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Manning’s roughness through the entropy parameter for steady open channel flows in low submergence

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Abstract

Entropy model application to river presents relevant aspects for theoretical and practical issues useful for cross section velocity distribution. The ratio between mean and maximum velocities is dependent on local branch morphology and remains quite uniform among similar location sections. Recent studies propose the formulation of Manning’s roughness, $n$, based on such ratio and the ratio between the positions where the velocity is zero and maximum, $y_0/y_{\text{max}}$, but falling for low depth. Basing on steady flows experiences, the paper analyses the dependence of $n$ on entropy parameter, $M$, for low depths, proposing an equation for $y_0/y_{\text{max}}$ knowing slope and submergence.

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Keywords: Entropy model; Manning’s roughness; relative submergence; steady flow; water discharge.

1. Introduction

River flow forecasting represents a crucial step for improving the management policy addressed to the right use of water resources as well as for conjugating prevention and defence actions against the environmental degradation. Moreover, the knowledge of velocity distribution in a river cross section is fundamental in hydraulic modelling of river, sediment and pollutant transport, channel design, river training works and hydraulic structure as well as in the implementation of rating curve. Thus, velocity measurements must be carried on at all flow stages, even in flood events, in a short time and reducing risks related to operative difficulties and dangerous location of personnel.

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involved and their equipment. Therefore, a velocity distribution law based on few sampling points is suggested and required. A further help is given applying mathematical models, derived from the application of maximization theories of the informational entropy to the collected data in order to evaluate the flow field and compute the water discharge (Chiu (1989), Chiu and Said (1995), Chiu and Hsu (2006)). The entropy velocity distribution, in fact, requires the assessment of one parameter, \( M \), which can be obtained through the knowledge of the mean-maximum flow velocities ratio. Moreover, the extension of such a law to natural flows allows sufficient reliability also in condition of geometric irregularity and for ordinary flow regime (Burnelli et al. (2008), Chiu et al. (2005), Greco (1998), Chiu and Tung (2002)). The application of the entropic profile to river flows deals with interesting results even for practical purposes, in order to define the velocity distribution in the cross section and provide an expeditive method to compute the water discharge reducing surveying and computation time (Greco and Mirauda (2004), Mirauda et al. (2011a,b)) as well as in two-dimensional velocity distributions for open channel flow modelling (Marini et al. (2011)). Further, the ratio between mean and maximum velocities, \( \Phi (M) \), seems to be strongly dependent on the local stream morphology but quite uniform among sections with similar location, i.e. along bends or along straight branches as discussed by Xia (1997) and Moramarco et al. (2004, 2008). This suggests to investigate the dependence of the entropy parameter on the hydraulic and geometric characteristics of the river cross section (Moramarco and Singh (2010)). Such studies, derived from the entropy velocity theory, proposed the formulation of the Manning’s roughness, \( n \), based on \( \Phi (M) \) and the positions in which the velocity is hypothetically zero, \( y_0 \), and maximum, \( y_{\text{max}} \), respectively. For low depth, this dependence needs to be rectified by the calibration of the ratio \( y_0/y_{\text{max}} \), inferring the possibility of an existing influence of some other entities on \( M \). A recent study by Mirauda et al. (2011c), carried out in laboratory on steady flows in low submergence, outlined the dependence of \( M \) on the relative submergence \( D/d \) (\( D = \) water depth; \( d = \) roughness dimension) and the local slope \( i \).

The objective of the present work, therefore, is to investigate the direct dependence of Manning’s roughness on the entropy parameter in the case of low depth and submergence. Thus, using the available data obtained through laboratory campaigns, an explicit equation to assess the ratio \( y_0/y_{\text{max}} \), knowing the local slope and the relative submergence, is proposed, enforcing the observed dependence of \( M \) on \( D/d \) and \( i \).

