Consolidity: Moving opposite to built-as-usual systems practices

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Abstract With the recent uncovering of the mystery of consolidity as an inner property of systems, it is demonstrated that this notion is an indispensable pillar of systems modeling, analysis, design and building. Based on the opposite mathematical relation between consolidity versus stability and controllability, a new conceptual life cycle (change pathway) graph for natural and man-made built-as-usual systems is presented and thoroughly discussed. For the conceptual cycle development progress, it is logically conceived that system behavior changes rate has not accidentally happened, but is relatively influenced at the point of progress by the associated direct system consolidity index corresponding to the acting on-the-spot varying environments or effects. Such conceptual graph represents a real research advancement indicating that we have to move opposite to current systems building practices for solving many real life enigmatic problems. It is illustrated using stabilization of inverted pendulum problem that it is amenable by cleverly manipulating systems structure and parameters to attain new designed systems with aggregates of superiority of consolidity, stability and controllability principle. It is recommended that we have to seek new generation of innovative non-conventional systems structures moving opposite to conventional built-as-usual system practices that can enable providing directly such three aggregates of superiority requirements as their built-in self property. This will open the door towards solving many real life challenging dilemmas in various sciences and disciplines, such as engineering, space sciences, medicine, pharmacology, biology, ecology, life sciences, economy, operations research, humanities and social sciences that are believed to be attributed due to their systems inferior consolidity.

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1. Introduction

The mystery of consolidity was uncovered in a recent paper as an inner hidden property of natural and man-made physical systems [1]. Under this notion, the systems are classified into consolidated, quasi-consolidated, neutrally consolidated, unconsolidated, quasi-unconsolidated and mixed types based on their output reaction to combined input and parameters action.
Consolidity (a new noun introduced to the literature that means “the act or quality of consolidation”) also uncovered the secrecy why strong stable and highly controllable systems are not invulnerable of falling and collapsing.

Consolidity theory provides an effective tool towards scrutinizing the inner behavior of systems [2,3]. Such property is an essential pre-requisite for proper operation if systems working in a varying environment. Moreover, consolidation is an inherent built-in property that can be defined within a fully fuzzy environment.1 For systems operating in a fully fuzzy environment, the implementation of the theory to some representative case studies was successfully presented. For real life systems, consolidation property is accompanied in the system through the natural physical laws governing their existence. For man-made systems, this property still till now is overlooked by researchers and developers, and is kept just as a by-product of the designed system.

Built-as-usual systems2 are common expressions indicating that the systems are made based on the normal course of activities and to the best of the systems builders’ knowledge. Nevertheless, as our knowledge is still insufficient, some parameters could be assigned unknowingly in an unsuitable manner during the building cycle. This could lead to a huge number of built-as-usual systems that possibly fall into the trap of undesired inferior unconsolidity zones jeopardizing the future smooth operation of such systems.

Four golden rules were highlighted for handing system consolidation of both natural and man-made systems [1]. The first rule is to refrain at all circumstances from any arbitrary assignment of system values as this may drag the system into a possible undesired unconsolidity state. The second rule is to select such arbitrary assigned values in an exhaustive way that allows the most appropriate consolidation while completely fulfilling its specified system functionality. The third rule is to interfere when possible into existing systems to change parameters values and environment to shift the consolidation to the most desired consolidation states. The fourth rule is based on entirely avoiding the use of empirical, regression, artificial or imaginary models for system consolidation decisions if these models coefficients do not correspond as one to one to the parameters of the original physical system. It was strongly recommended that the four golden rules of system consolidation should be enforced as universal strict regulations of systems modeling, analysis, design and building for different disciplines [1]. Such golden rules are fully abided in the investigations all over the paper.

It was demonstrated when analyzing systems parameters of case studies that the system consolidation changes are contrary (opposite in sign) to changes of both system stability and system controllability3 [1]. Such results were confirmed by many ad hoc examples of considered classes of linear and linearized real life state space-based systems of various orders and complexities. This is a very significant result emphasizing that our present built-as-usual systems practices of insisting on building our systems with strong stability and high controllability features could have given rise to the appearance of an ample family of systems with completely very poor consolidation. Therefore, the system developer has to perform now an additional task to attain a certain balancing point to compromise between the best functionality (such as stability, controllability, and performance) versus the most appropriate consolidation.

It was indicated also in the previous study that consolidation could provide necessary profound foundations that could give guidance towards solving many real life enigmatic problems. All these left unsolved problems are clear manifestation of stable normal systems that are deviated due to their excessive unconsolidity to other abnormal states [1,2]. Examples of these challenging enigmatic problems are the uncovering of the transfer of normal living cells into cancerous tissues, the transformation of dangerous infectious diseases and viruses, the mechanism of occurrences of human immune deficiency (HIV/AIDS), the dissolution of previously well-formed political or business organizations, the sudden collapse or falling of moving space aerospace vehicles, the blackout of running electricity grids, etc.

In the following section, the basics of consolidation and its different classifications are firstly summarized, then some selected crucial case studies will be presented for carrying out necessary consolidation analysis forced to be moving opposite to built-as-usual systems practices.

2. Materials and methods

2.1. Basic definition of system consolidation

Systems can be classified according to consolidation into three categories as follows,4 see Fig. 1 [1–3]:

(i) Consolidated Systems or well connected, under hold, under grasp, well linked, robust or well joined systems,
(ii) Neutrally Consolidated Systems, and
(iii) Unconsolidated Systems or weakly connected, separated, non-robust or isolated systems.

1 The term “Fully Fuzzy Environment” indicates that all systems input parameters and coefficients are fuzzy (have hazy or varying nature around their operating points) [4–10].

2 A general system is said to be “Built-as-Usual” if its model or design was built based on conventional, heuristic, empirical, statistical, artificial, optimal or any other standard design tools not taken into consideration its system consolidation behavior. This definition applies to the majority of systems developed before the appearance of the System Consolidity Theory [3].

3 The controllability measures the degree that the system can be controlled, such that a control exists that will transfer the system from any initial state $x(0)$ to some final state $x(t)$ in a finite time interval. For a linear, time invariant plant, we have $\dot{x} = Ax + Bu$ such as $x \in \mathbb{R}^{n \times 1}$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $u \in \mathbb{R}^{m \times 1}$. A sufficient condition for the complete state controllability is that the matrix $M \in \mathbb{R}^{m \times n}$, defined as $M = [B; AB; \ldots; A^{m-1}B]$ is of rank $n$, or equivalently $|M| > 0$. On the other hand, there is the related notion of observability which is satisfied if and only if the value of the initial state can be determined from the system output. In fact, the two concepts of controllability and observability are very similar, and there is usually a concrete dual principle relationship between the two notions.

4 Consolidity could be regarded as a general internal property of natural and man-made physical systems that can also be defined far from fuzzy logic. Other consolidation indices, however, could be defined by researchers but the concept still remains the same.
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2.2. The system consolidity index

The System Consolidity Index is now presented in this section as given by [1–3]. This index measures the system overall output fuzziness behavior versus the combined input and system parameters variations. It describes the degree of how the systems react against input and system variation actions. Let us assume a general system operating in a fully fuzzy environment, having the following elements:

**Input Parameters:**

\[ I_i = (V_{I_i}, \xi_{I_i}) \]  

such that \( V_{I_i}, i = 1, 2, \ldots, m \) describe the deterministic values of input component \( I_i \), and \( \xi_{I_i} \) indicates its corresponding fuzzy level.

**System Parameters:**

\[ S_j = (V_{S_j}, \xi_{S_j}) \]  

such that \( V_{S_j}, j = 1, 2, \ldots, n \) denote the deterministic values of system parameter \( S_j \), and \( \xi_{S_j} \) denotes its corresponding fuzzy level.

**Output Parameters:**

\[ O_i = (V_{O_i}, \xi_{O_i}) \]  

such that \( V_{O_i}, i = 1, 2, \ldots, k \) designate the deterministic values of output component \( O_i \), and \( \xi_{O_i} \) designates its corresponding fuzzy level.

We will apply in this investigation, the overall fuzzy levels notion, first for the combined input and system parameters, and second for output parameters. As the relation between combined input and system with output is close to (or of the like type of) the multiplicative relations, the multiplication fuzziness property is applied for combining the fuzziness of input and system parameters.

For the combined input and system parameters, we have for the weighted fuzzy level the combined Input and System Fuzziness Factor \( F_{I+S} \), given as:

\[ F_{I+S} = \frac{\sum_{i=1}^{m} V_{I_i} \cdot \xi_{I_i}}{\sum_{i=1}^{n} V_{S_j} \cdot \xi_{S_j}} \]

Similarly, for the Output Fuzziness Factor \( F_o \), we have

\[ F_o = \frac{\sum_{i=1}^{k} V_{O_i} \cdot \xi_{O_i}}{\sum_{i=1}^{m} V_{O_i}} \]

Let the positive ratio \( |F_o/F_{I+S}| \) defines the System Consolidity Index, to be denoted as \( F_{O/(I+S)} \). Based on \( F_{O/(I+S)} \) the system consolidity state can then be classified as [1–3]:

(i) **Consolidated** if \( F_{O/(I+S)} < 1 \), to be referred to as “Class C”.

