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Z-number-based alloy selection problem

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Abstract

Alloy selection is a very important problem for various fields. Despite that a series of various studies devoted to this problem exists, reliability of experimental and judgmental information is not taken into account in selection of an optimal alloy. However, reliability of information on alloys characteristics significantly affects the results of choosing an alloy, which satisfies a predefined objective in the best way. In this study, for the first time we consider application of the Z-number theory for optimal alloy selection. The suggested approach is based on an aggregation of Z-number-valued information, Z-number ranking procedures and Z-information processing. The suggested approach is applied for selection of an optimal alloy from the set of alloys created by the authors. This application shows validity, good interpretability and universality of the proposed approach.

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Keywords: material selection; multicriteria decision making; Z-number; partial reliability; ranking of Z-numbers

1. Introduction

Alloy selection is an important problem attracting theoretical and practical interest. One of the important applications concern aeronautical field. Materials for aeronautical application should work in various environments such as humidity, non-stable temperature, high-pressure etc. In these conditions the materials are subjected to some corrosive and mechanical impacts for instance cyclical forces, creep, tension, torsion, compression and bending.

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For the reasons mentioned above, materials for aeronautical application must be considered for structural application as: strength level, plastic deformation degree, crack grown resistance etc[1].

Nowadays, a lot of materials and alloys are designed. Before designing these alloys, the factors which mentioned above were taken into account. It should be mentioned that cost effectiveness is one of the important factors.

In most alloys some properties are good and in compliance with the requirements, but some of them are not acceptable. Generally, for material selection methods it is necessary to have unique synergy of theoretical knowledge and practical experiences data. Scientists used and developed some selection methods due to all of these.

One of the most used methods is a data system in material selection. For optimal material selection by this method, material designer needs two kinds of information:

a) Screening and ranking information

b) Supporting information[2].

Screening and ranking information is usually based on shifting through the database due to the technical and economical requirements of design. Supporting information is some kind of specialist knowledge which could include knowledge about microstructure, performance in specific environment or other phenomena [3].

Another way for selection materials is using expert systems method. In expert system method artificial intelligence application is applied for reproducing the knowledge of human experts. Expert system relies on inference rules to reason and come to conclusion[4].

In the weighted properties method, material selection is based on comparing properties of materials which is required by their importance. A weighted-property value is obtained by multiplying the value of the property by the corresponding list of importance factors[5].

As explained at[5] the digital logic method is used for determination of weighting factors for material requirements and properties.

Ashby method was invented and developed by Michael Ashby in the 1990's at Cambridge University[2,6,7]. From Refs. 6,7 it is known that the Ashby method is based on using exchange, performance metrics and selection charts.

Alloy selection problems are naturally characterized by imperfect information. To account for imprecision and vagueness of information in real-world problems, fuzzy set theory was suggested[2,8]. Zadeh suggested a concept of a Z-number[10] as a more adequate formal construct for real-world information. A Z-number is an ordered pair $Z=(A,B)$ of fuzzy numbers used to describe a value of a variable X, where A is an imprecise constraint on values of X and B is an imprecise estimation of reliability of A and is considered as a value of probability measure of A.

Unfortunately, in the existing works on alloy selection problems, reliability of available experimental and judgmental information is not taken into account. In this study, we propose an approach to alloy selection in Z-valued environment.

The rest of the paper is structured as follows. In Section 2, we present some prerequisite material which is used in the study including definitions of a discrete Z-number, weighted arithmetic mean of discrete Z-numbers, fuzzy optimality principle, pessimism-optimism principle for Z-numbers, ranking of Z-numbers etc. In Section 3 we give statement of alloy selection problem and the solution method. In Section 4 we illustrate an application of the suggested approach to real-world alloy selection problem. Section 5 offers conclusions.

2. Preliminaries

Definition 1. Discrete Z-number[11,12]. A discrete Z-number is an ordered pair $Z=(A,B)$ of discrete fuzzy numbers A and B. A plays a role of a fuzzy constraint on values that a random variable X may take. B is a discrete fuzzy number with a membership function $\mu_B:\{b_1,...,b_n\}\to[0,1]$, $\{b_1,...,b_n\}\subset[0,1]$, playing a role of a fuzzy constraint on the probability measure of A, $P(A)$.