2. Experimental data

The experimental tests were carried out in the Hydraulics Laboratory of Basilicata University, on a free surface flume of 9 m length and with a cross section of 0.5 x 0.5 m² (Fig. 1a), whose slope can vary from 0 % up to 1 %. At 4 m from the entrance section a set of wood spheres of 0.035 m in diameter (\( d \)) was placed, for a length \( L = 3 \) m, in order to simulate the behavior of a natural channel with homogenous roughness (Fig. 1b,c). These elements were located in order to obtain a roughness concentration, \( \lambda \), expressed as the ratio between the total projected area of the spheres (\( A \)) and the reference area (\( A_0 \)), equal to 0.15, corresponding to the maximum flow resistance according to Rouse (1965). The distance of 4 m was chosen to dampen large-scale disturbances and allow a quasi-constant water depth. In the end section of the flume, a grid was installed to regulate the water depth for each assigned discharge or rather to obtain a small longitudinal variation of the flow depth. The experiments were performed in steady flow conditions for different values of discharge (0.007 - 0.076 m³/s) and slope (0.05 - 1 %). Relying on the forecasted flow depth (≈ 0.25 m), the cross section was located in the middle of the rough reach in order to observe a fully developed flow, avoiding edge effects. The velocity was acquired through a micro current-meter with a measuring head diameter of 0.01 m. The flow depth was obtained as the average between the two values acquired through a couple of hydrometers placed at both the beginning and the end of the measurement reach. The water discharge was measured by a concentric orifice plate installed in the feeding pipe and compared to the value calculated according to the velocity-area method (ISO 748/1997), with a maximum error of around 1-2 %. In particular, the velocity-area method requires to divide the cross section area into several verticals (Fig. 1b) and a subdivision of each vertical into discrete points, in order to assess the mean velocity of the flow along each vertical. The number of measurement points on each vertical was chosen in order to have a good reconstruction of the flow field. In fact, in the adopted approach, the difference in velocity between two consecutive points was less than 20 %, compared
to the higher measured velocity value, and the points close to the channel bottom and the water surface was fixed according to the size of the micro-current meter.

Table 1 reports the ranges of flow characteristics of the laboratory tests in terms of flume slope $i\%$, number of tests $N$, water discharge $Q$, relative submergence $D/d$, mean cross section velocity $\bar{u}$, maximum cross section velocity $u_{\text{max}}$ and mean-maximum cross section velocities ratio, $\Phi(M)$.

<table>
<thead>
<tr>
<th>$i%$</th>
<th>$N$</th>
<th>Min $Q$ [m$^3$/s]</th>
<th>Max $Q$ [m$^3$/s]</th>
<th>Min $D/d$</th>
<th>Max $D/d$</th>
<th>Min $\bar{u}$ [m/s]</th>
<th>Max $\bar{u}$ [m/s]</th>
<th>Min $u_{\text{max}}$ [m/s]</th>
<th>Max $u_{\text{max}}$ [m/s]</th>
<th>Min $\Phi(M)$</th>
<th>Max $\Phi(M)$</th>
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</thead>
<tbody>
<tr>
<td>0.050</td>
<td>9</td>
<td>0.007</td>
<td>0.076</td>
<td>2.17</td>
<td>6.43</td>
<td>0.22</td>
<td>0.70</td>
<td>0.34</td>
<td>0.94</td>
<td>0.65</td>
<td>0.75</td>
</tr>
<tr>
<td>0.100</td>
<td>9</td>
<td>0.007</td>
<td>0.075</td>
<td>2.14</td>
<td>6.40</td>
<td>0.21</td>
<td>0.70</td>
<td>0.34</td>
<td>0.94</td>
<td>0.62</td>
<td>0.74</td>
</tr>
<tr>
<td>0.250</td>
<td>9</td>
<td>0.007</td>
<td>0.076</td>
<td>2.06</td>
<td>6.31</td>
<td>0.23</td>
<td>0.72</td>
<td>0.36</td>
<td>0.98</td>
<td>0.62</td>
<td>0.73</td>
</tr>
<tr>
<td>0.375</td>
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<td>0.007</td>
<td>0.075</td>
<td>2.03</td>
<td>6.23</td>
<td>0.23</td>
<td>0.72</td>
<td>0.36</td>
<td>0.98</td>
<td>0.63</td>
<td>0.73</td>
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<tr>
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<td>0.076</td>
<td>2.00</td>
<td>6.17</td>
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<td>0.41</td>
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<td>0.076</td>
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<td>6.09</td>
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<td>0.75</td>
<td>0.39</td>
<td>1.02</td>
<td>0.60</td>
<td>0.73</td>
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<tr>
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<td>0.075</td>
<td>1.91</td>
<td>6.00</td>
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<td>0.75</td>
<td>0.41</td>
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<td>0.074</td>
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<td>1.89</td>
<td>5.83</td>
<td>0.25</td>
<td>0.78</td>
<td>0.41</td>
<td>1.08</td>
<td>0.61</td>
<td>0.72</td>
</tr>
</tbody>
</table>