(ii) Neutrally Consolidated if \( F_{O/(I+S)} = 1 \), to be denoted by “Class N”.

(iii) **Unconsolidated** if \( F_{O/(I+S)} > 1 \), to be referred to as “Class U”.

For cases where the system consolidity indices lie jointly at both consolidated and unconsolidated parts, the system consolidity is designated as a mixed class or “Class M”.

It must be pointed out that the same concept of consolidity index can be also applied in a linguistic rather than numeric type for descriptive systems that are not expressible in mathematical forms.

2.3. Different classifications of system consolidity

Various classifications of consolidity classes are defined based on corresponding consolidity zones as shown in Fig. 2. In the figure, each point designates the associated direct value of consolidity index corresponding to certain acting on-the-spot varying environments or effects. In the classifications, two other minor classes are added within the Mixed Class. These are the Quasi-Consolidated and Quasi-Unconsolidated zones. This makes the total six consolidity classes.

It can also appear from Fig. 2 that every system can have a certain consolidity pattern for each operation point. The search for such consolidity pattern is essential for understanding with some depth the inner behavior of the system. The thorough examining of the possible trends of such consolidity patterns will require testing an ample number of applications under different situations in an open fully fuzzy environment and to classify their results into various appropriate groups. We can even think further to try to design our future systems in a manner fulfilling certain pre-specified consolidity patterns.

We need therefore to develop soft computing-based algorithms for determining the external boundary of the

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Footnote: 1 An “open fully fuzzy environment” is defined as that all fuzzy levels can freely change all over the positive and negative values of the environment. A subclass of this environment is the bounded fuzzy environment where all fuzzy levels can only change within restricted positive and negative ranges of the environment.
consolidity signature, and its center of gravity. This step can be carried out using two different approaches. The first is to examine an exhaustive number of trials scanning a mesh of all possible input and system fuzziness, then trace the external boundary or envelope of the results. The second approach is to build intelligent searching algorithm that attempts to allocate and follow up the external boundary of the consolidity signature (zone). For some situations, it is possible for standard mathematical functions and statistical formula to develop compact forms of their corresponding consolidity indices $|F_O/F_{(I+S)}|$.

2.4. Stairwise ranking of various consolidated systems

Systems in real life vary according to their consolidity based on their score of Consolidity Index $F_O/(I+S)$. For most applications, several systems can be built with a wide variety of this index. These systems could relatively be ranked based on their overall consolidity indices in a stairwise form as follows [1]:

(i) **Superior consolidated system** offering the lowest index score $F_O/(I+S) << 1$.
(ii) **Neutrally consolidated system** with index $F_O/(I+S) \approx 1$.
(iii) **Inferior consolidated system** having the highest score of index $F_O/(I+S) >> 1$.
(iv) **Natural or Built-as-usual systems** that could assume consolidity values between the superior and inferior consolidated extremes.

In real life, it is the main intention to exert our efforts to move the natural or built-as-usual systems based on their desired consolidity as appropriate to one of the two extremes of the superior or inferior consolidated points.

3. Why should we need to move opposite to built-as-usual systems practices?

The important question now arises is that why when considering consolidity we should have to move opposite to current built-as-usual systems practices? Some brainstorming explanations for this important question are given hereafter.

Built-as-usual current methods ignore consolidity as an indispensable pillar for system design and can only provide consolidity as a final by-product at the end after concluding system design and building cycles. Moreover, current built-as-usual systems practices in many design applications depend on assuming certain weighting matrices to attain specified targeted performance. Experimental investigations have demonstrated that these arbitrary matrices are the driving guiding forces for directing the system design towards a certain degree of consolidity. Examples are the arbitrary selection of Kalman Riccati matrix in solving the linear quadratic regulator problem commonly used in the literature since year 1960 [11] as illustrated by the example shown in Appendix A, and also the arbitrary selection of Lyapunov matrix used in deriving the required Lyapunov stability conditions applied extensively in the literature since year 1892 as presented by solving the drug concentration problem in Appendix B [12,13]. These arbitrary selections of such matrices have opened the door for long decades towards the unavoidable possibility of making improper choices of built-as-usual systems designs from the consolidity point of view.

The same argument can also be extended for the appropriate selection of the arbitrary weighting matrices ($Q$ and $R$) of the objective functions of wide classes of global optimization techniques, or incorporated with most polices objective functions in various practical optimization problems.
Current built-as-usual systems practices give in addition the utmost emphasis on ongoing *time-driven* operation frameworks. They do not sufficiently incorporate in their analysis the inevitable changes of parameters during their pathway course of life as affected by varying environments and events. These continual parameters changes can be only handled through *event-driven (activity-driven)* frameworks that are *logically* influenced by the consolidity behavior of the systems. Such consolidity-based change behavior constitutes an essential part of the systems inner property during their pathways of operation. Nonetheless, such very important aspect is only now acknowledged, but its in-depth investigation is postponed to a future stage.

In all respects, system operation of *natural* and *man-made* built-as-usual systems depends mainly on the three pillars: **Consolidity, Stability, and Controllability.** For any system of good standing operation, it is essential that the system should possess excellent stand for each individual pillar. As the relation between consolidity is contrary to the two other pillars, it is rendered difficult under present built-as-usual system practices to have such overall good standing system. This gives rise towards searching for another form of non-conventional systems that could provide as their built-in self property direct supporting relations between the three different pillars. That is why, if we need consolidity to foster without affecting other properties, we have to move in an *opposite way* to these current built-as-usual systems practices. Thus, there is an urgent need to address this aspect and to find some way for developing new generation of systems that have aggregates of *superiority of consolidity, stability and controllability* at their original set-points.

### 4. Systems life cycle (change pathway) based on current built-as-usual systems practices

#### 4.1. Description of life cycle of built-as-usual systems

Current practices for developing built-as-usual systems are based on giving sole emphasis on designing the systems with *strong* stability and *high* controllability. Consolidity in this practice was but a direct by-product of the finished designed system, and thus the built-as-usual methods have a high possibility towards mostly moving in the *undesirable* direction towards the *inferior* consolidity end. Such current practice situations have been depicted for the *natural* or *man-made* produced physical systems as illustrated in the new conceptual graph of Fig. 3. In the figure, the abbreviations used are as follows: VH (very high), H (high), M (moderate), L (low) and VL (very low). In the majority of real life situations, however, each system’s change pathway follows a *zigzagging pattern* with many *downs and ups*, but the prevailing tendency will always be towards a definite final (or end) state(s).

Real life *natural and man-made* systems developed using built-as-usual practices usually undergo during their life span into similar cycles based on their status of consolidity. Original normal systems usually exist with *superior* stability and controllability. Such stand, however, will be directly accompanied by *inferior* system consolidity that makes these systems very susceptible to parameters changes under operation in real life *fully fuzzy environment*. If the fuzziness by one way or another are carefully observed and controlled, the system will remain always within its *normal* specified operation or set-point. Such condition, however, is not viable as these systems are operating as part of larger systems of the universe and must interact and be interacted by these other systems.

Due to such unconsolidity of the normal systems, the parameters of the systems tend to change very slightly far from their set-points towards more improved consolidity, leading to degrading their corresponding levels of stability and controllability. The mechanism of the conceptual cycle development is based on that the system behavior changes rate has not accidentally happened, but is logically conceived to be *relatively* influenced at the point of progress with the associated direct (on-the-spot) value of system consolidity index. They can also be caused by system parameters slight aging and deteriorations. The systems in this case are still in their good standing condition fulfilling all their functionalities in almost similar to their specified original target degrees.

With the continual changes of parameters, the systems start approaching less stable and controllable states but at the same time much improved consolidated status. It is similar to going to their far end. This leads the systems to go into abnormal status which is not amenable to possible *stability or controllability* enhancements. In the same time, the system consolidity could have reached very strong stand and the system began to be almost insusceptible to parameters changes. It looks like that the systems in this case fall in a complete sink (or trap) and could possibly be subject to failure or collapse. As the last part of the cycle is accompanied by very poor stability, the rate of system degradation within this part usually gets slower as it is associated with its corresponding consolidity status compared to the beginning of the cycle of these systems. Each cycle length varies significantly from one system to another based on their original unconsolidity level and rate of changing of the environment affecting their operation.