Definition 2. Z-number valued weighted arithmetic mean of discrete Z-numbers[12]. Let a Z-valued vector $\boldsymbol{Z}=(Z_1,Z_2,...,Z_n)$ be given. The Z-number valued weighted arithmetic mean (ZWAM) operator $M()$ assigns to any vector Z a unique Z-number $Z_M=M(Z_1,Z_2,...,Z_n)=(A_M,B_M)$:

$$M(Z_1, Z_2, ..., Z_n) = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{1}$$

Definition 3. Fuzzy Pareto Optimality principle based comparison of Z-numbers[11,12,13].Fuzzy Pareto Optimality (FPO)[13] principle allows to determine degrees of Pareto Optimality of multiattribute alternatives. We apply this principle to compare Z-numbers as multiattribute alternatives – one attribute measures value of a variable, the other one measures the associated reliability. According to this approach, by directly comparing Z-numbers $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ one arrives at total degrees of optimality of Z-numbers: $do(Z_1)$ and $do(Z_2)$. These degrees are determined on the basis of a number of components (the minimum is 0, the maximum is 2) with respect to which one Z-numbers dominates another one. Z_1 is considered higher than Z_2 if $do(Z_1) > do(Z_2)$.

Definition 4. Pessimism-Optimism degree based comparison of Z-numbers[1,12]**.** In order to improve FPO-based comparison of Z-numbers, we suggest taking into account degree of pessimism $\beta \in [0,1]$ as a mental factor which influences a choice of a preferred Z-number. The degree of pessimism is submitted by a human observer who does not completely rely on the results obtained by the above mentioned FPO approach. In this viewpoint, given $do(Z_j) \le do(Z_i)$, we define for two Z-numbers Z_1 and Z_2:

$$r(Z_i, Z_j) = \beta do(Z_j) + (1-\beta) do(Z_i) . \tag{2}$$

Then

$$\left.\begin{array}{l} Z_i > Z_j \text{ iff } r(Z_i, Z_j) > \frac{1}{2}(do(Z_i) + do(Z_j)) \\ Z_i < Z_j \text{ iff } r(Z_i, Z_j) < \frac{1}{2}(do(Z_i) + do(Z_j)) \\ \text{and} \\ Z_i = Z_j \text{ otherwise} \end{array}\right\} \tag{3}$$

3. Statement of problem and solution method

Assume that a set of *m* alloys (alternatives) are given, $F = \{f_1, f_2, ..., f_n\}$. Every alternative $f_i, i = 1,...,n$ is characterized by *n* criteria, for example, mechanical properties, electrical properties etc. The problem is to choose the best alloy, which simultaneously fits all the criteria $E_j, j = 1,...,m$ more optimally. This problem is a multicriteria decision making problem. We consider this problem under Z-environment because experimental and expert-driven information usually is characterized with partial reliability. At the same time, weights for all the criteria $E_j, j = 1,...,m$, are usually obtained from experts or a decision maker and are vague with partial truth. All the criteria E_j, and the criteria importance weights $V_j, j = 1,...,m$ are Z-numbers. Decision processes for choosing the best alternative from a set of alternatives (alloys) is based on decision matrix $D_{n\times m}$ or payoff table given below:

$$D_{n\times m} = \left\| \begin{array}{lcccc} & E_1 & E_2 & ... & E_m \\ f_1 & (A_{11}, B_{11}) & (A_{12}, B_{12}) & ... & (A_{1m}, B_{1m}) \\ f_2 & (A_{21}, B_{21}) & (A_{22}, B_{22}) & ... & (A_{2m}, B_{2m}) \\ ... & ... & ... & ... & ... \\ f_n & (A_{n1}, B_{n1}) & (A_{n2}, B_{n2}) & ... & (A_{nm}, B_{nm}) \end{array} \right\| \tag{4}$$

where a Z-number valued criteria evaluation $\left(A_{ij}, B_{ij}\right), i=1,...,n,\ j=1,...,m$ are Z-number valued criteria evaluations. The considered problem of multiattribute choice is to determine the best alloy:

$$\text{Find}\quad f^{*}\in F \text{ such that } f^{*}\succ f_i,\ \forall f_i\in F, \tag{5}$$

where $\succ$ is a preference relation.