3. Relating Manning’s roughness $n$ to the entropy parameter $M$

The mean velocity, $\bar{u}$, and the maximum one, $u_{\text{max}}$, sampled in a cross section of an open channel flow can be expressed as (Chiu and Said (1995)):

$$\bar{u} = \Phi(M)u_{\text{max}}$$  \hspace{1cm} (1)

where:

$$\Phi(M) = \left(\frac{e^M}{e^M - 1} - \frac{1}{M}\right)$$  \hspace{1cm} (2)

in which $M$ represents the entropy parameter.

In Eq. (1), according to the approach of Moramarco and Singh (2010), the mean velocity can be evaluated using the classical Manning’s formula:

$$\bar{u} = \frac{1}{n} R_h^{2/3} S_f^{1/2}$$  \hspace{1cm} (3)

where $n$ is Manning’s roughness, $R_h$ the hydraulic radius and $S_f$ the energy slope.
Instead, to determine the maximum velocity of the cross-section, \( u_{\text{max}} \), along the y axis assumed perpendicular to the bottom, the dip-modified logarithmic law for the velocity distribution in a smooth uniform open channel flow, proposed by Yang et al. (2004), is considered:

\[
    u(y) = u_* \left[ \frac{1}{k} \ln \frac{y}{y_0} + \alpha \ln \left(1 - \frac{y}{D}\right) \right]
\]

(4)

where \( u_* = (gR_hS_f)^{0.5} \) is the shear velocity (\( g \) = gravity acceleration); \( k \) is the von Karman constant equal to 0.41; \( y_0 \) is the distance at which the velocity is hypothetically equal to zero; \( \alpha \) is the dip-correction factor, depending only on the ratio between the relative distance of the maximum velocity location from the river bed, \( y_{\text{max}} \), and the water depth, \( D \), along the y axis, where \( u_{\text{max}} \) is sampled.

The location of the maximum velocity, supporting the dip-phenomenon hypothesis, can be obtained by differentiating Eq. (4) and equating \( du/dy = 0 \), which gives:

\[
    \frac{y_{\text{max}}}{D} = \frac{1}{1 + \alpha}
\]

(5)

Experimental studies by Greco and Mirauda (2002) have shown that, for channels at different shapes of the cross section, the maximum velocity is below the free surface around the 20÷25% of the maximum depth. This result is also confirmed from the \( y_{\text{max}} \) values collected in experimental tests of the present work and shown in Fig. 2, in function of the water depth, \( D \).

![Fig. 2. Relation between the location of maximum velocity, \( y_{\text{max}} \), and the water depth, \( D \), for all laboratory tests.](image)

Thus, considering \( y_{\text{max}} \) equal to \( \frac{3}{4} \) of the water depth, \( D \), according to Eq. (5), \( \alpha \) becomes equal to \( 1/3 \). Replacing the value of \( \alpha \) in Eq. (4), and after a little algebraic manipulation, the maximum flow velocity can be expressed as:

\[
    u_{\text{max}} = \frac{\sqrt{gR_hS_f}}{k} \ln \left( \frac{y_{\text{max}}}{y_0} - 0.4621 \right)
\]

(6)

Substituting Eq. (3) and Eq. (6) in Eq. (1), it is possible to derive the following relationship:
\[ \Phi(M) = \left( \frac{e^M}{e^M - 1} - \frac{1}{M} \right) = \frac{1}{n} R_h^{1/6} / \sqrt{g} \]

\[ = \frac{1}{k} \left[ \ln \frac{y_{\text{max}}}{y_0} - 0.4621 \right] \]

which allows to relate \( \Phi(M) \) to the hydraulic and geometric characteristics of the flow. Finally, from Eq. (7) it is possible to value the Manning’s roughness:

\[ n = \frac{R_h^{1/6} / \sqrt{g}}{\Phi(M) \frac{1}{k} \left[ \ln \frac{y_{\text{max}}}{y_0} - 0.4621 \right]} \]  