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6. Fig. 3 represents a new conceptual graph developed based on the opposite mathematical relation between *consolidity* versus *stability* and *controllability.* The graph provides a new research advancement that could give some profound foundations for solving many enigmas of real life.

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8. The possibility that the system tends to move opposite to the shown life cycle direction of Fig. 3 is not normal and could lead to transferring the system to a state of higher unconsolidity and much higher stability. The direct (on-the-spot) behavior changes of the reverse trend will follow then an ascending rate closely connected to the associated consolidity index at the point of progress. Two possible examples of such reverse trend were given in [1] describing the AIDS epidemic problem and the spread of infectious diseases and viruses problem. The systems in these cases will be extremely susceptible to changes, highly probable to move towards chaotic situations. Such situations lie within the other far chaotic side of unconsolidity and is left as an important aspect for future research.

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6 It must be remarked that the consolidity index \( F_{0/1-s} \) as a measure of absolute ratio of change can assume the same value for any batch of parameters fuzzy changes versus their corresponding negative value of parameters changes. This is due to the *linearity* property of the suggested fuzzy theory [4-8]. Moreover, it must be observed that the consolidity index should not be *literally* applied if the fuzziness of system output or input cannot be easily determined, and could thus be replaced by other existing indicative indirect aspects reflecting systems behavior under the varying environment.
The onus of this life cycle (change pathway) is that we have to start from the beginning to build our systems not only on the basis of excellent functionality but also with appreciable high consolidity. For natural systems we should exert our efforts to develop through some innovative tools new generation of natural and man-made systems that possess original superior consolidity. Such new systems will be able to withstand parameters changes even if they are operating in harsh fully fuzzy environment. Therefore, they can maintain their strong span much longer than their corresponding present ones. The task is not that difficult as far as its clue was uncovered.

In summary, it can be stated that the natural and man-made built-as-usual systems are self progressing far from their original set-points due to the opposite mathematical relations between consolidity pillar versus stability and controllability pillars. Such cycle progress is relatively influenced by the system consolidity situation and cannot be stopped except by fully controlling external (fuzzy) factors affecting the operation of these systems. Furthermore, the prevention of such progress following the current built-as-usual systems practices is not reachable as the systems loose gradually part of their vital properties with each progress step.

4.2. The aggregate system consolidity, stability, and controllability principle

It follows from the above discussion that the present built-as-usual systems structures can not permit building real life systems with combined superior consolidity, stability and controllability. Changes of consolidity are in fact contrary (reverse in sign) to changes of both stability and controllability. Therefore, we have to search for new innovative generation of systems that enable overall original superiority of all these three aggregated principle. Such system will be of excellent standing as it can strongly withstand better external disturbing forces and changing fuzzy parameters variations, while fulfilling completely its specified functionality. Therefore, the moving opposite to the built-as-usual systems practices by developing innovative non-conventional consolidity supported systems appears now to be of a real necessity.

In the following section, the stabilization of inverted pendulum problem is designed as a proof of concept to satisfy the aggregate consolidity, stability, and controllability principle. The analysis for the consolidity part is based on keeping the problem parameters during solution as symbols and not to be substituted (with fuzziness defined as their pairs or shadows). Conventional mathematics is then applied to the basic variables while the appropriate fuzzy algebra is implemented on their corresponding pairs or shadows (fuzziness). Parameters substitutions are made at the end step of solution leading to the calculation of the consolidity factors as specified by the problem analyst.

5. Case study of system design based on the aggregate superiority principle

In this section, the suggested consolidity approach will be implemented for the fuzzy design and stabilization of the inverted pendulum system as represented in Fig. 4. The inverted pendulum problem is a classic example of producing a stable closed-loop control system from an unstable plant. Since the system can be modeled, it is possible to design a controller using the pole placement techniques. Neglecting friction at the pivot and the wheels, the equations of motion of the inverted pendulum can be expressed as [10,14–16].
\[ \ddot{x} = F + m \cdot L \left( \frac{\dot{\theta} \cdot \sin \theta - \dot{\theta} \cdot \cos \theta}{M' + m} \right) \]  

\text{(6)}

and

\[ \ddot{\theta} = \frac{g \cdot \sin \theta + \cos \theta \cdot \left( -\frac{F - m \cdot L \cdot \dot{\theta}^2 \cdot \sin \theta}{M' + m} \right)}{L \cdot \left( \frac{4}{5} \cdot \frac{m \cdot \cos^2 \theta}{M' + m} \right)} \]  

\text{(7)}

In (6) and (7) \( m \) is the inverted pendulum mass, \( L \) denotes the half-length of the pendulum, \( M' \) indicates the mass of the trolley, \( g \) is the gravitation constant, and \( F(t) \) represents the applied force to the trolley in the \( x \)-direction. If it is assumed that \( \dot{\theta} \) is small and second-order terms \( (\dot{\theta}^2) \) can be neglected, then [14]:

\[ \ddot{x} = \frac{F - m \cdot L \cdot \dot{\theta}}{M' + m} \]  

\text{(8)}

and

\[ \ddot{\theta} = \frac{g \cdot \theta + \left( -\frac{F}{M' + m} \right)}{L \cdot \left( \frac{4}{5} \cdot \frac{m}{M' + m} \right)} \]  

\text{(9)}

If the state variables are

\[ x_1 = \theta, \quad x_2 = \dot{\theta}, \quad x_3 = x \quad \text{and} \quad x_4 = \dot{x} \]  

\text{(10)}

then the state equations become

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
a_{41} & 0 & 0 & 0 \\
a_{42} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
b_2 \\
b_4
\end{bmatrix} \cdot u
\]  

\text{(12)}

where

\[ a_{21} = \frac{3 \cdot g \cdot (M' + m)}{L \cdot [4 \cdot (M' + m) - 3 \cdot m]} \]  

\[ a_{41} = \frac{-3g \cdot m}{4 \cdot (M' + m) - 3 \cdot m} \]  

\[ b_2 = \frac{3 \cdot m}{L \cdot [4 \cdot (M' + m) - 3 \cdot m]} \]  

\[ b_4 = \left( \frac{1}{M' + m} \right) \cdot \left( 1 + \frac{3 \cdot m}{4 \cdot (M' + m) - 3 \cdot m} \right) . \]  

\text{(13)}

Input original set data of the inverted system are represented in Table 1 for various selected scenarios corresponding to four different scenarios of trolley mass \( M' \), namely 0.6, 3.0, 6.0 and 12.0. Each trolley mass correspond to specific scenario (I, II, III and IV) respectively as shown in Fig. 4.

The output equation is

\[ y = C \cdot x \]  

\text{(14)}

where \( C \) is the output matrix. For a regulator, we have for the scalar control variable

\[ u = -K \cdot x \]  

\text{(15)}
The elements of the designed output feedback gain $K$ can be obtained by selecting a set of desired closed-loop poles and applying a suitable pole assignment technique.

The Ackermann’s formula [14] is a direct evaluation method for system stabilization through the pole placement technique. It is only applicable to SISO systems and therefore is the system matrix and $K$ is the controller matrix.

Several designs of the stabilized pendulum system are selected depending on the target level of stabilized system. The first two design cases are given as follows:

**Case I**: Design for stability poles of $s = -2 \pm j2$ for the pendulum, and $s = -4 \pm j4$ for the trolley

The required closed-loop poles are $s = -2 \pm j2$ for the pendulum, and $s = -4 \pm j4$ for the trolley. This yields the closed-loop characteristic equation expressed as:

$$s^4 + 12s^3 + 72s^2 + 192s + 256 = 0.$$  \hspace{1cm} (18)

**Case II**: High stabilized system

The required closed-loop poles are $s = -3 \pm j3$ for the pendulum, and $s = -5 \pm j5$ for the trolley then the closed-loop characteristic equation can be obtained as

$$s^4 + 16s^3 + 128s^2 + 480s + 900 = 0.$$  \hspace{1cm} (19)

Applying the Arithmetic fuzzy logic-based operations to (16) and (17), we can arrange the results of the designed output fuzzy gain vector as shown in Table 1 for the two cases and four different sizes considered for the trolley mass $M'$.

The equations are solved using the Gaussian-Jordan Elimination Technique [7,8].
It follows from the above analysis that for both cases of specified system stabilized characteristic equations, the smallest trolley mass yields the superior consolidated solution. This demonstrates that it is amenable by manipulating systems structure and parameters to attain new design systems with aggregates of superior consolidity and superior stability. Nevertheless, there is still a real need to seek new innovative non-conventional systems structures that can directly enable such aggregate superiority as their built-in internal self property.