For solving the considered problem, we suggest an approach based on computation with Z-numbers and human-oriented ranking of Z-numbers. The algorithm for applying this approach is as follows.

Step 1. Scaling of Z-number-valued criteria evaluation information and creation of scaled Z-number-valued decision matrix.

Step 2. Computation of aggregated Z-number-valued criteria evaluation for each alloy by using Z-number-valued weighted arithmetic mean according to Definition 2.

Step 3. Determination of degrees of optimality of Z-numbers computed at the previous step by FPO-principle according to Definition 3

Step 4. Final ranking of Z-numbers computed at step 2 by using the degrees of optimality and pessimism-optimism degrees according to Definition 4.

4. Numerical example

A set of alloys (alternatives) includes, but not limited by, three alloys created by the authors: Ti12Mo2Sn alloy (f_1), Ti12Mo4Sn alloy (f_2), Ti12Mo6Sn alloy (f_3). The optimality of the alloys are considered in terms of the following characteristics: Strength Level, Plastic Deformation Degree, Tensile strength. Due to partial reliability and imprecision of experimental and judgmental information on the characteristics of the considered alloys, Z-numbers-based formalization is used (Table 1).

Table 1. Z-number valued information on characteristics of alloys

	Strength Level, MPa	Plastic Deformation Degree,%	Tensile Strength, MPa
Ti12Mo2Sn	((490,510,530), (0.94,0.95,0.96))	((30,35,40), (0.94,0.95,0.96))	((850,910,970), (0.94,0.95,0.96))
Ti12Mo4Sn	((550,595,640), (0.9,0.935,0.97))	((25,27.5,30), (0.9,0.935,0.97))	((815,865,915), (0.9,0.935,0.97))
Ti12Mo6Sn	((705,718,730), (0.91,0.935,0.96))	((16,20,25), (0.91,0.935,0.96))	((896,976,1056), (0.91,0.935,0.96))

The considered characteristics play the role of choice criteria for selection an optimal alloy. Experts' opinion based information on criteria importance weights is also represented by Z-numbers $V_i, i=1,...,3$:

$$V_1 = ((0.4, 0.5, 0.6), (0.95, 0.97, 0.98)),$$
$$V_2 = ((0.25, 0.3, 0.35), (0.95, 0.97, 0.98)),$$
$$V_3 = ((0.05, 0.2, 0.35), (0.95, 0.97, 0.98)).$$

According to the methodology given in Section 3, we create the scaled decision matrix given information in Table 1 and codebooks in Tables 2,3,4:

$$D_{3\times 3} = \left\| \begin{matrix} & E_1 & E_2 & E_3 \\ f_1 & (very\ low, very\ sure) & (high, very\ sure) & (low\ \ average, very\ sure) \\ f_2 & (below\ \ average, very\ sure) & (average, very\ sure) & (low, very\ sure) \\ f_3 & (very\ \ high, very\ sure) & (low, very\ sure) & (above\ \ average, very\ sure) \end{matrix} \right\| \tag{6}$$

Table 2. The codebook for A parts of Z-number-valued criteria evaluations

Linguistic term	Fuzzy number
Very low	(0.01,0.1,0.15)
Low	(0.15,0.2,0.25)
Low average	(0.2,0.3,0.4)
Below average	(0.3,0.4,0.5)
Average	(0.4,0.5,0.6)
Above average	(0.5,0.6,0.7)
High average	(0.6,0.7,0.8)
High	(0.7,0.8,0.9)
Very high	(0.8,0.9,1)

Table 3. The codebook for A parts of criteria importance weights

Linguistic term	Fuzzy number
Very Low	(0.01,0.1,0.2)
Low	(0.1,0.2,0.2)
Average	(0.2,0.3,0.4)
High	(0.4,0.5,0.6)
Very High	(0.6,0.7,0.8)

Table 4. The codebook for B parts of criteria evaluations and importance weights

Linguistic term	Fuzzy number
Almost Sure	(0.6,0.7,0.8)
Sure	(0.7,0.8,0.9)
Very sure	(0.8,0.9,1)

For example, $D_{n\times m}$ shows that if we choose the 1st alternative then the value of Strength Level will be (very low, very sure), Plastic Deformation Degree will be (high, very sure), Tensile strength will be (low average, very sure).