4. Experimental data analysis

Equation 8 infers to compute Manning’s roughness once calculated the values of \( \Phi(M) \) as well as calibrate the value of \( y_0/y_{\text{max}} \). In fact, knowing \( \Phi(M) \) for each test and imposing the value of \( n \) obtained through Eq. (3) in Eq. (8), should be possible to derive \( (y_0/y_{\text{max}})_{\text{computed}} \). Figure 3 reports the values of \( y_0/y_{\text{max}} \) computed by Eq. (8) versus the observed relative submergence, \( D/d \), for each fixed slope, \( i \). As it is possible to observe from the high values of the correlation coefficients \( (R^2) \) in Fig. 3, the set of data sorted by slope can be well fitted by a power law, whose general equation can be formulated as follows:

\[ \left( \frac{y_0}{y_{\text{max}}} \right)_{\text{obs}} = a_{\%} \cdot e^{b_{\%} (D/d)} \]

with \( a_{\%} \) and \( b_{\%} \) depending on the slope as plotted in Fig. 4, where the coefficient \( a_{\%} \) has a linear dependence; while the coefficient \( b_{\%} \) presents a logarithmic distribution.

![Fig. 3. \( y_0/y_{\text{max}} \) versus the observed relative submergence, \( D/d \), for each investigated slope, \( i \% \).](image-url)
Thus, the ratio $y_0/y_{\text{max}}$ can be calibrated by the following equation:

$$
\left( \frac{y_0}{y_{\text{max}}} \right)_{\text{calibrated}} = 0.094i\% \cdot e^{(0.445\ln(i\%)-0.278)\left[D/d\right]}
$$

(10)

with the obvious meaning of symbols. The validity of Eq. (10) is verified substituting the value $y_0/y_{\text{max}}$ obtained by this equation in Eq. (8) and deriving $n_{\text{computed}}$, and then comparing it to the observed Manning’s roughness, $n_{\text{Manning}}$. The results are plotted in Fig. 5 and show a good agreement among the two quantities. Moreover, Fig. 5 infers a slight overestimation of the Manning’s roughness for those tests in which the slope is close to 1 %, which can be due to the accuracy of logarithmic curve interpolating $b_{i\%}$ coefficients (Fig. 4).
Therefore, for low depth, the use of Eq. (10) together to the verified hypothesis of $y_{\text{max}}$ at $\frac{2}{3}D$ from the channel bottom, give a better and more immediate assessment of the Manning’s roughness in respect to one calculated by Moramarco and Singh (2010) with a constant value of $y_0$ and with the observed values of $y_{\text{max}}$, which are difficult to evaluate in field measurements. Further, it is important to underline, for low depth and relative submergence, such result enforces the dependence of the $M$ parameter on the geometric and hydraulic characteristics of the flow, giving a valid enlightenment through the analyses performed on the experimental data here presented.

5. Conclusion

The application of the entropic velocity profile to river, as expeditive model for the evaluation of the water discharge, reduces times and difficulties of the fluvial control and monitoring activities.

Besides, the formulation of the Manning’s roughness, $n$, based on entropy parameter, $M$, and on the ratio between the positions in which the velocity is zero and maximum, $y_0/y_{\text{max}}$, could be useful to overcome the uncertainties in the evaluation of the resistance parameter, especially in presence of large relative roughness.

Basing on experimental data collected in a flume on a steady flow in low submergence, the research proposes an explicit formulation of the ratio $y_0/y_{\text{max}}$ taking into account the relative submergence, $D/d$, and the channel slope, $i$.

The analysis shows how the dependence of $y_0/y_{\text{max}}$ on $D/d$ and $i$ improves the assessment of Manning’s roughness, $n$, through the formulation proposed by Moramarco and Singh (2010) and modified considering $y_{\text{max}}$ equal to $\frac{2}{3}$ of the water depth, $D$. Such result further enforces the relationship between the entropy parameter and the hydraulic and geometric characteristics of the flow.

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References