The controllability matrix $M$ of the inverted pendulum problem of (12) can be expressed as:

$$
M = \begin{bmatrix}
0 & b_2 & 0 & a_{21}b_2 \\
b_2 & 0 & a_{22}b_2 & 0 \\
0 & b_4 & 0 & a_{41}b_2 \\
b_4 & 0 & a_{42}b_2 & 0
\end{bmatrix}.
$$

For the controllability of the inverted pendulum problem given by (20), we have the corresponding values of the controllability determinant $|M|$ and its consolidity indices for different values given in Table 1. Different cases and scenarios are calculated as shown in the same table. It can be seen from the table that the best solution is also corresponding in each case to the smallest trolley scenario.

The same finding can be further confirmed by considering two additional adjacent cases of weak and very strong inverted pendulum stabilization. The concluding results of these two cases are found to be similar to the two other cases and can be summarized as follows:

(i) Case 0: Low stabilized system

The required stability poles are $s = -1 \pm j1$ for the pendulum, and $s = -2 \pm j2$ for the trolley. The best consolidity results are obtained for the smallest trolley having stabilized feedback gain $K$, calculated as $K^T = [-30.29-8.95-1.58-2.36]$, controllability determinant $|M| = 110.2125$, and overall value of consolidity index of $K$ is $F_{O(f+S)} = 1.0503$.

(ii) Case III: Very high stabilized system

The required stability poles are $s = -4 \pm j4$ for the pendulum, and $s = -6 \pm j6$ for the trolley. The best consolidity results are obtained also for the smallest trolley having stabilized feedback gain $K$, calculated as $K^T = [-506.83-145.46-227.03-94.59]$, controllability determinant $|M| = 110.2125$, and overall value of consolidity index of $K$ is $F_{O(f+S)} = 0.4071$.

The overall results of the above design process of the inverted pendulum problem with different trolley masses for attaining the aggregate superiority solution of consolidity, stability and controllability are illustrated by Fig. 5. In the figure, the legends of stability are: “L” is low, “M” denotes moderate, “H” means high, and “VH” indicates very high stabilized system. It follows also from the above design trend that with aging or deterioration of the trolley, if we have $M' = 0.6$ mass is reduced to 0.55 for the very high stabilized scenario, the corresponding inverted pendulum consolidity index will with move further towards the origin from 0.4071 to 0.4240 indicating only slight consolidity drop of the designed system.

![Figure 5](https://example.com/f5.png)

**Figure 5** Sketch showing procedure for attaining aggregate superiority design for the inverted pendulum design problem $(L = 1, m = 0.5)$. 
This completes the proof of concept of the feasibility of attaining real life design with aggregate superiority of the three systems basic pillars of consolidity, stability and controllability.

6. Additional discussions and applications

6.1. Additional discussions on implementation of consolidity theory

It must be pointed out that using the suggested fuzzy approach, it is now amenable to derive the consolidity indices in compact mathematical forms for the majority of well known basic functions, such as the trigonometric, hyperbolic, and exponential, as shown in Appendix C. Similar implementations to fuzzy matrices and to standard fuzzy probabilistic functions and expressions are also a straightforward endeavor. In fact, using the suggested fuzzy approach the derivations of compact form expressions for consolidity indices of standard basic probabilistic functions and expressions are also straightforward operations as illustrated in the same appendix.

It is important to note once more that the required fuzzy know-how for performing consolidity analysis lie within basic college mathematics and statistics and can be applied in a straightforward manner, enabling wide classes of developers and researches to use each in their own field.

In general, the developed consolidity approach in this paper can be applied for the mathematical problems by simply using spreadsheet representation with Visual Basic Applications (VBAs) programming. However, the approach is general and can be applied to other unlimited forms of representations and complicated fuzzy symbolic manipulations (and computations) using other known programming software such as MATLAB and Mathematica.

6.2. General applications areas of system consolidity theory

The applications of the consolidity theory cover almost all facets of existing sciences [1]. A brief account of these applications is provided in Table 2. In fact, we cannot think of any other real life discipline without having consolidity playing the central role including visual arts, performing arts, and athletics [17,18]. The treatment in each discipline could be carried out either in numeric or linguistic type based on the considered type of system’s representation [19].

Buckley and Jowers [20] presented in a very comprehensive way an ample number of fuzzy models that were solved by continuous simulation. The majority of these studied models together with other reported ones in the literature could represent an excellent forum for the methodological experimentation of the systems’ consolidity theory using the suggested fuzzy logic-based representation algebra [4–10]. These models cover very interesting areas of biology, ecology, medicine, pharmacology, space sciences, electrical, mechanical and chemical engineering, economy, operations research, social science, etc. In all situations, only real life methodological, field investigations and in-depth simulation analyses are encouraged for systems developed under the framework of consolidity theory. The simulation analysis can be devised based on a sequential behavior of the system where the system behavior changes rate is assigned relatively connected at the point of progress with the associated direct (on-the-spot) system consolidity index.
In general, consolidity is an internal property of systems that enables giving an in-depth look inside such systems, regardless of their field of applications. Such property will lead to giving a new forum for better understanding of various sciences. With the developed know-how for consolidity calculations, new classes of advanced systems with strong consolidity while fulfilling fully their required performance will be born and will be taken for granted as the future standard of systems in various disciplines.

7. Conclusions

A comprehensive new conceptual graph describing the life cycle (change pathway) for natural and man-made built-as-usual systems was proposed based on the opposite mathematical relation between consolidity versus stability and controllability. The mechanism of the conceptual cycle development progress was logically conceived that the system behavior changes rate has not accidently happened, but is relatively influenced at the point of progress with the associated direct system consolidity index corresponding to the acting on-the-spot varying environments or effects. This is a very significant advancement towards solving many enigmas of real life problems. Many unsolved pressing problems will definitely benefit of this new trend of aggregate superiorities. Examples of real life enigmatic problems are the prevention of human immune deficiency, transformation of dangerous diseases and viruses, transfer of normal living cells into cancerous tissues, etc. All these left unsolved enigmatic problems are clear manifestation of stable normal systems that are deviated due to their excessive unconsolidity to other abnormal states, and can fall within the scope of the presented consolidity analysis.

The discovery of the mystery of consolidity has been accompanied with the revealing of other very significant and intriguing findings. For instance, two clear case studies are highlighted demonstrating that the arbitrary matrices normally assigned in the literature during systems analysis and design are in fact the principal guiding force towards attaining designed systems with appropriate levels of consolidity. The two case studies are the arbitrary selection of Kalman Riccati matrix in solving the linear quadratic regulator problem commonly used in the literature since year 1960, and also the arbitrary selection of Lyapunov matrix used in deriving the required Lyapunov stability conditions applied extensively in the literature since year 1892. These arbitrary selections of such matrices have unfortunately opened the door for long decades towards the unavoidable possibility of making improper choices of built-as-usual systems designs from the consolidity point of view.

It is illustrated as a proof of concept using the stabilization of inverted pendulum problem that it is amenable by manipulating systems structure and parameters to attain new design systems with original set-points of aggregates of superior consolidity, superior stability and superior controllability. Through manipulating physical systems structures and parameters in a manner moving opposite to current built-as-usual systems procedures, many remained solutions of real life dilemmas attributed due to systems unconsolidity can now be gradually approached. Thus, such inverted pendulum problem can remarkably be considered as the first successful mathematical developed system reported in the literature attaining the aggregate superiority of the three systems pillars. In fact, the only way now to solve many open dilemmas is our life is to start moving opposite to our current built-as-usual system practices. In this case, we have to seek new generation of systems with original set-points of aggregate superiority of the three system basic pillars. This can only be effectively achieved through striving for innovative non-conventional systems structures that can directly enable such three aggregates requirements as their built-in self property.

A quick effort for achieving such target can firstly be attempted through cleverly manipulating the physical structures and parameters of the real life natural or man-made physical systems. A parallel process is to search within the wealthy literature of automatic control and system theory for new matching systems structures fulfilling the required self property of aggregate superiority of the three pillars. Tremendous number of versatile structures and techniques have been developed and elaborated since the publishing of the celebrated Kalman theory paper of 1960, that could provide impetus for our search for innovative consolidity supported systems structures [11]. Moreover, reverse engineering analysis and thorough investigation of nonlinear dynamical structures of chaotic systems [21,22] could also shed some light towards what should not (or should) be the form of the anticipated innovative aggregate superiority structures.

It must be strongly emphasized at this point that the current efforts to solve the above described enigmas and real life and many other challenging dilemmas in various sciences and disciplines such as engineering, space sciences, medicine, pharmacology, biology, ecology, life sciences, economy, operations research and social sciences that are attributed due to their unconsolidity through only extensive field investigation and experimentation will not give effective quick results as they are searching for something non-appreciable in their experimentation fronts. In this aspect, they should have to combine their experimentation with appropriate mathematical profound consolidity-based mathematical tools for constructing novel systems structures/parameters capable of providing their necessary level of superior consolidity accompanied simultaneously with superior stability and controllability.