Now, according to Step 2 (see Section 3) we compute ZWAM by using Definition 2:

$$M(Z_{11},Z_{12},Z_{13}) = V_1Z_{11} + V_2Z_{12} + V_3Z_{13} = (A_1,B_1)$$

$$M(Z_{21},Z_{22},Z_{23}) = V_1Z_{21} + V_2Z_{22} + V_3Z_{23} = (A_2,B_2)\ ,$$

$$M(Z_{31},Z_{32},Z_{33}) = V_1Z_{31} + V_2Z_{32} + V_3Z_{33} = (A_3,B_3)\ .$$

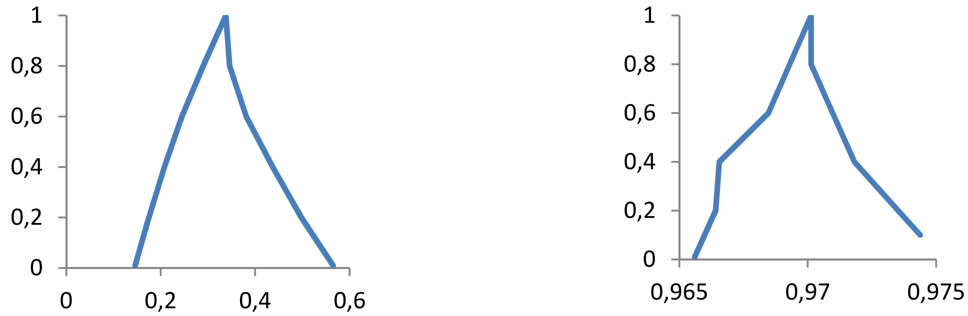
The Z-number-valued criteria evaluations $M(Z_{i1},Z_{i2},Z_{i3}) = (A_i,B_i),\ i=1,...,3$ for alloys are given below:

$$A_1 = 0/0.15 + 0.4/0.2 + 1/0.34 + 0.4/0.437 + 0/0.57,\ \ B_1 = 0/0.96 + 0.5/0.968 + 1/0.97 + 0.5/0.972 + 0/0.974;$$

$$A_2 = 0/19 + 0.5/0.26 + 1/0.4 + 0.5/0.52 + 0/0.65,\ \ B_2 = 0/0.85 + 0.5/0.875 + 1/0.9 + 0.5/0.925 + 0/0.95.$$

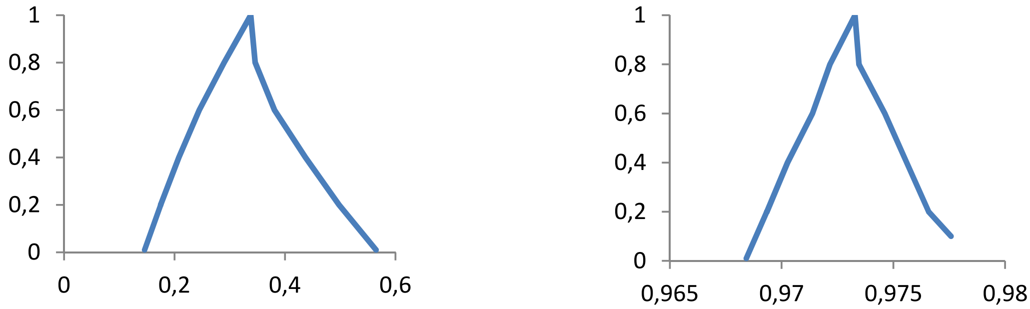
$$A_3 = 0/0.36 + 0.4/0.45 + 1/0.62 + 0.4/0.74 + 0/0.895,\ \ B_3 = 0/0.85 + 0.5/0.875 + 1/0.9 + 0.5/0.925 + 0/0.95.$$

Graphical representation of the obtained Z-numbers is given in Figs. 1-3.



