtually coincides with the bound (5) for the choice $L = H^{-1}$. Thus any choice for L being larger than the Hubble distance at the time of reheating leads to a weaker bound. This arguably demonstrates how strongly the choice for the 'size' of the universe can influence our bounds. There are of course other choices for L being relevant for the bound (5); for instance, the particle horizon. For this choice for L one would expect a bound similar to (8); however, the employed model in this case can no longer be responsible for the current accelerated phase of the universe.

Summing up, we have made use of a reasonable expectation that thermal epochs in the universe are successfully describable by conventional QFTs. We have considered the field-theoretical realization of noncommutative gauge field theories and shown, making use of the specific UV/IR relationship attributive to such an approach, that adequate description of a thermal system entails an upper bound on the scale of noncommutativity. We have also discussed such a bound in conjunction with the holographic bound. For the reheating stage after inflation we have obtained a bound of order of 10^3 TeV. The radiation-dominated epochs at higher temperatures, if existent, would provide much better bounds, possibly all the way up to the Planck mass. It is important to notice that

[5] The modification of the right-hand side of the Einstein equations arising from the fuzziness of space induced by $\Lambda_{\rm NC}$ [40] can be shown to be unimportant for the radiation-dominated epochs of interest here.

only existence of such epochs matters and not the physics responsible for their emergence, since in thermal equilibrium the physical system loses memory of its initial state. We have also demonstrated how the size and nature of the thermal system may crucially affect our bounds.

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References

[1] S. Minwalla, M. Van Raamsdonk, N. Seiberg, JHEP 0002 (2000) 020.
[2] V.V. Khoze, G. Travaglini, JHEP 0101 (2001) 026, arXiv:hep-th/0011218.
[3] T.J. Hollowood, V.V. Khoze, G. Travaglini, JHEP 0105 (2001) 051, arXiv:hep-th/0102045.
[4] S.A. Abel, J. Jaeckel, V.V. Khoze, A. Ringwald, JHEP 0609 (2006) 074, arXiv:hep-ph/0607188.
[5] S. Abel, C.-S. Chu, M. Goodsell, JHEP 0611 (2006) 058, arXiv:hep-th/0606248.
[6] R. Horvat, J. Trampetic, JHEP 1101 (2011) 112, arXiv:1009.2933 [hep-ph].
[7] M.R. Douglas, N.A. Nekrasov, Rev. Mod. Phys. 73 (2001) 977, arXiv:hep-th/0106048.
[8] R.J. Szabo, Phys. Rep. 378 (2003) 207, arXiv:hep-th/0109162.
[9] T. Filk, Phys. Lett. B 376 (1996) 53.
[10] N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa, Nucl. Phys. B 573 (2000) 573, arXiv:hep-th/9910004.
[11] R. Horvat, A. Ilakovac, J. Trampetic, J. You, JHEP 1112 (2011) 081, arXiv:1109.2485 [hep-th].
[12] R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic, J. You, Neutrino propagation in noncommutative spacetimes, arXiv:1111.4951 [hep-th].
[13] A. Matusis, L. Susskind, N. Toumbas, JHEP 0012 (2000) 002, arXiv:hep-th/0002075.
[14] D. Bigatti, L. Susskind, Phys. Rev. D 62 (2000) 066004, arXiv:hep-th/9908056.
[15] M.M. Sheikh-Jabbari, Phys. Lett. B 455 (1999) 129, arXiv:hep-th/9901080.
[16] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B 216 (1989) 41.
[17] E.W. Kolb, M.S. Turner, Front. Phys. 69 (1990) 1.
[18] A. Mazumdar, J. Rocher, Phys. Rep. 497 (2011) 85, arXiv:1001.0993 [hep-ph].
[19] L. Kofman, A.D. Linde, A.A. Starobinsky, Phys. Rev. Lett. 73 (1994) 3195, arXiv:hep-th/9405187.
[20] L. Kofman, A.D. Linde, A.A. Starobinsky, Phys. Rev. D 56 (1997) 3258, arXiv:hep-ph/9704452.
[21] R.H. Cyburt, J.R. Ellis, B.D. Fields, K.A. Olive, Phys. Rev. D 67 (2003) 103521, arXiv:astro-ph/0211258.
[22] E.W. Kolb, A. Notari, A. Riotto, Phys. Rev. D 68 (2003) 123505, arXiv:hep-ph/0307241.
[23] R. Horvat, D. Kekez, J. Trampetić, Phys. Rev. D 83 (2011) 065013, arXiv:1005.3209 [hep-ph].
[24] J.D. Bekenstein, Phys. Rev. D 23 (1981) 287.
[25] R. Banerjee, B.R. Majhi, S. Samanta, Phys. Rev. D 77 (2008) 124035, arXiv:0801.3583 [hep-th].
[26] P. Nicolini, A. Smailagic, E. Spallucci, Phys. Lett. B 632 (2006) 547, arXiv:gr-qc/0510112.
[27] G.D. Barbosa, Phys. Rev. D 71 (2005) 063511, arXiv:hep-th/0408071.
[28] G.D. Barbosa, N. Pinto-Neto, Phys. Rev. D 70 (2004) 103512, arXiv:hep-th/0407111.
[29] B. Vakili, P. Pedram, S. Jalalzadeh, Phys. Lett. B 687 (2010) 119, arXiv:1003.1194 [gr-qc].
[30] O. Obregon, I. Quiros, Phys. Rev. D 84 (2011) 044005, arXiv:1011.3896 [gr-qc].
[31] C. Neves, G.A. Monerat, E.V.C. Silva, L.G.F. Filho, Can noncommutativity affect the whole history of the universe?, arXiv:1109.3514 [gr-qc].
[32] S. Alexander, R. Brandenberger, J. Magueijo, Phys. Rev. D 67 (2003) 081301, arXiv:hep-th/0108190.
[33] A. Vilenkin, L.H. Ford, Phys. Rev. D 26 (1982) 1231.
[34] B.A. Powell, W.H. Kinney, Phys. Rev. D 76 (2007) 063512, arXiv:astro-ph/0612006.
[35] I.-C. Wang, K.-W. Ng, Phys. Rev. D 77 (2008) 083501, arXiv:0704.2095 [astro-ph].
[36] S.D.H. Hsu, Phys. Lett. B 594 (2004) 13, arXiv:hep-th/0403052.
[37] M. Li, Phys. Lett. B 603 (2004) 1, arXiv:hep-th/0403127.
[38] R. Horvat, Phys. Rev. D 70 (2004) 087301, arXiv:astro-ph/0404204.
[39] B. Wang, Y.-g. Gong, E. Abdalla, Phys. Lett. B 624 (2005) 141, arXiv:hep-th/0506069.
[40] M. Rinaldi, Class. Quant. Grav. 28 (2011) 105022, arXiv:0908.1949 [gr-qc].