Intensive real life methodological, field investigations and in-depth simulation analyses are encouraged for natural or man-made physical systems which could be either carefully adjusted or fully developed under the framework of consolidity theory. The simulation analysis can be devised based on a sequential behavior of the system where the system behavior changes rate is assigned direct (on-the-spot) relatively connected at the point of progress with the associated system consolidity index.

In conclusion we can state now after fully diagnosing the consolidity versus both stability and controllability dilemma that we should be quick in moving opposite to current built-as-usual systems practices. In this regards we should seek building innovative type of system structures with aggregate superiority. Finally, the study of the essential role of the associated direct (on-the-spot) consolidity factor in influencing the

---

*Chaos is the science of the global nature of all systems around us. It extends the system theory to include the cause and effect phenomena that lie outside of our normal limit of experience. Chaotic systems could have both stable and unstable components, and sometimes give rise to astonishing structures of large-scale orders.*
progress of system life cycle (change pathway) for natural and man-made systems is a very important issue that is left for immediate future research. Such research should be directed towards integrating the consolidation-based mathematical formulations together with experimental observations for deriving relevant consolidation-based change mechanisms for systems in different sciences and disciplines, taking into account the event-driven (activity-driven) nature of the problem.

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Appendix A. Consolidity analysis of optimal design of the fuzzy linear quadratic regulator problem

A.1. Definition of the optimal linear quadratic regulator problem

In this section, the consolidity analysis for the optimal design of the fuzzy linear quadratic regulator problem is considered as described in Fig. A.1. For the regulator control problem, it is required that the system initially displaced from equilibrium will return the system to the equilibrium state in such a manner so as to minimize a given performance index. In general, the linear quadratic regulator (LQR) provides an optimal control law for a linear system with a quadratic performance index.

For a linear, time invariant plant, we have [11]:

\[ \dot{x} = Ax + Bu \]  
(A.1)

such as \( x \in \mathbb{R}^{n_1} \), \( A \in \mathbb{R}^{n_1 \times n_1} \), \( B \in \mathbb{R}^{n_1 \times n_2} \), and \( u \in \mathbb{R}^{n_2} \).

The linear quadratic regulator will be designed based on the quadratic performance index, described as:

\[ J = \int_{t_0}^{t_1} (x^TQx + u^TRu)\,dt \]  
(A.2)

such as \((\cdot)^T\) denotes the transpose of \((\cdot)\). The two matrices \( Q \) and \( R \) are arbitrarily selected.

The optimal law for this linear quadratic regulator can be expressed as:

\[ u_{opt} = -R^{-1}B^TPx. \]  
(A.3)

Or equivalently

\[ u_{opt} = -Kx \]  
(A.4)

where

\[ K = R^{-1}B^TP \]  
(A.5)

and \( P \) indicates the Riccati Matrix, given as

\[ \dot{P} = -PA - A^TP - Q + PBR^{-1}B^TP. \]  
(A.6)

Carrying out the integration in reverse time proceeds, the solutions of \( P(t) \) converge to constant values. Should \( t_0 \) be infinite, or far removed from \( t_1 \), the Matrix Riccati equations (A.6) reduce to a set of simultaneous equations, expressed by [14]:

\[ PA + A^TP + Q - PBR^{-1}B^TP = 0. \]  
(A.7)

This equation was developed by Kalman in year 1960 and is commonly used since its development in many disciplines and applications [14].

A.2. Example of optimal fuzzy linear quadratic regulator problem

Let us consider the design of the optimal fuzzy linear quadratic regulator problem of the following second order model, representing a real original physical system, expressed as [11]:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
b_2
\end{bmatrix} u
\]  
(A.8)

\[ y = [c_1 \quad c_2]x \]

where \( a_{21}, a_{22}, \) and \( b_2 \) are fuzzy parameters. The parameters \( a_{21} \) and \( a_{22} \) are related to the physical system, while parameter \( b_2 \) is a constant related to the input. The output coefficients \( c_1 \) and \( c_2 \) are also fuzzy parameters. Eq. (A.8) can be written in the general state space form of (A.1). The controllability \( M \) of the linear quadratic regulator problem expressed by (A.8) is such that \( |M| = -b_2^2 \), which means that controllability magnitude increases with the increase of the value of parameter \( b_2 \).

Let us introduce the performance index of this problem as follows:
Table A.1  Consolidity results of selected scenarios of the optimal linear quadratic regulator problem.

<table>
<thead>
<tr>
<th>Design scenario No.</th>
<th>Selected performance index coefficients</th>
<th>Riccati matrix ( P )</th>
<th>State feedback matrix ( K )</th>
<th>Eigen-values ( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q_{11} ) ( q_{22} ) ( r )</td>
<td>( p_{11} ) ( p_{12} )</td>
<td>( p_{21} ) ( p_{22} )</td>
<td>( k_1 ) ( k_2 )</td>
</tr>
<tr>
<td>(a) Design inputs of testing scenarios</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>2 0.5 0.1</td>
<td>1.6425 0.3583</td>
<td>0.3583 0.2021</td>
<td>0.3583 0.2021</td>
</tr>
<tr>
<td>II</td>
<td>2 0.3 1</td>
<td>2.1584 0.7321</td>
<td>0.7321 0.4009</td>
<td>0.7321 0.4009</td>
</tr>
<tr>
<td>III*</td>
<td>2 1 3</td>
<td>2.5866 0.8730</td>
<td>0.8730 0.6512</td>
<td>0.8730 0.6512</td>
</tr>
<tr>
<td>IV*</td>
<td>2 1 15</td>
<td>2.6695 0.9373</td>
<td>0.9373 0.7011</td>
<td>0.9373 0.7011</td>
</tr>
<tr>
<td>V**</td>
<td>3 1 15</td>
<td>3.9049 1.4317</td>
<td>1.4317 0.9508</td>
<td>1.4317 0.9508</td>
</tr>
</tbody>
</table>

(b) Consolidity results of selected testing scenarios

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Input parameters</th>
<th>Consolidity indices of eigenvalues ( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_{21} ) ( a_{22} )</td>
<td>Scenario I</td>
</tr>
<tr>
<td>Fuzzy levels</td>
<td></td>
<td>( \alpha )</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.1528</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>0.7493</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>1.3833</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0.9566</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.9991</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>1.1656</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.9222</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.5994</td>
</tr>
<tr>
<td>Average value of index ( F_{O/( I-8) } )</td>
<td>( M )</td>
<td>( U )</td>
</tr>
<tr>
<td>Overall consolidity class</td>
<td>( 0.9885 )</td>
<td>( 1.6485 )</td>
</tr>
</tbody>
</table>

\[
J = \int_0^\infty \left[ x^T \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} x + r x^2 \right] dt \tag{A.9}
\]

such that parameters \( q_{11}, q_{22}, \) and \( r \) are scalars to be arbitrarily selected. Now it is required to design the optimal fuzzy linear regulator of this example by determining the Riccati Matrix \( P \), the state feedback matrix \( K \), and the closed-loop eigenvalues.

For this example, we have for the Riccati matrix \( P \) as described in (A.7) after some substitutions and manipulations, the following elements [11]:

\[
p_{11} = \frac{b_2^2}{r} \cdot p_{12}p_{22} - p_{22}a_{21}t - p_{11}a_{22}
\]

\[
p_{12} = p_{21} = \frac{r}{b_2^2} \cdot \left[ a_{21} \pm \sqrt{a_{21}^2 + \frac{q_{11}b_2^2}{r}} \right]
\] (A.10)

and

\[
p_{22} = \frac{r}{b_2^2} \cdot \left[ a_{22} \pm \sqrt{a_{22}^2 + \frac{2p_{12} + q_{22}}{r}} \right].
\]

The state feedback matrix \( K \) of this problem is given as:

\[
K = R^{-1}B^TP = \begin{bmatrix} 0 & b_2 \end{bmatrix} \cdot \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}. \tag{A.11}
\]

From (A.1) and (A.4), the closed-loop characteristic equation can be expressed as:

\[
|sI - A + B \cdot K| = 0. \tag{A.12}
\]

This yields the system characteristic equation for the optimal linear quadratic regulator problem, described as:

\[
x^2 + (b_2k_2 - a_{22})sx + (b_2k_1 - a_{21}) = 0. \tag{A.13}
\]

The characteristic equation (A.13) has two eigenvalues to be denoted by \( \lambda = [\lambda_1, \lambda_2]^T \). These eigenvalues will be regarded as the output of the system for calculating the overall system consolidity indices.

Five scenarios are selected for the problem including the scenario of built-as-usual, by varying the selected coefficients the Performance Index, namely \( q_{11}, q_{22}, \) and \( r \) as shown in Table A.1. Other scenarios could lead to more superior or less inferior than the ones obtained in this limited illustrative selections.