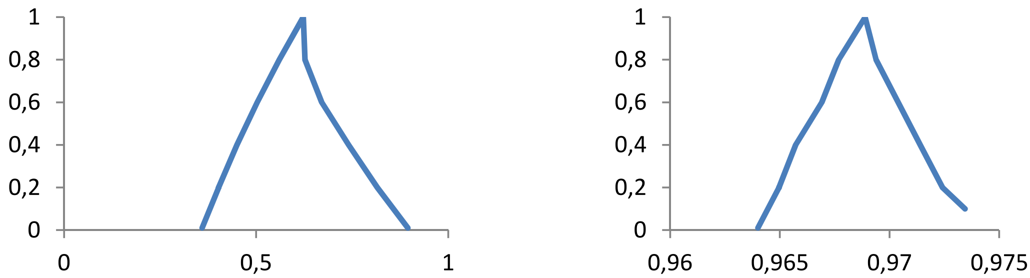
(a) A_1 (b) B_1

Fig. 1. Z-number valued aggregated criteria evaluation for f_1



(a) A_2 (b) B_2

Fig. 2. Z-number valued aggregated criteria evaluation for f_2



(a) A_3 (b) B_3

Fig. 3. Z-number valued aggregated criteria evaluation for f_3

Now, according to Step 3 (Section 3) we need to compare the obtained Z-numbers $Z_i = (A_i, B_i),\ i = 1,...,3$ by using FPO-based approach. For this purpose, first we calculated values of the functions n_b, n_e, n_w which measure the graded number of components with respect to which one Z-number dominates another one:

$$n_b(Z_1,Z_2)=0,\ n_b(Z_2,Z_1)=0.1,\ n_e(Z_1,Z_2)=1,\ n_e(Z_2,Z_1)=1.9, n_w(Z_1,Z_2)=0,\ n_w(Z_2,Z_1)=0,$$
$$n_b(Z_1,Z_3)=0,\ n_b(Z_3,Z_1)=0.54,\ n_e(Z_1,Z_3)=1,\ n_e(Z_3,Z_1)=1.5,\ n_w(Z_1,Z_3)=1,\ n_w(Z_3,Z_1)=0.$$
$$n_b(Z_2,Z_3)=0.1,\ n_b(Z_3,Z_2)=0.26,\ n_e(Z_2,Z_3)=0.9,\ n_e(Z_3,Z_2)=0.74,\ n_w(Z_2,Z_3)=1,\ n_w(Z_3,Z_2)=0.$$

Next, a function d is calculated which measures degree of dominance of one Z-number over another one:

$$d(Z_1,Z_2)=0, d(Z_1,Z_3)=0,$$
$$d(Z_2,Z_1)=1, d(Z_2,Z_3)=0,$$
$$d(Z_3,Z_2)=1;\ d(Z_3,Z_1)=1.$$

Next, the optimality degrees for the Z-numbers are obtained $do(Z_1)=0$, $do(Z_2)=0$, $do(Z_3)=1$.

Finally, according to the step 4, we adjust FPO-based ranking of Z-numbers by using pessimism-optimism degrees. We will consider only Z-numbers $Z_2=(A_2,B_2)$ and $Z_3=(A_3,B_3)$ (Z_1 is the worst as $d(Z_1,Z_2)=0, d(Z_1,Z_3)=0$). Let us use degree of pessimism $\beta=0.4$. For this case we will have: $r(Z_1,Z_2)=0.6>\frac{1}{2}(0+1)=0.5$. Therefore, $Z_3>Z_2$.

Thus, the best alternative as the most optimal alloy, which satisfies all the criteria, is Alloy 3.

5. Conclusion

In the existing works on material selection problems partial reliability of relevant information is not taken into account. In this paper for the first time a material selection problem in Z-environment is considered. A new approach for solving this problem, which is based on aggregation of Z-number-valued information and ranking of Z-numbers, is proposed. Application of the proposed approach to selection of an optimal titanium alloy on the basis of three criteria illustrates validity and applicability of the approach.

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