Using the selected input parameters values and their corresponding fuzzy levels variations shown in Table A.1, the results of the consolidity indices of the characteristic equation eigenvalues \( \lambda \) reflecting designed system output behavior for different scenarios are calculated as shown in the same table. In the table, the symbol “+” denotes the built-as-usual scenarios and “**” designates the inferior scenario. The selected variations of the input fuzzy level are non-exhaustive, but are given only as a demonstration of the implementation of the consolidity theory. The results of the exhaustive selection of the fuzzy levels could only alter slightly the specific values of overall results, without affecting the relative ranking or the overall consolidity conclusions of such scenarios.

It must be remarked that the above analysis will lead to the evaluation of the overall or average system consolidity index that can be invoked only as a general guide for the system inner behavior. Nonetheless, in real circumstances, the associated direct value of the consolidity index is the one to be calculated corresponding to the acting on-the-spot varying environments or effects.
It follows from Table A.1 that the four scenarios can be ranked as follows:

(i) Superior consolidated scenario which is scenario I having the best consolidity index $F_{O/(1-S)}$ score.

(ii) Inferior consolidated scenario which is scenario V with the worst consolidity index $F_{O/(1-S)}$ score.

(iii) Built-as-usual scenarios III and IV are very poor scenarios from the consolidity point of view. Such design will be highly susceptible in its operation and could cause unexpectedly many operational problems.

(iv) Scenario II is still another poor designed scenario, though it is much better than the built-as-usual scenario.

It is very interesting to note once more that both superior and inferior consolidated systems and the ones in between should not differ significantly in their implementation costs [1]. It is only due to the cleverness of the designer to select the appropriate parameters for his system.

It clearly appears again from the consolidity results of the eigenvalues (system poles) $\lambda$ in Table A.1 that the system consolidity changes are opposite in sign of changes in system stability. As the real value of the Left Half Poles (LHPS) $\lambda$ move from the negative left half side of the s-plane to the origin, the system gets less stable but has a consequent improved consolidity status.

As for consolidity relation versus controllability, it can be observed that if $b_2$ is assumed equal to 2 in Table A.1, the controllability determinant $|M| = -b_2^2 = -4$ (that is four times the original value). At the same time the corresponding consolidity indices for different scenarios become (1.0075, 1.7192, 3.0844, 5.9941, 11.0027) respectively. The corresponding values of updated consolidity indices for $b_2 = 2$ are now higher than the original indices of the case with $b_2 = 1$. If we increase $b_2$ further to 7, this means that controllability is increased 49 times the original, the corresponding consolidity indices analogously increase more to (1.02668, 1.8324, 3.3194, 6.4696, 12.2059) respectively. This confirms again the opposite relation between consolidity and controllability, and the unavoidable possibility of drifting the optimal solution into the unconsolidity due to the arbitrary selection of the LQR weighting matrices.

It can be observed that the overall or average consolidity index of the most inferior scenario V is exceeding the value of 10, which can be regarded as a high value of unconsolidity. Results of other solved ad hoc applications in life sciences and medicine indicated the possibility that some systems could possess even much higher overall consolidity indices roughly in the range from 20 to 40+. Their associated direct (on-the-spot) consolidity indices could even be much higher than the shown overall consolidity range. Such systems are thus highly susceptible to substantial parameters changes once subjected to any appreciable varying environments or events; resulting in making these systems highly vulnerable to unavoidable falling down or collapsing.

It must be pointed out that one of the highly intriguing (missing gaps) problems in optimal control theory regarding the best selection of the performance index weighting matrices ($Q$ and $R$) is now uncovered after many decades upon solving the consolidity problem of the above numerical example. These weighting matrices can guide the optimal solution from their extreme inferior to extreme superior consolidated designs. In the current built-as-usual practices, these matrices are arbitrarily selected by intuition without any mathematical justification. Such selection could unfortunately lead the solution towards the inferior side as shown in this example.

The results of this typical linear quadratic regulator problem design suggests that there is an urgent necessity now to start revising all our important systems designed based on the optimal control theory and its related subjects from the new view point of their system consolidity. These “built-as-usual” systems around us at the present time could have a high possibility to fall within the inferior consolidated zones. Definitely, the future could bring many astonishing results in this regards when implementing the consolidity theory to many other existing built-as-usual systems in various disciplines and sciences. Therefore, the moving opposite to the built-as-usual systems practices by developing innovative non-conventional consolidity supported methods appears now to be of a real necessity.

Appendix B. Consolidity analysis of fuzzy Lyapunov stability of drug concentration model

B.1. Consolidity of the fuzzy Lyapunov stability condition

The same system consolidation concept applied to the above fuzzy linear quadratic regulator problem can now be extended for the consolidity testing of the second method of Lyapunov stability condition. This will help in the appropriate selection of the Lyapunov function used for determining the stability condition. This function is energy-like functions based on a quadratic formula, and is not easy to find as they are not unique. For a state $x$ the typical form of this function is $V(x) = x^TPx$, where $P \in \mathbb{R}^{nxn}$ a symmetric positive definite matrix is ($P = P^T$). Taking the derivative of $V(x)$, we can get

$$\dot{V} = \frac{\partial V}{\partial x} \frac{dx}{dt} = x^T \cdot (A^TP + PA)x = -x^T \cdot Q \cdot x \quad (B.1)$$

which must be negative definite (for asymptotic stability).

This reduces the requirement that the matrix $Q$ should be selected arbitrarily to be positive definite. Thus, to find a Lyapunov function for the linear system it is sufficient to choose $Q > 0$, and solve the following linear equation for $P$ using linear algebra (developed originally by Lyapunov in year

![Figure B.1](image_url)

Figure B.1 A diagram showing operation of the drug concentration problem.
\[ \begin{align*}
\dot{x}_1 &= -b_{12} x_1 + b_{21} x_2 + u, \\
\dot{x}_2 &= b_{12} x_1 - (b_{22} + b_2) \cdot x_2 - u
\end{align*} \quad (B.4)\]

such that \( x(t) \) is the amount of the drug in compartment \( i = 1, 2 \) respectively at time \( t \). The system has two compartments of different values separated by a membrane. The drug can flow through such membrane from compartment 1 to 2, and vice versa. The variable \( u \) indicates the input parameter of the system. The problem can be arranged in the state space form, as follows:

\[
\begin{align*}
\dot{x}_1 &= -b_{12} x_1 + b_{21} x_2 + u, \\
\dot{x}_2 &= b_{12} x_1 - (b_{22} + b_2) \cdot x_2 - u
\end{align*} \quad (B.5)
\]

which can also be written in the general form of the state space equation.

The stability of this system using the fuzzy Lyapunov stability criterion will now be developed [12,13]. It is required by solving (B.2) to find an arbitrary symmetric positive definition. \( Q = \{q_{ij}\} \) that leads to calculating the Lyapunov matrix \( P = \{p_{ij}\} \). Upon substitution of (B.5) into (B.2), and simplifying, we can obtain the following relation:

\[
\begin{align*}
\begin{pmatrix}
-2b_{12} & b_{21} & b_{12} & 0 \\
b_{21} & -b_{22} - b_2 & -b_{21} & b_{12} \\
0 & -b_{21} - b_2 & -2b_{21} - 2b_2 & b_{21} \\
0 & -b_{22} & -b_{21} & -2b_{22}
\end{pmatrix}
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{p}_{11} \\
\dot{p}_{12}
\end{pmatrix}
&= 
\begin{pmatrix}
q_{11} \\
q_{12} \\
q_{21} \\
q_{22}
\end{pmatrix},
\end{align*}
\]

\quad (B.6)

Assuming arbitrary values of \( q_{ij} \) the elements of matrix \( Q \), the corresponding values of the Lyapunov matrix \( P \) and its determinant \( |P| \) reflecting designed system output behavior can be calculated for various fuzzy scenarios using direct fuzzy matrix operations. Five scenarios were considered in Table B.1, and their corresponding consolidity results of the matrix \( P \) are also shown in Table B.1. In the table, the symbol “∗” denotes the superior consolidated scenario, the symbol “∗∗” designates the built-as-usual scenarios, and the symbol “∗∗∗” indicates the inferior consolidated scenarios. The results indicated that the consolidity of the built-as-usual scenarios

Table B.1 Consistency results of selected scenarios of Lyapunov stability matrix of drug concentration problem.

<table>
<thead>
<tr>
<th>Scenario No.</th>
<th>Description</th>
<th>Selected matrix ( Q )</th>
<th>Calculated Lyapunov matrix ( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Superior consolidated system</td>
<td>(-2) (-1.95)</td>
<td>(1.95) (-2) (17.4917) (14.1583) (14.1583) (11.5417)</td>
</tr>
<tr>
<td>2</td>
<td>Marginal consolidated system</td>
<td>(-3) (-2.9)</td>
<td>(-2.9) (-3) (26.1500) (21.1500) (21.1500) (17.2500)</td>
</tr>
<tr>
<td>3</td>
<td>Built-as-usual system I</td>
<td>(-1) 0</td>
<td>0 (-2) (6.0833) (4.4167) (4.4167) (4.5833)</td>
</tr>
<tr>
<td>4</td>
<td>Built-as-usual system II</td>
<td>(-2) 0</td>
<td>0 (-1) (9.9167) (6.5833) (6.5833) (5.4167)</td>
</tr>
<tr>
<td>5</td>
<td>Inferior consolidated system</td>
<td>(-10) (-3)</td>
<td>(-3) (-1) (57.0833) (40.4167) (40.4167) (29.5833)</td>
</tr>
</tbody>
</table>

Aspect Input parameters Consistency indices of \( |P| \)

<table>
<thead>
<tr>
<th>b_{12}</th>
<th>b_{21}</th>
<th>b_{2}</th>
<th>Scenario I</th>
<th>Scenario II</th>
<th>Scenario III</th>
<th>Scenario IV</th>
<th>Scenario V***</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.2</td>
<td>1.4246</td>
<td>3.7650</td>
<td>8.3750</td>
<td>10.3750</td>
<td>55.2083</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>(-2)</td>
<td>0.9485</td>
<td>0.1391</td>
<td>3.8498</td>
<td>4.1977</td>
<td>8.2749</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>0.8108</td>
<td>1.7750</td>
<td>6.5843</td>
<td>7.4059</td>
<td>12.5059</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>6</td>
<td>1.0847</td>
<td>0.5341</td>
<td>2.1673</td>
<td>2.0891</td>
<td>4.9615</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
<td>0.6449</td>
<td>1.5760</td>
<td>6.1758</td>
<td>6.9642</td>
<td>11.8634</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
<td>1.1831</td>
<td>2.3051</td>
<td>7.8609</td>
<td>9.0276</td>
<td>15.0513</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>0.1026</td>
<td>0.6681</td>
<td>4.4699</td>
<td>4.8890</td>
<td>8.8176</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
<td>1.1275</td>
<td>2.1548</td>
<td>7.2393</td>
<td>8.2490</td>
<td>13.7324</td>
</tr>
<tr>
<td>(-5)</td>
<td>(-5)</td>
<td>(-7)</td>
<td>0.2175</td>
<td>1.0254</td>
<td>5.0146</td>
<td>5.5393</td>
<td>9.7058</td>
</tr>
<tr>
<td>(-2)</td>
<td>(-3)</td>
<td>(-1)</td>
<td>0.4167</td>
<td>1.3694</td>
<td>6.0787</td>
<td>6.8619</td>
<td>11.8638</td>
</tr>
</tbody>
</table>

Average value of index \( F_{Q/(I-8)} \)

Overall consolidity class

\( C \) \( U \) \( U \) \( U \) \( U \)
are highly unconsolidated as the arbitrary selection of the elements of matrix of $Q$ was previously made without any overall rational justification. For this example, it is possible to find other selections such as of scenarios I and II that could yield corresponding Lyapunov matrices with superior or marginal consolidated indices.

It is clear again from Table B.1 that the system consolidity changes move contrary to corresponding changes of the system controllability as measured by $|P|$. Upon changing system parameters and as the magnitude of $|P|$ decreases, the system consolidity index $F_{Q(M+S)}$ decreases indicating higher system consolidity.

The contrary stability and controllability versus consolidility relationship of the present drug concentration problem for different value of parameters $b_2$ was previously scrutinized in [1]. In such analysis, it was demonstrated that as $b_2$ increase, stability becomes stronger while consolidility gets weaker. On the other hand, for system controllability, it can be seen from (B.5), that the controllability determinant reduces to $|M| = b_2$. Thus, for any increase in $b_2$, controllability becomes higher while consolidility become smaller, confirming once more such opposite relations between these two pillars of the typical built-as-usual systems.

The lesson learned from the implementation of consolidility theory in the procedure of the stability analysis of the drug concentration model is highly significant. The case study demonstrated how to use the consolidility theory for the determination of the arbitrary assignment of the matrix $Q$ leading to the calculation of the essential Lyapunov matrix $P$. This represents a new contribution to solving a long left intriguing problem in Lyapunov stability theory left unsolved for long decades of the arbitrary selection of such matrix. Such concept can be extended to other many methods of system analysis and design where some weighting or auxiliary coefficients are arbitrarily assigned at one step or another during solution.

Appendix C. Derived consolidility indices of standard mathematical and statistical functions

C.1. Consolidity indices of standard fuzzy mathematical functions

The consolidity analysis is developed using the Arithmetic Fuzzy Logic-based Representation introduced in [4–9]. This representation is based on expressing each parameter $X$ by two components: $X_o$, the deterministic equivalence, and $X_f$

Table C.1 Analogy between algebra of conventional fuzzy and arithmetic fuzzy logic-based representation Approaches [9].

<table>
<thead>
<tr>
<th>Algebraic operation</th>
<th>Conventional fuzzy theory*</th>
<th>Arithmetic fuzzy logic-based representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = A + B$</td>
<td>$A + B = [a_1^{(s)} + b_1^{(s)}, a_2^{(s)} + b_2^{(s)}]$ or equivalently $\mu_C(z) \equiv \bigvee_{x,y \in A} [\mu_A(x) \wedge \mu_B(y)]$</td>
<td>$C_o = A_o + B_o$ and $\ell_x = \frac{\ell_{A_o} \cdot A_o + \ell_{B_o} \cdot B_o}{A_o + B_o}$</td>
</tr>
<tr>
<td>$C = A - B$</td>
<td>$A - B = [a_1^{(s)} - b_1^{(s)}, a_2^{(s)} - b_2^{(s)}]$ or equivalently $\mu_{A\cdot B}(z) \equiv \bigvee_{x,y \in A} [\mu_A(x) \wedge \mu_B(y)]$</td>
<td>$C_o = A_o - B_o$ and $\ell_x = \frac{\ell_{A_o} \cdot A_o - \ell_{B_o} \cdot B_o}{A_o - B_o}$</td>
</tr>
<tr>
<td>$C = A \cdot B$</td>
<td>$A \cdot B = [a_1^{(s)} \cdot b_1^{(s)}, a_2^{(s)} \cdot b_2^{(s)}]$ or equivalently $\mu_{A\cdot B}(z) \equiv \bigvee_{x,y \in A} [\mu_A(x) \wedge \mu_B(y)]$</td>
<td>$C_o = A_o \cdot B_o$ and $\ell_x = \ell_A \cdot \ell_B$</td>
</tr>
<tr>
<td>$C = A / B$</td>
<td>$A / B = [a_1^{(s)} / b_1^{(s)}, a_2^{(s)} / b_2^{(s)}]$ or equivalently $\mu_{A\cdot B}(z) \equiv \bigvee_{x,y \in A} [\mu_A(x) \wedge \mu_B(y)]$</td>
<td>$C_o = A_o / B_o$ and $\ell_x = \ell_A - \ell_B$</td>
</tr>
</tbody>
</table>

* $\lor$ (o) Designates maximum of (o), while $\land$ (o) indicates minimum of (o).

Table C.2 Examples of derived consolidility indices for standard fuzzy mathematical functions.

<table>
<thead>
<tr>
<th>Original function</th>
<th>Taylor’s series expansion</th>
<th>Calculated compact form of fuzzy level</th>
<th>Consolidity index (Compact form)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \sin x$</td>
<td>$y = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot \cos x_0 / \sin x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = \cos x$</td>
<td>$y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$</td>
<td>$\ell_y = -\ell_x \cdot x_0 \cdot \sin x_0 / \cos x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = \sinh x$</td>
<td>$y = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot \cosh x_0 / \sinh x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = \cosh x$</td>
<td>$y = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot \sinh x_0 / \cosh x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = e^x$</td>
<td>$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = e^x \sin x$</td>
<td>$y = x + x^3 + \frac{x^5}{3!} - \frac{x^7}{7!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot (1 + \cos x_0 / \sin x_0)$</td>
<td>$</td>
</tr>
<tr>
<td>$y = e^x \cos x$</td>
<td>$y = 1 - x + \frac{x^3}{3!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot (1 + \sin x_0 / \cos x_0)$</td>
<td>$</td>
</tr>
<tr>
<td>$y = e^{\text{trix}}$</td>
<td>$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot (1 - \tan^2 x_0)$</td>
<td>$</td>
</tr>
<tr>
<td>$y = \ln x$</td>
<td>$y = 2 - x + \frac{2x - 1}{3 (x + 1)^3} + \frac{2 (x - 1)^5}{5 (x + 1)} + \cdots$</td>
<td>$\ell_y = \ell_x / \ln x_0$</td>
<td>$</td>
</tr>
<tr>
<td>$y = e^{e^{a x}}$</td>
<td>$y = 1 + x \cdot \ln a + \frac{(x \cdot \ln a)^3}{2!} + \frac{(x \cdot \ln a)^5}{3!} + \cdots$</td>
<td>$\ell_y = \ell_x \cdot x_0 \cdot \ln a$</td>
<td>$</td>
</tr>
</tbody>
</table>
the fuzzy equivalence representing a small uncertainty or value

**Table C.3** Examples of some standard fuzzy probability density functions analysis.

<table>
<thead>
<tr>
<th>Name</th>
<th>Fuzzy probability density function</th>
<th>Fuzzy mean</th>
<th>Fuzzy variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>( P_x = \frac{1}{b - a} )</td>
<td>( \mu = \frac{(b_0 + a_0)}{2} )</td>
<td>( \nu = \frac{(b_0 - a_0)^2}{12} )</td>
</tr>
<tr>
<td>Normal</td>
<td>( P_x = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2} )</td>
<td>( \mu = \frac{a + b}{2} )</td>
<td>( \nu = \frac{(b - a)^2}{12} )</td>
</tr>
<tr>
<td>Exponential</td>
<td>( P_x = \lambda e^{-x} )</td>
<td>( \mu = \lambda )</td>
<td>( \nu = \lambda^2 )</td>
</tr>
<tr>
<td>Lognormal</td>
<td>( P_x = \frac{1}{x \sigma \sqrt{2\pi}} \exp \left( -\frac{(\ln(x) - \theta)^2}{2\sigma^2} \right) )</td>
<td>( \mu = e^{\mu_0 + \sigma_0^2/2} )</td>
<td>( \nu = e^{2\mu_0 + \sigma_0^2} )</td>
</tr>
<tr>
<td>Gamma</td>
<td>( P_x = \frac{\lambda x^{r-1} e^{-x}}{\Gamma(r)} )</td>
<td>( \mu = \frac{\lambda}{\Gamma(r)} )</td>
<td>( \nu = \frac{\lambda^2}{\Gamma(r)^2} )</td>
</tr>
<tr>
<td>Beta</td>
<td>( P_x = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{(x - \theta)^2}{2\sigma^2} \right) )</td>
<td>( \mu = x_0 )</td>
<td>( \nu = \frac{\lambda}{\Gamma(r)} )</td>
</tr>
<tr>
<td>Geometric</td>
<td>( P_x = \frac{1}{r - 1} \left( \frac{x}{r} \right)^{r-1} \left( \frac{r - 1}{x} \right)^{x-1} )</td>
<td>( \mu = \frac{1}{r} )</td>
<td>( \nu = \frac{1}{r^2} )</td>
</tr>
<tr>
<td>Erlang</td>
<td>( P_x = \frac{1}{\Gamma(r - 1)} \exp \left( -\frac{r}{x} \right) )</td>
<td>( \mu = \frac{r}{\Gamma(r)} )</td>
<td>( \nu = \frac{r^2}{\Gamma(r)^2} )</td>
</tr>
<tr>
<td>Poisson (Discrete Type)</td>
<td>( f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!} )</td>
<td>( \mu = \lambda )</td>
<td>( \nu = \lambda^2 )</td>
</tr>
<tr>
<td>Binomial (Discrete Type)</td>
<td>( f(x; n, p) = \binom{n}{x} p^x (1 - p)^{n-x} )</td>
<td>( \mu = np )</td>
<td>( \nu = np(1 - p) )</td>
</tr>
</tbody>
</table>

It was shown that the suggested approach under the main assumption that \( \lambda_1 < 1 \) is identical to that of the conventional fuzzy theory for addition, gives average fuzziness interval results for the subtraction operations, and yields similar results of multiplications and divisions operations after ignoring the second order relative variations terms [9]. Moreover, the Arithmetic Fuzzy Logic-based Representation approach possesses additional key features over the Conventional Fuzzy Theory, namely; linearity, reversibility, simplicity, and applicability. A comparison between the two approaches is presented in Table C.1.

In a systematic way, the concept of the arithmetic fuzzy logic-based representation was successfully applied to selected classes of linear, nonlinear, multivariate and dynamic systems [4–10]. The implementation included also many fuzzy applications such as functions of different dimensionalities and types, analytic geometry, vector analysis, functions of complex variables, formulas derivatives and integrals, partial fraction of polynomials, matrix operations and formulations, functions of matrices, etc. Such achievements have given the impetus
for forming the basic infrastructure for the development of a generalized fuzzy mathematics in this respect. This will render the consolidity theory highly extendable to very wide classes of real life systems.

Using this fuzzy concept, we can derive compact formulas for the consolidity indices of selected standard fuzzy functions such as the trigonometric, hyperbolic and exponential functions. The analysis starts by expressing each function by its equivalent of Taylor’s series expansion. In general, for the fuzzy series expressed as [23]:

\[ f(x) = \sum_{i=0}^{\infty} a_i x^i, \]  

we have used the Arithmetic Fuzzy Logic-based representation approach to obtain the following corresponding fuzzy level at original set point of \( x_0 \):

\[ \ell(f(x)) = \left\{ \sum_{i=0}^{\infty} a_i x_0^i \cdot i! \right\} / \sum_{i=0}^{\infty} a_i x_0^i. \]  

(C.1)

Applying formulas C.1 and C.2 to various selected functions, it is easy to reach after some straightforward derivations the compact forms of their consolidity indices as shown in Table C.2, providing corresponding functions consolidity by direct formulas substitutions.

The results of the implementation of the consolidity theory to some selected standard functions indicated that their consolidity indices vary from consolidated to unconsolidated forms according to the various setting points selected for these functions. It is remarked at this point that the derivation of the consolidity index of the standard functions in compact form represents a real impetus for pushing the new theory and will help in making its future implementation follows a neat and smooth path. Extensions are possible to other mathematical formulas of algebra, geometry and topology, calculus, dynamics, mechanics, etc.

For complicated symbolic manipulations (and computations) the use of MATLAB Symbolic Toolbox, Mathematica or similar like software packages could be highly effective to foster the consolidity theory through conducting its necessary derivations [24,25]. This will enable the implementation of the suggested consolidity analysis to wider classes of linear, non-linear, multivariable and dynamic problems with different types of complexities.

C.2. Fuzzy analysis of standard fuzzy probability functions

Most important applications in real life are dealing with fuzzy data. These data will lead to generating corresponding probability density functions with fuzzy coefficients. Some examples of these functions are given in Table C.3 with corresponding fuzzy levels of their means and variances \( \ell_{\mu} \) and \( \ell_{\sigma} \), respectively [26,27]. The compact form of the derivation of the fuzzy means and variances will directly lead to simple calculations of their consolidity indices. Such compact form realization of the consolidity indices will represent another impetus for fostering the new theory in handling fuzzy probability and statistics problems.

Similar analysis can be generalized for multivariate probability density, distribution and conditional functions of various continuous or discrete types. Preliminary investigations indicated that the consolidity indices of each mean \( \mu \) and variance \( \sigma \) of the selected probability density functions of Table C.3 are of diverse patterns belonging to different consolidity zones. Such mathematical treatment can also be extended for the implementation of consolidity theory to other fuzzy statistical functions such as correlation and covariance matrices, moments of fuzzy random variables, multivariable fuzzy statistics, and entropy of fuzzy random variables.

The visual representation of the fuzzy probability functions and statistical expressions using fuzzy positive or negative colors [4–6] could provide additional effective simplified tool in the consolidity analysis of some specific disciplines. Examples of these disciplines are humanities and social sciences with limited scope of implementations of advanced mathematics and statistics.

The developed functions of both Tables C.2 and C.3 can be programmed as built-in function in special computational Toolbox in MATLAB or to be created as special functions inside other software languages [24]. The building of such library will markedly strengthen the capability of consolidity theory to effectively handle fuzzy problems expressed by more complicated mathematical and statistical expressions.

In all respects, the success of using the suggested consolidity theory will encourage transferring the concept to other fields of mathematics and statistics, and building necessary software toolboxes for their implementation. This will provide the necessary mathematical infrastructure for the wide implementation of the consolidity theory. The transfer can start systematically piece by piece by the different researchers and analysts who are anxious to experiment the consolidity behavior of their applications. This will help in accelerating following steps towards complete conversion of needed ordinary mathematics and statistics to their corresponding fuzzy form suitable for direct use with consolidity theory.

References


Consolidity: Moving opposite to built-as-usual systems practices


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